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SUPERSYMMETRIC QUANTUM THEORY OF SUPERCONDUCTIVITY AT HIGH TC

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The study of supersymmetric quantum theory of superconductivity has been undertaken in a broader sense. The main features of supersymmetric quantum mechanics have been derived in the straight forward manner and the consequences of supersymmetric breaking have been analyzed in terms of possibility of occurrence of superconductivity, dual superconductivity and color superconductivity.

INTRODUCTION

The discovery of high temperature superconductivity in several Cu-oxide systems have caused great excitement and led to many proposals and models to explain this phenomenon. There is still no consensus among the experts about the model representing correct starting point and as such the microscopic theory of superconductivity still remains controversial. On the other hand, the phenomenological theory of superconductivity at microscopic level has been carried out[1-5] in the framework of local gauge invariance, Quantum Chromo Dynamics (QCD)[6-9] and dynamical as well as spontaneous breaking[10-16] of symmetry (DSB and SSB). Through these efforts it became clear that there exists a sort of parallelism between the Condensed Matter Physics (CMP) and High Energy Physics (HEP). Gauge theories in four dimensions can manifest themselves in three phases, Coulomb, Higgs and Confinement Phases. As the parameters of the theory are varied, a phase transition between them can take place. A gauge in Coulombian phase has a massless photon and hence it is subject to standard electric-magnetic duality.

Quantum Chromo Dynamics (QCD) is the most favored color gauge theory of strong interactions whereas superconductivity is a remarkable manifestation of quantum mechanics on a truly macroscopic scale[17]. In the process of current understanding of superconductivity[18,19], Rajput *et al* have conceived the notion of its hopeful analogy with QCD. The essential clues for gauge symmetry breaking emerged from the crucial theoretical framework of superconductivity formulated by Bardeen, Cooper and Schrieffer (BCS)[20] by demonstrating that the ground state of an assembly of mutually attracting fermions is separated by an energy gap from the lower excited level of energy spectrum. Moreover, other salient features of superconductivity *i.e.*, the Meissner effect and the flux quantization, provided the vivid models for actual confinement mechanism in QCD. In this connection.Nambu and others[3,4,21] suggested that the color confinement could occur in QCD in a way similar to magnetic flux confinement in superconductors.

In general the thermal fluctuations restore various spontaneously broken inter-symmetries but the fermions and bosons respond differently to thermal fluctuations and hence it is natural to speculate that any supersymmetric system immersed in a thermal bath should loose supersymmetry[22,23]. This problem has drawn many controversies [24-26] in connection with the resolution of mingled quest of supersymmetry breaking at finite temperature. The desire to restore this beautiful symmetry at a finite temperature and then to break it spontaneously to explore the superconductivity at high temperature has motivated some authors[27-29] to propose various models of restoration of supersymmetry at finite non-zero temperature. Keeping in view the proposal[30] that the high temperature superconductivity must have its origin in the gauge theoretical formulations, the superconductivity at high temperature may be identified as the consequence of spontaneous breaking of supersymmetry restored at non-zero temperature. This question of restoration of super-symmetry (and hence manifestation of superconductivity) has been examined in principle[27] by constructing the temperature dependent supercharge and Hamiltonian from their expressions at zero temperature. This study demonstrates that the study of supersymmetry at finite temperature provides a convincing argument to understand the behavior of superconductor at high temperature. In the present paper the study of supersymmetric quantum theory of superconductivity has been undertaken in a broader sense. The main features of supersymmetric quantum mechanics have been derived in the straightforward manner and the consequences of supersymmetric breaking have been analyzed in terms of possibility of occurring of superconductivity, dual superconductivity and color superconductivity.

SUPERSYMMETRIC QUANTUM MECHANICS

Supersymmetry is a relativistic symmetry between fermions (*i.e*, the mathematical objects which on quantization are associated with an anti-commutation algebra) and bosons (*i.e.* the objects where quantization is associated with commutator algebra). In its simplest form, supersymmetric algebra is an extension of usual Poincare albegra. It is such a graded Lie algebra[31,32] which involves both commutation and anti-commutation relations, plays a unique role in particle physics and provides a fusion between space-time and internal symmetries overcoming no-go theorm[33] about the possible symmetries of *S*-matrix. This graded algebra or superalgebra, in its simplest form, is generated by fourteen Hermition operators which include ten generators of Poincare group (P_μ and $J_{\mu\nu}$) and four self-conjugate spin –1/2 anti-commuting generators corresponding to Majorana spinor charge Q_α ($\alpha = 1\ldots4$)

known as super-translations. Apart from usual commutation rules for Poincare generator, these generators also satisfy the following relations [31,32].

$$[Q_\alpha, P_\mu] = 0;$$

$$[Q_\alpha^i, J_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta \, Q_\beta^i$$

$$[Q_\alpha^i, J_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta \, Q_\beta^i$$

$$\{Q_\alpha^i, J_\beta^j\} = 2\,(\gamma_\mu C)_{\alpha\beta}\, \delta^{ij} P^\mu \qquad \text{... (2.1)}$$

where bracket [ ] denotes commutation while { } denotes anti-commutation and

$$\sigma_{\mu\nu} = \frac{1}{2}[\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu] \qquad \text{... (2.2)}$$

and C is charge conjugation matrix. The other symbols have their usual meaning. Of all the graded Lie algebra, only the super-algebra, described by equation (2.1), generates the symmetries of S-matrix consistent with relativistic quantum field theory.

Starting with the pioneer work of Witten[34], it has been recognized that supersymmetry could be applied to quantum mechanics as a limiting case ($N = 1$) of field theory and the subsequent development of supersymmetric quantum mechanics provides us with the realistic model of particle physics which do not suffer with gauge hierarchy problem besides its intrinsic mathematical interest. Rajput *et al*[35-37], have developed supersymmetric quantum mechanics in complex space-time. In order to develop it in a straight forward manner, let us construct super space formulation by extending ordinary time variable t to new super-time involving two time variables θ and $\bar{\theta}$, where the covariant derivatives are constructed as follows:

$$D = \frac{\partial}{\partial_{\bar{\theta}}} - i\theta \frac{\partial}{\partial t}$$

$$\bar{D} = \frac{\partial}{\partial_\theta} - i\bar{\theta} \frac{\partial}{\partial t} \qquad \text{... (2.3)}$$

The corresponding supersymmetric transformations are

$$t \to t' = t - i\,(\bar{\theta} \in - \bar{\in}\, \theta)$$

$$\theta \to \theta' = \theta + \in$$

$$\bar{\theta} \to \bar{\theta} = \bar{\theta} + \bar{\in} \qquad \text{... (2.4)}$$

where $\in$ and $\bar{\in}$ are constant anti-commuting parameters. We can then construct the supersymmetry generators as

$$G = \exp\{i\,(\bar{\in}\, Q^+ + \in Q)\} \qquad \text{... (2.5)}$$

where $$Q = i\frac{\partial}{\partial\theta} - \bar{\theta}\frac{\partial}{\partial t} \qquad ... (2.6)$$

and $$Q^{+} = i\frac{\partial}{\partial\bar{\theta}} + \theta\frac{\partial}{\partial t} \qquad ... (2.7)$$

are non-Hermitian supercharge operators. From equations (2.6) and (2.7), we obviously have

$$\{q, Q^{\dagger}\} = QQ^{\dagger} + Q^{\dagger}Q = 2i\frac{\partial}{\partial t} = 2H, \qquad ... (2.8)$$

where H is super symmetric Hamiltonian. The super position is then constructed as

$$Z\ (t, \theta, \bar{\theta}\} = q(t) + i\bar{\theta}\psi(t) + i\theta\bar{\psi}(t) + \theta\bar{\theta}\ A(t) \qquad ... (2.9)$$

where the usual position variable $q(t)$ and the function $A(t)$ are bosonic variables while $\Psi(t)$ and $\bar{\psi}(t)$ are the fermionic variables. This superposition and the corresponding super-field $\phi^{i}(t, \theta, \bar{\theta})$ with i as anti-symmetric index, along with the covariant derivatives (2.3) acting on them, are sufficient to form supersymmetric Lagrangian. With single real super field ϕ (*i.e.* $N = 1$), we have following Lagrangian density which is invariant under super symmetric transformations (2.4):

$$L = \frac{1}{2}[\bar{D}\phi][D\phi] - W(\phi) \qquad ... (2.10)$$

where the super potential $W(\phi)$ is an arbitrary function of the super field ϕ. Its Taylor's expansion in power of θ and $\bar{\theta}$ may be written in the following term keeping in view equation (2.10);

$$W(\theta) = W(q) + i\theta W'(q)\bar{\psi} + i\bar{\theta}W'(q)\ \psi + \theta\bar{\theta}W' \,|\, q \,|\, A + W''\,(q)\ \bar{\psi}\psi \qquad ... (2.11)$$

where primes denote the derivatives with respect to q.

Using Berezin integration rules[38] for integration of the anti- commuting variables, we write the action of equation (2.10) as

$$S = \int dtL\ ... \qquad ... (2.12)$$

where L is the Lagrangian given by

$$L = \frac{1}{2}\dot{q}^2 + \frac{1}{2}A^2 + \frac{i}{2}(\psi\dot{\bar{\psi}} - \dot{\psi}\bar{\psi}) - AW'(q) = -\frac{1}{2}W''(q)\ (\bar{\psi}\psi - \psi\bar{\psi}) \qquad ... (2.13)$$

Since the coordinate A does not have a kinetic term, it may be eliminated from this equation by using the condition.

$$\frac{\partial L}{\partial A} = AW'(q) = 0 \qquad ... (2.14)$$

and then equation (2.13) reduces to

$$L = \frac{1}{2}\dot{q}^2 + \frac{i}{2}(\psi\dot{\bar{\psi}} - \dot{\psi}\bar{\psi}) - \frac{W'^2}{2} - \frac{1}{2}W''(\bar{\psi}\psi - \psi\bar{\psi}) \qquad ... (2.15)$$

which corresponds to Witten model[39]. The corresponding expression for Hamiltonian is

$$H = \frac{1}{2} p^2 + \frac{1}{2}[W'(q)]^2 + \frac{1}{2} W''(q)[\bar{\psi}, \psi] \quad \text{... (2.16)}$$

which may be decomposed into bosonic part H_B (which does not contain any fermionic degree of freedom) and the fermionic part H_F (which does not contain any bosonic degree of freedom);

$$H = H_B + H_F \quad \text{... (2.17)}$$

where $$H_B = \frac{1}{2} p^2 + \frac{1}{2}[W'(q)]^2$$

and $$H_F = -iW''(q)Y \quad \text{... (2.18)}$$

with $$Y = \frac{i}{2}[\bar{\psi}, \psi] \quad \text{... (2.19)}$$

Using these equations, we obviously have

$$[Y, Q] = -iQ,$$

$$[Y, Q^\dagger] = -iQ^\dagger$$

$$\{Q, Q^\dagger\} = 2H \quad \text{... (2.20)}$$

and $$[Q, Q^\dagger] = [H, Q^\dagger] = 0,$$

$$Q^2 = Q^{\dagger 2} = 0$$

Choosing different types of supersymmetric potentials in equation (2.16) for the Hamiltonian, energy eigen values and eigen functions may be derived for the quantum mechanical states of supersymmetric harmonic oscillator[40], supersymmetric hydrogen atom[41] and first order Dirac equation[42]. Without making any specific choice of this potential in any specific model, we demonstrate here some characteristics of a supersymmetric quantum mechanical system.

Equations (2.17) and (2.20) require

$$Q = [p - iW'(q)]\bar{\psi}$$

and $$Q^\dagger = [p + iQW'(q)]\psi \quad \text{... (2.23)}$$

Let us assume that E_n is an eigen value of H with the corresponding eigen state $| n >$,

$$Q^\dagger = [p + iQW'(q)]\psi$$

$$\frac{1}{2}[QQ^\dagger + Q^\dagger Q] | n > = E_n n >$$

or $$\frac{1}{2}[Q | n >_+ + Q^\dagger | n >_- = E_n | n > \quad \text{... (2.24)}$$

where $$| n >_+ = Q^\dagger | n > \quad \text{... (2.25)}$$

and $$| n >_- = Q | n >$$

Equation (2.24) yields

$$E_n = \frac{1}{2}[_+ < n | n >_+ +_- (n | n >_-] \geq 0 \quad \text{... (2.26)}$$

showing that all eigen values of the operator H are non-negative and hence the energy in supersymmetric theory is always a positive quantity. This eigen value is zero only when

$$Q | n > = Q^\dagger | n > = 0 \quad \text{... (2.27)}$$

which is necessary condition for the supersymmetric ground state and hence the ground state of a supersymmetric system is the true vacuum (*i.e.*, zero particle state). The super symmetry is spontaneously broken when the ground state energy is non-zero.

Any state $| B >$, satisfying the conditions

$$Q | B > 0 \quad \text{... (2.28)}$$

and $$Q^\dagger | B > \neq 0,$$

is bosonic state for which we have

$$H | B > = \frac{1}{2} Q Q^\dagger | B > \quad \text{... (2.29)}$$

The fermionic state $| F >$ satisfies the conditions

$$Q^\dagger | F > = 0$$

and $$Q | F > \neq 0 \quad \text{... (2.30)}$$

which give

$$H | F > = \frac{1}{2} Q Q^\dagger | F > \quad \text{... (2.31)}$$

Using these relations, it may readily be *i.e.*

$$H | F > = E^{1/2} | B > \quad \text{... (2.32)}$$

and $$Q^\dagger | B > = E^{1/2} | F >$$

showing that the supersymmetry pairs the bosonic and fermionic states of all positive energy states of H. On the other hand, the zero energy states are not paired in this way. Each state annihilated by H is also annihilated by Q. These states form trivial one dimensional supersymmetric multiplets. There exists the unpaired state (ground state) if and only if the supersymmetry is an exact symmetry of the system. In other words, the ground states of zero energy preserve super symmetry, while those of positive energy break it spontaneously. Thus supersymmetry is unbroken if and only if the energy of vacuum is exactly zero. In other words in supersymmetric theories, the energy E is equal to or greater than the magnitude of the momentum $\bar{p}$ for any state. Zero energy states must therefore have $\vec{p} = 0.$ In super space of states of zero momentum, the supersymmetric algebra is particularly simple.

In general, there may be an arbitrary number $n_B^{E=0}$ zero-energy bosonic states and arbitrary number $n_F^{E=0}$ zero-energy fermionic states. In the most general allowed form of the spectrum, there are paired states of positive energy and there may be states, not necessarily paired, of zero energy. As we vary the parameters, the states of non-zero energy move around in energy. They move of course, in Bose-Fermi pairs. One of these pairs ($E \neq 0$) may move down to E = 0. In this case $n_B^{E=0}$ and $n_F^{E=0}$ both increase by one. On the other hand, with the variation of parameters, some states of zero-energy may gain non-zero energy. It is possible for a single zero-energy state to acquire a non-zero energy. As soon as it has a non-zero energy, it must have a supersymmetric partner. What can occur is that a pair of states (one boson and one fermion) can move from $E = 0$ to $E \neq 0$. In this case $n_B^{E=0}$ and $n_F^{E=0}$ both decrease by one. In either case the difference

$$n_B^{E=0} - n_F^{E=0} = 0$$

is not changed as one varies the parameters. If $n_B^{E=0} - n_F^{E=0} \equiv 0$, supersymmetry is not broken spontaneously since either $n_B^{E=0}$ or $n_F^{E=0} \neq 0$ or both are non-zero. In any case, there are some states of zero energy and hence supersymmetry is unbroken.

Formally, the quantity $n_B^{E=0} - n_F^{E=0}$ may be regarded as trace of the operator

$$(-1)^f = \exp(2\pi i J_z) \qquad \text{... (2.33)}$$

where J_z is the third component of angular momentum associated with the state concerned. The states of non-zero energy do not contribute to the trace of $(-1)^f$ because for every bosonic state that contributes + 1 to this trace there is a fermion state of non-zero energy that contributes –1 and cancels the bosonic contribution. Therefore $(-1)^f$ can be evaluated among the non-zero states only and it is equal to $n_B^{E=0} - n_F^{E=0}$;

$$tr(-1)^f = n_B^{E=0} - n_F^{E=0} = \Delta \qquad \text{... (2.34)}$$

where Δ is known as Witten index.

Supersymmetry is unbroken if $\Delta \neq 0$. It is also unbroken if $\Delta = 0$ but $n_B^{E=0} = n_F^{E=0} \neq 0$.

If the supersymmetry charge is Hermitian operator written as

$$Q = \begin{pmatrix} 0 & M^* \\ M & 0 \end{pmatrix} \qquad \text{... (2.35)}$$

and states are arranged as

$$\begin{pmatrix} |B> \\ |F> \end{pmatrix},$$

we have $H = Q^2$ and then the zero-energy states are precisely the states annihilated by Q. In this case the Hilbert space H of the theory can be splitted into bosonic and fermionic subspaces H_B and H_F. Then the bosonic states annihilated by Q are states Ψ in H_B that satisfy

$$M\Psi = 0 \quad \text{... (2.36)}$$

and the fermionic states annihilated by Q are states Ψ in H_F that satisfy

$$M^*\Psi = 0 \quad \text{... (2.37)}$$

The quantity Δ is therefore the difference of number of solutions of equation (2.36) and number of solutions of equation (2.37)

Let us make use of two Hermitian supersymmetric charges Q_1 and Q_2 and define

$$Q_{\pm} = \frac{1}{\sqrt{2}}[Q_1 \pm iQ_2] \quad \text{... (2.38)}$$

Then the supersymmetric algebra, in the zero momentum sector of Hilbert space, takes the simple form given by

$$Q_+^2 = Q_-^2 = 0$$

$$Q_+Q_- + Q_-Q_+ = H \quad \text{... (2.39)}$$

where Q_+ annihilates at least half of the states in Hilbert space. *A* state Ψ is either annihilated itself by Q_+, *i.e*

$$Q_+\Psi = 0$$

or its supersymmetric partner $\chi = Q_+\Psi$ is annihilated by Q_+, *i.e.*,

$$Q_+\chi = Q_+^2\psi = 0$$

Zero energy states are precisely the states χ such that

$$Q_+\chi = 0 \text{ but } \chi \neq Q_+\Psi \text{ for any } \Psi.$$

The algebra (2.39) yields

$$[Q_i, H] = 0$$

and

$$[Q_i, Q_j] = \delta_{ij}H \quad \text{... (2.40)}$$

where $i, j = 1, 2$. The simplest such system has $N = 2$ and involves a spin – ½ particle moving on a line. The wave function is therefore a two component Pauli spinor,

$$\psi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \quad \text{... (2.41)}$$

The algebra (2.40) is satisfied for

$$Q_1 = ½\,[\sigma_1\, p + \sigma_2\, W(x)]$$

$$Q_2 = ½\,[\sigma_2\, p - \sigma_1\, W(x)] \quad \text{... (2.42)}$$

and $$H = \frac{1}{2}\left[p^2 + W^2(x) + \hbar\sigma_3 \frac{dW}{dx} \right] \quad \text{... (2.43)}$$

where σ_i are Pauli spinor matrix $p = -i\hbar\, \partial/_{\partial x}$. A super symmetric state must satisfy.

$$Q_i \Psi = 0$$

Because of the general relation

$$Q_1^2 = Q_2^2 = \frac{1}{2}H,$$

it is enough to satisfy the following relations by a super symmetric state Ψ;

$$Q_1 \Psi = 0$$

or $$\sigma_1 p\, \Psi = -\sigma_2 W \Psi \quad \text{... (2.44)}$$

Multiplying it by σ_1 and using the property

$$\sigma_1\sigma_2 = i\sigma_3,$$

we get $$\frac{d\psi}{dx} = \frac{1}{\hbar} W(x)\, \sigma_3 \psi(x) \quad \text{... (2.45)}$$

Solution of this equation is

$$\psi(x) = \left[\exp \int_0^x dy \frac{W(y)}{\hbar} \sigma_3 \right] \psi(0) \quad \text{... (2.46)}$$

which defines a supersymmetric state provided that $\Psi(x)$ is normalizable.

The supersymmetric Largangian (2.15), containing kinetic term (*D*-type), mass term (*F*-type) and interaction term (*F*-type), does not remain invariant when one introduces the gauge transformation. It must be reconstructed in a gauge covariant manner by introducing a gauge field. Usually, gauge invariant interactions are introduced on replacing ordinary derivative by gauge covariant derivative. But the kinetic term of chiral superfield does not have any explicit derivative term visible in the space of super-fields. This fact precludes the method of replacing ordinary derivatives by gauge covariant derivatives for the introduction of local gauge invariance in supersymmetric theory. The simplest supersymmetric generalization of spin –1 gauge boson is that of a vector super- field consisting of the spin – 1 along with spin – ½ component fields.

SUPERCONDUCTIVITY DUE TO SUPER SYMMETRY BREAKING

Supersymmetry relates particles having different spins following different statistics, demanding that there is equivalence between fermions and bosons. Such equivalence is not observed in nature and hence supersymmetry (SUSY) should be broken spontaneously preserving all its nice properties and attractive aspects of the theory and also suggesting the presence of super-partners (with predictable properties) of all the fundamental particles of Nature. Supersymmetric quantum mechanics serves as theoretical laboratory for testing various ideas of supersymmetry breaking mechanism in high energy physics and it has enhanced the hope to get better insight in to the mechanisms of supersymmetry breaking. In unbroken supersymmetry there are no quadratic divergences and the finite induced mass splittings are determined through the mechanism of supersymmetric break down.

A superconductive phase transition is always accounted for as a spontaneous breaking of supersymmetry. It has recently been shown[43] that color superconductivity dynamically takes place at non-SUSY vacuum due to spontaneous breaking of baryon number symmetry as the consequence of $SU(2)$ strong gauge dynamics in the vacuum structure of $N = 2$ supersymmetric QCD based on the gauge group $SU(2)$ in the presence of flavours of hypermultiplet quarks.

In general any boundary condition or any environment which would distinguish between bosons and fermions, would break supersymmetry. In particular, since bosons and fermions respond differently to temperature, a supersymmetric system immersed in a thermal bath would loosesupersymmetry[22,23]. Finite temperature supersymmetry breaking can also be visualized by applying the properties of thermo-field-dynamics (TFD) which is an alternative way[44] of making calculations in quantum mechanics. One of the order parameter for SUSY breaking at finite temperature is the thermal ground state energy. Its non-vanishing value is a sure test of breakdown of SUSY and hence the occurrence of high – T_C superconductivity. One of the important criteria for spontaneous breakdown of SUSY at non-zero temperature is that the thermal average of at least one auxiliary field of supersymmetric theory does not vanish[45]. This criterion has enhanced the hope to restore SUSY at finite temperature. The spontaneous breaking of this restored SUSY at finite temperature would lead to high-T_c superconductivity.

DISCUSSION

It is interesting to note that of all the graded Lie algebra, only the super-algebra, described by equation (2.1), generates the symmetries of S-matrix consistent with relativistic quantum field theory. Equations (2.6) and (2.7) give the supercharge operators and equation (2.8) give the general supersymmetric Hamiltonian. Supersymmetric Lagrangian density is given by equation (2.10) with the supersymmetric [potential given by eqn. (2.11). Under the condition (2.14), this Lagrangian density leads to the form given by eqn. (2.15) which corresponds to Witten model[39]. Equation (2.26) show that all eigen values of the supersymmetric Hamiltonian operator H, given by eqn. (2.16), are non-negative and hence the energy in supersymmetric theory is always a positive quantity. Equation (2.27) is the necessary condition for the supersymmetric ground state and hence the ground state of a supersymmetric system is the true vacuum (i.e. zero particle state) and the super symmetry is spontaneously broken when the ground state energy is non-zero. Any state satisfying condition (2.28) is bosonic state for which we have equation (2.29) while any fermionic state, given by eqn. (2.31) satisfies conditions (2.30). Equations (2.32) demonstrate that the operator

Q transforms states | F > into states | B > of the eigen energy E and the operator $Q^\dagger$ transforms states | B > into the states | F >, showing that the supersymmetry pairs the bosonic and fermionic states of all positive energy states of H. These equations also show that the ground states of zero energy preserve super symmetry, while those of positive energy break it spontaneously. Thus supersymmetry is unbroken if and only if the energy of vacuum is exactly zero. Equation (2.34) gives Witten Index, the nonzero of value of which ensures the supersymmetry.

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