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SUPERCONDUCTIVITY IN RESTRICTED CHROMODYNAMICS IN *SU*(3) GAUGE THEORY

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The study of condensation of monopoles and the resultant state of chromo-magnetic superconductivity has been undertaken in restriced chromodynamics (RCD) in *SU*(3) gauge theory. It has been shown that the resultant Lagrangian leads to dyonic condensation, color confinement and dual superconductivity with the presence of two scalar modes and two vector modes.

INTRODUCTION

The condensation of monopoles incorporates the state of magnetic superconductivity[1] and the notion of chromo-magnetic superconductor[2] where the Meissner effect confining magnetic field in ordinary superconductivity would be replaced by dual Meissner effect which would confine the color electric field. It leads to a correspondence between quantum chromo-dynamic situation and chromo-magnetic superconductor, where the Abelian electric field is squeezed by solenoidal monopole current[3,4] and the color confinement takes place due to dual Meissner effect caused by monopole condensation.Using this idea of confinement of electric flux due to condensation of magnetic monopoles, a dual gauge theory called restricted chromodynamics (*RCD*) has been constructed out of *QCD* in *SU*(2) theory[5-8]. This dual gauge theory incorporates a dynamical dyoniccondensation[9-10] and exhibits the desired dual dynamics that guarantees the confinement of dyonic quark through generalized Meissner effect. This *RCD* has been extracted from *QCD* by imposing an additional internal symmetry named magnetic symmetry[5-11] which reduces the dynamical degrees of freedom. Attempts have been made[12,13] to establish an analogy between superconductivity and the dynamical breaking of magnetic symmetry, which incorporates the confinement phase in *RCD* vacuum.

In this paper the formulation of *RCD* has been extended in the light of the concept of chromo-dyonic superconductor and it has been shown that in the confinement phase the dyonic condensation of vacuum gives rise to the complex screening current which confines both the chromo-electric and chromo-magnetic fluxes through the mechanism of generalized Meissner effect (the usual one and its dual).Extending the *RCD* in the realistic color gauge group *SU*(3) by using two internal killing vectors as λ_3-like octet and λ_8-like octet, the *RCD*

Lagrangian of *SU*(3) theory has been obtained in magnetic gauge and it has been shown to lead to dyoniccondensation, color confinement and the resulting superconductivity in *SU*(3) theory with the presence of two scalar modes and two vector modes as the consequence of the presence of two magnetic octets (λ_3-like and λ_2-like). It has been shown that due to the dynamical breaking of magnetic symmetry the vacuum acquires the properties similar to those of relativistic superconductor where the quantum fields generate non-zero expectation values and induce screening currents.

MAGNETIC SYMMETRY AND RESTRICTED CHROMODYNAMICS (RCD) IN *SU*(3) GAUGE THEORY.

Mathematical foundation of restricted chromodynamics (RCD) is based on the fact that a non-Abelian gauge theory permits some additional internal symmetry *i.e.*, magnetic symmetry[5-8]. Unified space P of non-Abelian gauge theory may be thought of as

$$P = M \otimes G \qquad \text{... (2.1)}$$

which is (4 + n) dimensional manifold where M is 4-dimensional external space and G, in general, is the n-dimensional internal space, generated by n Killing vectors ξ_i satisfying the conditions

$$[\xi_i, \xi_j] = f_{ij}^k \xi_k \qquad \text{... (2.2)}$$

and

$$\pounds_{\xi_i} g_{AB} = 0 \qquad \text{... (2.3)}$$

where g_{AB} (A, B = 0, … n + 3) is the metric of manifold P with gauge symmetry as n dimensional isometry[14,15] and $\pounds_{\xi_i}$ is the Lie derivative along ξ_i. In equation (2.2) f_{ij}^k is internal structure parameter, the four dimensional quotient space $M = P/G$ is the base manifold and P is the principal fibre bundle. It has been conjectured that the dynamics of magnetic monopole is effectively described by a gauge theory based on magnetic symmetry which has the topological meaning. This magnetic symmetry is an additional internal isometry H having some additional Killing vector fields of generalized gauge theory. These additional Killing vectors are purely internal ones and hence commute with already existing fields ξ_i of G. The internal isometry H is Cartan's subgroup of G and commutes with it. Let the additional Killing vector fields be m_a (a = 1, 2, ... k = dim H). Then we have

$$\begin{aligned} m_a &= m_a^i \xi_i, \ (i = 1,2,3) \\ (\xi_i, m_a) &= 0, \\ (m_a, m_b) &= -f_{ab}^{(H)c} m_c \\ \pounds_{ma} g_{AB} &= 0 \end{aligned} \qquad \text{... (2.4)}$$

where $\pounds_{ma}$ is the Lie derivative along the direction of magnetic symmetry. Since the isometry H commutes with the right isometry G, it is called the left isometry. The topological magnetic charge associated with monopoles corresponds to the elements of second homotopic group $\pi_2(G/H)$.

Let us start with the construction of the restricted chromodynamics in *SU*(3) limit. The magnetic structure of this theory may be described by two internal Killing vectors which commute with each other and also with the gauge symmetry itself and are normalized to unity according to the following equations :

$$\hat{m}^2 = 1 \text{ and } \hat{m}'^2 = 1 \qquad \text{... (2.5)}$$

These Killing vectors are a λ_3- like octet $\hat{m}$ and its symmetric product

$$\hat{m}' = \sqrt{3}\,(\hat{m} \times \hat{m}) \qquad \text{... (2.6)}$$

which is λ_8-like. The restricted theory (RCD) may be extracted from the full QCD by imposing the extra internal symmetries. Let us restrict the dynamical degrees of freedom of the theory (while keeping the full gauge degrees of freedom intact) by imposing the extra magnetic symmetry which restricts the generalized non-Abelian gauge potential $\vec{V}_\mu$ to satisfy the constraints given by

$$D_\mu\,\hat{m} = \partial_\mu \hat{m} + i\,|\,q\,|\,\vec{V}_\mu \times \hat{m} = 0 \qquad \text{... (2.7)}$$

and $$D_\mu\,\hat{m}' = \partial_\mu \hat{m}' + i\,|\,q\,|\,\vec{V}_\mu \times \hat{m}' = 0$$

where D_μ is covariant derivative for the gauge group. Then the dyonic generalized four-potential

$$\vec{V}_\mu = \vec{A}_\mu - \vec{B}_\mu$$

of QCD in *SU*(3) gauge theory may written as follows in RCD *SU*(3) gauge theory:

$$\vec{V}_\mu = -iV^*{}_\mu \hat{m} - iV'^*_\mu \hat{m}' + \left(\frac{i}{|\,q\,|}\right)\hat{m} \times \partial_\mu \hat{m} + \left(\frac{i}{|\,q\,|}\right)\hat{m}' \times \partial_\mu \hat{m}' \qquad \text{... (2.8)}$$

where $$\hat{m}.\vec{V}_\mu = -iV^*_\mu \qquad \text{... (2.9)}$$

and $$\hat{m}'.\vec{V}_\mu = -iV'^*{}_\mu \qquad \text{... (2.10)}$$

are, respectively, λ_3-like and λ_8-like unrestricted Abelian components of the restricted potential. In the magnetic gauge $\hat{m}$ and $\hat{m}'$ become the space-time independent $\hat{\xi}_3$ and $\hat{\xi}_8$ respectively, where

$$\hat{\xi}_3 = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} \text{ and } \hat{\xi}_8 = \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} \qquad \text{... (2.10)}$$

Then the generalized potential of equation (2.8) may be written as

$$\vec{V}_\mu = (-iV_\mu^* + W_\mu)\,\hat{\xi}_3 + (-iV_\mu'^* + W'_\varpi)\,\hat{\xi}_8 \qquad \text{... (2.11)}$$

where W_μ and W'_μ may be identified as the potentials of topological dyons in magnetic symmetry of *SU* (3) gauge theory. These are entirely fixed by $\hat{m}$ and $\hat{m}'$, respectively, up to Abelian gauge degrees of freedom. The generalized field strength can, then, be constructed as

$$\vec{G}_{\mu\nu} = \vec{G}_{\mu\nu} + \frac{i}{|q|}[\vec{V}_\mu \times \vec{V}_\nu]$$

$$= (-iF_{\mu\nu} + H_{\mu\nu})\,\hat{\xi}_3 + (-iF'_{\mu\nu} + H'_{\mu\nu})\,\hat{\xi}'_8 \qquad \text{... (2.12)}$$

where $\vec{G}_{\mu\nu} = \vec{V}_{\nu,\mu} - \vec{V}_{\mu,\nu}$,

$$F_{\mu\nu} = V^*_{\nu,\mu} - V^*_{\mu,\nu}; \qquad \text{... (2.13)}$$

$$H_{\mu\nu} = \left(\frac{i}{|q|}\right)\hat{m}\,.[\partial_\mu \hat{m} \times \partial_\nu \hat{m}]; \qquad \text{... (2.14)}$$

$$\vec{G}_{\mu\nu} = \partial_\nu \vec{V}_\mu - \partial_\mu \vec{V}_\nu; \qquad \text{... (2.15)}$$

$$F'_{\mu\nu} = \partial_\mu V'_\nu - \partial_\nu V'^*_\mu;$$

and $H'_{\mu\nu} = \partial_\mu W'_\nu - \partial_\nu W'_\mu$ ... (2.16)

In this theory the gauge fields are expressible in terms of purely time-like non-singular potentials V_μ^* and V'^*_μ, W_μ and W'_μ. Then in the absence of quarks or any colored object, the RCD Lagrangian of *SU*(3) theory in magnetic gauge may be written as

$$L = \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{4}H'_{\mu\nu}H'^{\mu\nu} + \frac{1}{4}[H_{\mu\nu}H'^{*\mu\nu} + H'_{\mu\nu}H^{*\mu\nu}] + \frac{1}{2}|D_\mu\varphi|^2$$

$$+ \frac{1}{2}|D'_\mu\phi'|^2 - V(\phi^*\phi, \phi'^*\phi') \qquad \text{... (2.17)}$$

where $D_\mu\varphi = (\partial_\mu + i\,|q|\,W_\mu)\varphi$;

$$D'_\mu\,\varphi' = (\partial_\mu + i\,|q|\,W'_\mu)\varphi'; \qquad \text{... (2.18)}$$

and the dyonic field operators ϕ and ϕ' correspond to m and m' respectively. Here $V(\phi^*\phi, \phi'^*\phi')$ is the effective potential introduced to induce the dynamical breaking of the magnetic symmetry.

DYONIC CONDENSATION AND SUPERCONDUCTIVITY IN RCD IN *SU*(3) GAUGE THEORY

The Langarian (2.17) of RCD in magnetic gauge in the absence of quark or any colored object looks like Ginsburg–Landau Lagrangian for the theory of superconductivity if we identify the dyon field as an order parameter and the generalized potential W_μ as the electric potential. The dynamical breaking of the magnetic symmetry, due to the effective potential $V(\phi^*\phi, \phi'^*\phi')$, induces the dyonic condensation of the vacuum. This gives rise to the dyonic supercurrent, the real part of which (electric constituent) screens the electric flux which confines the magnetic color charge (through usual Meissner effect) and the imaginary part (i.e. magnetic constituent) of this supercurrent screens the magnetic flux that confines the electric color iso-charges (due to dual Meissner effect). In other words, the dual Meissner effect expels the electric field between static coloured charges into a narrow flux tube, giving rise to a linearly rising potential and to confinement. This Lagrangian leads to dyonic condensation, color confinement and the resulting dual superconductivity in *SU*(3) theory. Lagrangian (2.17) has been obtained from the standard *SU*(3) Lagrangian and hence the desired dynamical breaking of magnetic symmetry is obtained by fixing the following form of the effective potential

$$V(\phi^*\phi, \phi'^*\phi') = -\eta(\,|\phi|^2 + |\phi'|^2 - \nu^2 - \nu'^2) \qquad \text{... (3.1)}$$

where η is a constant, ν and ν' are the expectation values of Higgs fields ϕ and ϕ' :

$$\nu = \langle\phi\rangle_0 \qquad \text{... (3.2)}$$

and

$$\nu' = \langle\phi'\rangle_0 \qquad \text{... (3.3)}$$

In Prasad-Sommerfeld limit[16]

$$V(\phi^*\phi, \phi'^*\phi') = 0 \qquad \text{... (3.4)}$$

but $\nu \neq 0$ and $\nu' \neq 0$ ... (3.5)

In this limit, the dyons have lowest possible energy for given electric and magnetic charges e and g respectively. Due to the presence of two magnetic vectors $\hat{m}$ and $\hat{m}'$ in *SU*(3) theory, we have here two scalar modes with masses

$$M_\phi = \sqrt{(8\eta)}\,\nu \quad \text{and} \quad M_{\phi'} = \sqrt{(8\eta)}\,\nu' \qquad \text{... (3.6)}$$

and two vector modes with masses given by

$$M_D = |q|\nu \text{ and } M_{D'} = |q|\nu' \qquad \text{... (3.7)}$$

respectively.

With these two mass scales the coherence lengths ε and ε' and the penetration length λ and λ',corresponding to the two magnetic vectors $\hat{m}$ and $\hat{m}'$, are given by

$$\varepsilon = 1/M_\phi = 1/[\sqrt{(8\eta)}\,\nu]\,;\; \varepsilon' = 1/M_\phi' = 1/[\sqrt{(8\eta)}\nu'] \qquad \text{... (3.8)}$$

and $\lambda = 1/M_D = 1/(|q|v)$; $\lambda' = 1/M_D' = 1/(|q|v')$ ... (3.9)

The region in phase diagram space, where $\varepsilon = \lambda$ and $\varepsilon' = \lambda'$ constitutes the border between type-I and type-II superconductors. It may be achieved for the following values of constant η of the effective potential given by eqn. (3.1):

$$\eta = \frac{|q|^2}{8} = \frac{e^2+g^2}{8} \quad \text{... (3.10)}$$

where e and g are the electric and magnetic charges of dyon.

The superconductivity provides vivid model for the actual confinement mechanism and the color confinement is due to the generalized Meissner effect caused by dyonic condensation. The dual superconductivity model proposed by Alessandro *et al*[17] places the Yang-Mills vacuum close to the border between type-I and type-II superconductors and marginally on the type-II side.

DISCUSSION

Equation (2.7) give the magnetic structure of restricted chromo-dynamics in *SU*(3) theory where two internal Killing vectors λ_3-like octet and λ_8-octet given by equation (2.6) have been introduced keeping in view the facts that any system possessing a *SU*(3) symmetry suffers with a non-Abelian magnetic instability for the 4-7th gluons[18] and the 8th gluon corresponds to the diagonal generator in color space[5, 6]. Equation (2.11) and (2.12) give restricted generalized potential and gauge field strength respectively in the magnetic symmetry of *SU*(3) gauge theory, where the space-time independent octet ε_3 and ε_8 are given by equations (2.10). The RCD Lagrangian of *SU*(3) theory in the absence of quarks or an colored object, is given by equation (2.17). This Lagrangian leads to dyonic condensation, color confinement and the resulting dual superconductivity in *SU*(3) theory with the presence of two scalar modes and two vector modes as the consequence of the presence of two magnetic octet (λ_3-like and λ_8-like) in RCD of *SU*(3) theory. The masses of these two scalars and two vector modes are given by equations (3.6) and equations (3.7) respectively. The coherence lengths ε and ε' and the penetration length λ and λ' are given by equations (3.8) and equations (3.9) respectively. The mases of scalar modes M_ϕ and M_ϕ' determine how fast the perturbative vacuum around a color source reaches condensation and the masses M_D and M_D' of vector modes determine the penetration lengths of the colored flux.

The Lagrangian, given by equation (2.17) for RCD in magnetic gauge in the absence of quarks or any coloured objects, establishes an analogy between superconductivity and the dynamical breaking of magnetic symmetry which incorporates the confinement phase in RCD vacuum where the effective potential $V(\phi^*\phi, \phi'^*\phi')$, given by equation (3.1), induces the dyonic condensation of vacuum. This gives rise to dyonic supercurrent. The electric constituent of this current (*i.e.,* its real part) screens the electric flux and confines the magnetic charges due to usual Meissner effect while its imaginary part (*i.e.* its magnetic constituent) screens the magnetic flux and confines the color iso-charges as the result of dual Meissner effect. Thus the dynamical breaking of the magnetic symmetry in this theory ultimately induces the generalized Meissner effect with electric constituent as the usual Meissner effect and its magnetic constituent as the dual Meissner effect. It dictates the mechanism for the confinement of the electric and magnetic fluxes associated with dyonic quarks[19] in the present theory. The confinement of colour is due to the spontaneous breaking of magnetic symmetry which yields a non-vanishing magnetically charged Higg's condensate, where the broken magnetic group is chosen by Abelianization process demonstrated by equation (2.12) to equation (2.16). It shows that the dyonic condensation mechanism of confinement in RCD is dominated by Abelian degrees of freedom. Such Abelian dominance in connection with monopole condensation has been demonstrated by Boykov *et al*[19]. The similar result has also been obtained in a dual superconductivity model[20] which places the Y-M vacuum close to the border between Type – I and Type–II superconductors and marginally on the Type–II side.

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