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SUPERCONDUCTIVITY DUE TO DYONICCONDENSATION

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Constructing the effective Lagrangian for dyonic field in Abelian projection of QCD, an Abelian Higgs model, incorporating dual superconductivity and confinement, has been constructed. It has been demonstrated that the non-Abelian dyons give rise to Abelian dyons in this Abelian projection (Abelianization). In this model the partition function in the Euclidean space-time has also been obtained. It has been shown that the dyonically condensed vacuum is characterized by two massive modes where the scalar mode determines how fast the perturbation vacuum around a colored source reaches condensation and the vector mode determines the penetration length of the colored flux. It has also been shown that the superconductivity provides a model for actual confinement mechanism caused by dyonic condensation.

INTRODUCTION

Rajput *et al*[1,2,3] conceived theexcellent analogy of superconductivity at high-T_c with QCD and demonstrated that the essential features of superconductivity provided the vivid models[4-9] for actual confinement mechanism in *QCD*. Izawa and Iwazaki made an attempt[10] to analyze a mechanism of quark confinement and demonstrated that most important role of monopoles and dyons in physics is their participation in the mechanism of confinement through their condensation[11-14], leading to efficient microscopic theories of superconductivity[15-18], dual-superconductivity[19-20] and color superconductivity [21].However, the crucial ingredient for condensation in a chromo-magnetic superconductor would be the non-Abelian force in contrast to the Abelian ones in ordinary superconductivity. Topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles[22]. The method of Abelian projection is one of the popular approaches to confinement problem, together with dual superconductivity[23,24] picture, in non-Abelian gauge theories. The condensation non-Abelian Monopole as a mechanism of confinement (together with dual superconductivity) implies that long-range physics is dominated by Abelian degrees of freedom [25,26] (Abelian dominance). The conjecture that the dual Meissner effect is the color confinement mechanism is realized if we perform Abelian projection in the maximal gauge where the Abelian component of gluon field and Abelian monopoles are found to be dominant[27-28]. The vacuum of gluon-dynamics behaves as a dual superconductor and the key role in dual superconductor model of *QCD* is played by Abelian monopole. For the self-dual fields, the Abelian monopoles become Abelian dyons [29]. There exists the model [30-33] of *QCD* vacuum in which the non-Abelian dyons are responsible for the confinement. The non-Abelian dyons give rise to Abelian dyons in the Abelian projection and hence an important problem, before studying the vacuum properties of non-Abelian theories, is to Abelianize them.

In the present paper the effective Lagrangian for dyonic field has been constructed in Abelian projection of *QCD* in *SU*(2) and *SU*(3) gauge theories and the Abelian Higgs model, incorporating dual superconductivity and confinement, has been developed. It has been demonstrated that the non-Abelian dyons give rise to Abelian dyons in this Abelian projection (Abelianization). In this model the partition function in the Euclidean space-time has also been obtained. It has been shown that the dyonically condensed vacuum is characterized by two massive modes where the scalar mode determines how fast the perturbation vacuum around a colored source reaches condensation and the vector mode determines the penetration length of the colored flux. The condition of border between type-I and type-II superconductors has been derived and it has been shown the superconductivity provides a model for actual confinement mechanism where the color confinement is due to the generalized Meissner effect caused by dyonic condensation.

2 CONDENSATION OF DYONS IN SU(2) GAUGE THEORY

The non-abeliannature of gauge group [*SU*(3) or *SU*(2)] is quite crucial to dyon condensation as mechanism of confinement.A general non-Abelian theory of dyons consists of usual four-space (external) and *n*-dimensional internal group space, where the field associated with dyons has *n*-fold internal multiplicity and the multiplets of gauge field transform as the basis of adjoint representation of n-dimensional non-Abelian gauge symmetry group. The field equations and equation of motion for abelian dyons preserve the invariance under the local non-Abelian gauge transformations:

$$\psi \to \psi' = S^{-1}\psi \qquad \text{... (2.1)}$$

where *S* is the local element of non-Abelian gauge group. Here the local gauge theory makes the presence of interacting field necessary.

Choosing the internal gauge group as SU(2), the generalized dyonic field tensor may be constructed as

$$\vec{G}_{\mu\nu} = G^a_{\mu\nu} T_a \qquad \text{... (2.2)}$$

with the generalized four-potential defined as

$$\vec{V_\mu} = V_\mu^a T_a \qquad \text{... (2.3)}$$

where repeated indices are summed over 1, 2 and 3 (internal degrees of freedom) , vector sign is denoted in the internal group space and the matrices T_a are three infinitesimal generators of group *SU*(2), satisfying the commutation relation

$$[T_a, T_b] = i\varepsilon_{abc} T_c$$

with ε_{abc} as structure constant of internal group.

We may connect $\vec{G}_{\mu\nu}$ and $\vec{V}_\mu$ through the following relation

$$G^a_{\mu\nu} = \partial_\nu V^a_\mu - \partial_\mu V^a_\nu + |q|\, \varepsilon^{abc} V_{\mu b} V_{\nu c} \qquad \text{... (2.4)}$$

where the dyonic generalized charge q is given as following complex quantity with electric and magnetic constituents as its real and imaginary parts

$$q = e - ig \qquad \text{... (2.5)}$$

and the generalized four-potential V_μ constitutes electric and magnetic constituents, A_μ and B_μ respectively as follows :

$$V_\mu = A_\mu - iB_\mu \qquad \text{... (2.5a)}$$

A suitable Lagrangian density of a spontaneously broken non-Abelian gauge theory $SU(2)$, yielding the classical dyonic solutions, may be constructed as

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2}(D_\mu \varphi)^a (D^\mu \varphi)_a - V(\varphi) = L_{dyon}(A_\mu, B_\mu, \varphi)$$

where $$D_\mu \varphi = \partial_\mu \varphi - i\,\mathrm{Re}(q * V_\mu)\varphi = (\partial_\mu - ieA_\mu - igB_\mu)\varphi \qquad \text{... (2.6)}$$

with Re denoting the real part and

$$V(\varphi) = \frac{1}{4}(\varphi^a \varphi_a)^2 - \frac{1}{2} v^2 (\varphi^a \varphi_a)$$

with $$v = \langle \varphi \rangle = \langle 0 | \varphi | 0 \rangle \qquad \text{... (2.7)}$$

which determines the vacuum expectation value of Higgs field. In simplest manner this equation may be written as

$$V(\varphi) = -\eta\, (|\varphi|^2 - v^2)^2 \qquad \text{... (2.8)}$$

with η as a constant.

The gauge dependent part of Lagrangian *i.e.*, first term of rhs in eqn. (2.6) is invariant under the following transformations of the fields A_μ and B_μ ;

$$V_\mu = \begin{pmatrix} A_\mu \\ B_\mu \end{pmatrix} \rightarrow \begin{pmatrix} A'_\mu \\ B'_\mu \end{pmatrix} = V'_\mu \qquad \text{... (2.9)}$$

with $$A'_\mu = A_\mu \cos\delta + B_\mu \sin\delta$$

and $$B'_\mu = -A_\mu \sin\delta + B_\mu \cos\delta$$

where $$\delta = \tan^{-1}\left(\frac{g}{e}\right) \qquad \text{... (2.10)}$$

Using the Lagrangian density given by eqn. (2.6) the electric and magnetic fields of dyons may be calculated by imposing the Julia- Zee ansatz[34]:

$$V_{ia} = \varepsilon_{aij} (\vec{r})^j \frac{K(r)-1}{|q| r^2}$$

$$V_{0a} = (\vec{r})_a \frac{J(r)}{|q| r^2}$$

$$\varphi_a = (\vec{r})_a \frac{H(r)}{|q| r^2} \quad \text{... (2.11)}$$

where the functions $K(r)$, $J(r)$ and $H(r)$ satisfy the following equations

$$r^2 H''(r) = 2HK^2$$
$$r^2 J''(r) = 2JK^2 \quad \text{... (2.12)}$$
$$r^2 K''(r) = K(K^2 - 1) + K(H^2 - J^2)$$

A solution of these equations may be written as follows :

$$J(r) = \alpha\varphi(r); H(r) = \beta\varphi(r); K(r) = \frac{Cr}{\sinh Cr}$$

where $\beta^2 - \alpha^2 = 1$

and $\phi(r) = C(r)\coth Cr - 1$ ... (2.13)

In the Prasad-Sommer field limit[35]

$$V(\varphi) = 0;$$

but $v = \langle \varphi \rangle \neq 0$ ... (2.14)

In this limit the dyons have lowest possible energy for given electric and magnetic charges e and g respectively. Thus we get the following expression for dyonic mass

$$M = v(e^2 + g^2)^{\frac{1}{2}} = v|q| \quad \text{... (2.15)}$$

where the electric and magnetic fields associated with dyons obey the first order equations

$$E^a{}_i = G^a{}_{0i} = \partial^i V^a{}_0 + |q| \varepsilon^{abc} V_{ib} V_{0c} = (D_i\varphi)^a \sin\alpha,$$
$$B^a{}_i = \varepsilon_{ijk} G^{jka} = (D_i\varphi)^a \cos\alpha$$

and $D_0(\varphi)^a = 0$

where $\alpha = \tan^{-1}\frac{e}{g}$ ... (2.16)

In these equations i and 0 indicate space and time directions and a is an $SU(2)$ vector index. These electric and magnetic fields associated with dyons are non-Abelian in nature having external as well as internal components. Using Gauss's law and these expressions for fields, we have the following expressions for electric and magnetic charges on dyon:

$$e = \frac{1}{v}\int d^3\, x\partial_i(\emptyset^a G^a_{0i})\ ;$$
$$g = \frac{1}{2v}\int d^3\, x\epsilon_{ijk}\partial_i(\emptyset^a G^a_{jk}) \qquad \text{... (2.17)}$$

In the case of pure monopole V^a_0 and α given by relation (2.16) vanishes and the eqns. (2.16) reduce to

$$E_i = 0;$$
$$B_i = D_i\emptyset \qquad \text{... (2.18)}$$

and eqn. (2.14) give the static energy which follows the Bogomol'nyi bond:

$$V(\emptyset) \geq |k| \qquad \text{... (2.19)}$$

where *k* is the monopole number given by

$$k = \int d^3\, x\ \partial_i\ tr(B_i\emptyset) \qquad \text{... (2.20)}$$

Condition (2.14) does not allow static dyonic solution but in this case the dyons emerge as time dependent solution and the ansatz given by eqn. (2.11) reduces to Prasad-Sommerfield condition (2.14) and then the solution of Bogomol'nyi equation (2.18) give Bogomol'ny-Prasad-Sommerfield (BPS) monopole as a static spherically symmetric solution with smooth field and finite mass. The Bogomol'nyi equation (2.18) is equivalent to the self-duality equation of pure Yang-Mills in R^4 space restricted to be translationally invariant in one direction. Let us construct a connection on R^4 that is invariant in the x^0 direction via

$$W^a_i = A^a_i;\ W^a_0 = \emptyset^a \qquad \text{... (2.21)}$$

If $\overrightarrow{F_{\mu\nu}}$ is the field strength corresponding to $\overrightarrow{W_\mu}$ in *SU*(2) theory, then the self- duality equation

$$\overrightarrow{F_{\mu\nu}} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\overrightarrow{F^{\rho\sigma}} \qquad \text{... (2.22)}$$

is equivalent to Bogomol'nyi equation (2.18). Introducing covariant derivative D_μ, given by eqn. (2.6), on R^4 with $\overrightarrow{V_\mu}$ replaced by $\overrightarrow{W_\mu}$ for *e* = 0, we may write Gauss's law as

$$D_\mu\overrightarrow{\dot{W}_\mu} = 0 \qquad \text{... (2.23)}$$

It follows from this relation that BPS monopoles are topological solitons in a Yang-Mills gauge theoryin three space dimensions. Such a monopole has four collective coordinates which include three position coordinates and a phase angle. When these four coordinates are time dependent, the monopole acquires momentum and electric charge and hence becomes a moving dyon. It is equivalent to Abelian projection (Abelian Higgs Model).

In the Abelian projection, the fields given by eqns. (2.16) reduce to the following form in the asymptotic limit;

$$E_j{}^a = -\frac{3b}{|q|r^4}(\vec{r})^a(\vec{r})_j - \frac{2c}{|q|r^3}(\vec{r})^a(\vec{r})_j;$$

$$B_j{}^a = -\frac{(\vec{r})_j(\vec{r})^a}{|q|r^4} \qquad \text{... (2.24)}$$

where *b* and *c* are positive constants having the dimensions of charge and mass respectively

For vanishing *c* (*i.e.*, vanishing mass) these fields corresponds to point-like mass-less dyons with electric charge $\frac{3b}{|q|}$ and magnetic charge $\frac{1}{|q|}$. Then the generalized charge of the dyon may be written as

$$q = \frac{1}{|q|}\ (3b - i) \qquad \text{... (2.25)}$$

Thus non-Abelian dyons give rise to the Abelian dyons in the Abelian projection. The infra-red properties of *QCD* in the Abelian projection can be described in the Abelian Higgs Model (AHM) [2] in which dyons are condensed. In this model the relevant degrees of freedom are two massive gluons $W_\mu^{\pm}$, *a* $U(1)$ gluon (associated with generalized field V_μ) and a dyon which we take to be scalar represented by complex field φ. Then the Lagrangian (2.6) reduces to

$$L_{dyon}(A_\mu, B_\mu, \varphi) = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\left|\ (\partial_\mu - ieA_\mu - igB_\mu)\ \varphi\ \right|^2 + \eta\ (|\ \varphi\ |^2 - v^2)^2 \qquad \text{... (2.26)}$$

In terms of this Lagrangian, the partition function in the Euclidean space-time may be written as

$$Z_{dyon} = \int DA_\mu DB_\mu D\varphi \exp\{-\int d^4x L_{dyon}(A_\mu, B_\mu, \varphi)\} \qquad \text{... (2.27)}$$

Applying the transformation (2.9) and integrating over the field A'_μ, this partition function reduces to the following form in Abelian Higgs Model AHM;

$$Z_{dyon} = \int DB'_\mu D\varphi \exp\{-\int d^4x L_{AHM}(B'_\mu, \varphi)\}$$

with $$L_{AHM}(B'_\mu, \varphi) = -\frac{1}{4}H'_{\mu\nu}H'^{\mu\nu} + \frac{1}{2}\left|(\partial_\mu - i\bar{g}B'_\mu)\varphi\right|^2 + \eta(|\varphi|^2 - v^2)^2 \qquad \text{... (2.28)}$$

where the Higgs field ϕ has the magnetic charge

$$\bar{g} = |\ q\ |$$

and $$H'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu \qquad \text{... (2.29)}$$

This model (AHM) incorporates dual superconductivity and hence confinement as the consequence of dyonic condensation since the Higgs type mechanism arises here. With the first of conditions (2.29), the field $B'_\mu$of equation (2.28) is the dual gauge field and ϕ carries the magnetic charge.

In equation (2.28) B'_μ is the dual gauge field where ϕ carries magnetic charge given by first of equations (2.29). Thus the Lagrangian L_{AHM}, given by equation (2.28), provides the effective model of *QCD* vacuum. If we set $e = 0$ in this model then monopoles are condensed with the magnetic charge of the Higg's field given as follows from first of equations (2.29).

$$\bar{g} = g$$

and $$H'_{\mu\nu} = H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \qquad \text{... (2.30)}$$

In this case equations (2.28) and (2.30) give only an effective theory valid for energies less than some scale typical of the monopole size, equivalently the mass M of the massive gluon $W_\mu^\pm$. Then the Lagrangian (2.28) carries with it a physical cut off ≈ M. Then this theory incorporates the following inferences:

(i) $A\frac{1}{k^4}$ - propagator for gluon is dual equivalent to the statement that the gluon is propagating in a chromagnetic superconductor.

(ii) The transition from $<0 \mid \phi \mid 0> = 0$ to $<0 \mid \phi \mid 0> \neq 0$ is first order and leads, in an analogy with the Higg's- Ginsburg – Landau theory of superconductivity, to the vacuum becoming a chromo-magnetic superconductor.

The vacuum of gluodymanics behaves as dual superconductor and the key role in dual superconductor model of Abelian Higgs mechanism, presented here with, is played by Abelian monopoles. For the self dual fields, the Abelian monopoles become Abelian dyons.

The infrared properties of *QCD* in the Abelian projection can be described by the Abelian Higgs model [AHM] in which dyons are condensed. The conjecture, that the dual Meissner effect is the color confinement mechanism, is realized if we perform Abelian projection in the maximal gauge where the Abelian component of the gluon field and Abelian monopoles are found to be dominant. Then the Abelian electric field is squeezed by solenoidal monopole current. Monopole condensation is confirmed by the energy-entropy balance of the monopole trajectories and by evaluation of the monopole operator. All these facts support the conjecture that color confinement is due to the dual Meissner effect caused by the dyonic condensation.

DYONIC CONDENSATION IN *SU*(3) GAUGE THEORY

In the presence of dyons, the presence of second potential through the eqn. (2.5a) is actually compensated by an enlargement of the group of gauge transformations from *SU*(2) to *SU*(3). It is well known that *SU*(3) gauge symmetry, spontaneously broken by an octet Higg's filed, exhibits *SU*(2)*X U*(1) symmetry with the non-zero expectation value of Higg's field. As per general topological argument [36], the very presence of *U*(1) factor in the unbroken gauge group guarantees the existence of smooth, finite energy, stable solution with quantized magnetic charge and chirality quantized dyons. In pure *SU*(2) gauge theory without Higg's field, the monopole field is unstable while the effect of adding Higg's field in *SU*(2) is to establish a *U*(1) gauge symmetry at large distance, reproducing the topology which stabilizes monopoles and dyons in ordinary electrodynamics by permitting them to have a finite size finite mass as given by eqn (2.15). In the internal two-dimensional complex space introduced at each point of Minkowski space-time, the charged field described by ψ through eqn.(2.1) in *SU*(2) is replaced by

$$\psi' = \exp\{i\Lambda^0(x)\}\ \psi \qquad \text{... (3.1)}$$

in $SU(2) \times U(1)$, where $\Lambda^0(x)$ is a phase factor. Then the basic spinors of internal space are acted upon by the following element $\hat{S}(x)$of SU(3):

$$\hat{S}(x) = S(x)\ \exp\{-i\Lambda^0(x)\} \qquad \text{... (3.2)}$$

where $S(x)$is a local group element of SU(2). Under this gauge transformation the generalized four-potential $\overrightarrow{V_\mu}$, given by eqn. (2.3), and the generalized field tensor associated with Abelian dyons as given by eqn (2.2) , transform as follows :

$$V'^a_\mu = S^{-1}V^a_\mu S - S^{-1}\partial^a_\mu S \qquad \text{... (3.3)}$$

and

$$G'^a_{\mu\nu} = S^{-1}G^a_{\mu\nu}S \qquad \text{... (3.4)}$$

where $G^a_{\mu\nu}$ and V^a_μ satisfying eqns. (2.2) and (2.3) respectively, are coupled by eqn. (2.4) in SU(3) theory also where the operators T_a are eight generators of SU(3) group and indices

$$a, b, c = 1,2, \ldots .8$$

In SU(3) theory, relation (2.8) may be generalized into the following form :

$$\overrightarrow{G_{\mu\nu}} = \partial_\nu\overrightarrow{V_\mu} - \partial_\mu\overrightarrow{V_\nu} + |q|[\overrightarrow{V_\mu},\overrightarrow{V_\nu}] \qquad \text{... (3.5)}$$

where →is denoted in the internal space of gauge group SU(3). This relation may also be written as

$$G^a_{\mu\nu} = G'^a_{\mu\nu} + \epsilon^{abc}V_{b\mu}V_{c\nu} \qquad \text{... (3.6)}$$

with ϵ^{abc} as structure constant of internal gauge group SU(3) and

$$G'^a_{\mu\nu} = \partial_\nu V^a_\mu - \partial_\mu V^a_\nu \qquad \text{... (3.7)}$$

Applying the operator D^ν given by equation (2.6) on eqn.(3.6), we get

$$D^\nu G^a_{\mu\nu} = \partial^\nu G'^a_{\mu\nu} + |q|\epsilon^{abc}V^\nu_b G'_{c\mu\nu} = J^a_\mu \qquad \text{... (3.8)}$$

where J^a_μ are the components of generalized non-Abelian four-current $\vec{J}_\mu$ in eight dimensional internal space of Gauge group SU(3). This equation may also be written as

$$J^a_\mu = j'^a_\mu + |q|\epsilon^{abc}V^\nu_b G'_{c\mu\nu} \qquad \text{... (3.9)}$$

where

$$\overrightarrow{J'_\mu} = \overrightarrow{J_\mu} - i\overrightarrow{k_\mu} = \quad \partial^\nu\overrightarrow{G'}_{\mu\nu} \qquad \text{... (3.10)}$$

with $\overrightarrow{J_\mu}$ and $\vec{k}_\mu$ as electric and magnetic four currents, is the generalized four current associated with dyon in the Abelian Higgs Model (after Abelianization).

Equation (3.8) is manifestly conserved equation since

$$D^\mu\vec{J}_\mu = 0 \qquad \text{... (3.11)}$$

but

$$\partial^\mu\vec{J}_\mu \neq 0 \qquad \text{... (3.12)}$$

On the other hand equation (3.10) gives

$$\partial^\mu\overrightarrow{J'}_\mu = 0 \qquad \text{... (3.13)}$$

which is usual Noetherian conservation lawof the Noetherian current $\overrightarrow{J'}_\mu$. These equation show that while Noetherian current $\overrightarrow{J'}_\mu$ of Abelian model is conserved in strict sense, the generalized non-Abelian current $\vec{J}_\mu$ in $SU(3)$ theory is not so but satisfy generalized conservation law (manifestly conserved equation) when ordinary derivative is replaced by the covariant derivative as shown by equation (3.11).

In the Abelian Higg With the development of non-Abelian gauge theories, Dirac monopole has mutated in another way as we have to take into account not only electromagnetic $U(1)$ gauge group but also the color gauge group $SU(3)_C$ describing strong interaction. At energy around 100 GeV electromagnetism merges in the electroweak interaction with the gauge group $SU(2)$ $XU(1)$. These gauge theories still have monopoles of Dirac type, but the ordinary magnetic fields of the monopoles, in general, will be accompanied by color magnetic or magneto-weak fields. Topologically, the most important difference between a non-Abelian gauge theory and a set of Abelian (*QED* type) gauge fields is the compactness of the non-Abelian gauge group. Thus in *QCD*, because $SU(3)$ is compact, the color electric charges defined with respect to any maximal Abelian subgroup are quantized. It implies that we can write down gauge field configurations that asymptotically look like magnetic monopole of any chosen Abelian direction. The confinement of color electric charge corresponds to the screening of color magnetic charge. In particular, for distances beyond 1 Fm the energy of the color magnetic field drops exponentially. This means that beyond 1 Fm one can neglect the difference between realistic monopoles and Dirac ones. Thus there are monopole field configurations in any non-Abelian gauge theory. To prove the phase structure of the theory, we can add a scalar field (*i.e.* Higg's field) in the adjoint representation so long as this does not change the nature of flow of the coupling constant with energy. For asymptotically free theories, the low energy behavior is dominated by the Abelian monopoles of zero mass which are almost point-like. The interaction of point- like monopoles with gluons and charged particles can be studied as a dual analog of point-like charged particle interactions. It leads to condensation of monopole[16]. Topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles which undergo condensation. This condensation leads to confinement. The scalar fields (*i.e.* Higg's fields) have all decoupled by now and hence this field ϕ plays a role of a regulator only. This theory also has massless gluons denoted by A_μ, charged massive gluons W_μ and monopoles which are coupled minimally to massles B_μ and electrically charged particle W_μ. These Abelian monopoles play the key role in the dual superconductor model of the *QCD* vacuum. In this process of Abelianization (*i.e.*, the Abelian projection) the quark are electrically charged particles and if the monopoles are condensed the dual Abrikosov string carrying the electric flux is formed between quarks and antiquark. Due to a non- zero string tension the quarks are confined by the linear potential. For the self dual fields the Abelian monopoles become Abelian dyons[19]. These dyons are coupled minimally to the massless V_μ given by equation (2.3) and electrically charged particles W_μ. In *QCD* for low energy the dyons interactions are saturated by duality. Thus the infrared properties of the *QCD* in the Abelian projection can be described by the Abelian Higgs Model (*AHM*) where dyons are condensed leading to confinement. As such, the non-Abelian confinement of dyonic charge is related to linear Abelian theory in a dyonic superconductor.

The non-Abelian dyonic field Lagrangian of $SU(3)$ gauge theory also reduces to the similar form as given by eqn. (2.26) and the condensation of dyon takes place through the process of Abelian Higgs Mechanism.

DUAL SUPERCONDUCTIVITY DUE TO DYONIC CONDENSATION

S model developed in the previous section in terms of magnetic gauge, defined by equation (2.29), B'_μ is the dual gauge field with mass of dual gauge boson (vector mode) given by

$$M_D = |q|v, \quad \text{... (4.1)}$$

which is the the same as the mass of dyon given by eqn. (2.15), and ϕ is the dyonic field (scalar mode) with mass given by

$$M_\phi = \sqrt{(8\eta)}\, v \quad \text{... (4.2)}$$

where η is a constant introduced in eqn. (2.8). In terms of these two mass scales we may get the coherence length and the penetration length in the following manner:

$$\varepsilon = \frac{1}{M_\phi} = \frac{1}{\sqrt{(8\eta)}\, v} \text{ (Coherence length)} \quad \text{... (4.3)}$$

$$\lambda = \frac{1}{M_D} = \frac{1}{|q|v} \text{ (Penetration Length)} \quad \text{... (4.4)}$$

The region in the phase space, where

$$\varepsilon = \lambda, \quad \text{... (4.5)}$$

constitutes the border between type-I and type-II superconductors. Thus the superconductivity provides a model for actual confinement mechanism where the color confinement is due to the generalized Meissner effect caused by dyonic condensation , where dyonic electric charge produces the screening effect for A_μ-propagator and anti-screening effect for B_μ-propagator, while the dyonic magnetic charge produces screening effect for B_μ-propagator and anti-screening effect for A_μ-propagator. The dyonically condensed vacuum is characterized by the presence of two massive modes where the mass M_ϕ of scalar mode given by eqn. (3.3) determines how fast the perturbation vacuum around a colored source reaches the condensation and the mass M_D of vector mode, given by eqn. (3.1) determines the penetration length of the colored flux.

DISCUSSION

The gauge depended part of the Lagrangian density, given by eqn. (2.6) for the fields associated with the non-Abelian dyons in the minimal gauge theory, is invariant under the linear transformation (2.9). Equations (2.12) and (2.13) demonstrate that the non-Abelian dyons give rise to Abelian dyons in the Abelian projection (Abelianization). It follows from equation (2.23) that BPS monopoles are topological solitons in a Yang-Mills gauge theory in three space dimensions. Such a monopole has four collective coordinates which include three position coordinates and a phase angle. When these four coordinates are time dependent, the monopole acquires momentum and electric charge and hence becomes a moving dyon. It is equivalent to Abelian projection (Abelian Higgs Model). The infrared properties of *QCD* in this Abelian projection can be described by the Abelian Higgs model with Lagrangian density given by eqn. (2.26) in which dyons are condensed. In this model the partition function in the Euclidean space-time is given by the first part of eqns. (2.26). This model incorporates dual superconductivity and confinement as the consequence of dyonic condensation. Equation (4.3) gives the Coherence length as the inverse of mass of dyonicfield (scalar mode) and the eqn. (4.4) gives the penetration length as the inverse of mass of dual gauge boson (vector mode). The former length (*i.e.* coherence length) determines how fast the perturbation vacuum around a colored source reaches the condensation and the later one (penetration length) gives the depth of penetration of colored flux as the consequence of dyonic condensation.Equation (4.5) constitutes the border between type-I and type-II superconductors. Thus the superconductivity provides a model for actual confinement mechanism where the color confinement is due to the generalized Meissner effect caused by dyonic condensation. On the similar lines, it has been, very recently, demonstrated[37] that in the background of a strong magnetic field the electroweak sector of vacuum experiences two consecutive crossover transitions associated with zero temperature dynamics of W-bosons and the scalar Higgs particle respectively, where above the first cross over, the presence of W and Z condensate supports the existence of exotic superconductivity and second transition restores the electroweak symmetry. Color confinement (and resulting Superconductivity), Chiral symmetry breaking and Catalytic effect induced by monople condensation have also been discussed[38] recently.

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