

CONDENSATION OF MONOPOLES AND CHROMOMAGNETIC SUPERCONDUCTING EFFECTS IN NON-ABELIAN GAUGE THEORIES

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In this paper it has been shown that topologically a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles which undergo condensation which leads to confinement and incorporates a dual superconductivity model of the QCD vacuum where the Higgs field plays the role of a regulator only.

INTRODUCTION

With the development of non-Abelian gauge theories, Dirac monopole^[1,2] has mutated in another way as we have to take into account not only electromagnetic $U(1)$ gauge group but also the color gauge group $SU(3)_C$ describing strong interaction. At energy around 100 GeV electromagnetism merges in the electroweak interaction with the gauge group $SU(2) \times U(1)$. These gauge theories still have monopoles of Dirac type, but the ordinary magnetic fields of the monopoles, in general, will be accompanied by color magnetic or magneto weak fields.^[3-6] The results of usual gauge group $U(1)$ may be generalized to an ordinary gauge group H when potentials are defined in the Lie algebra of H , i.e.,

$$B_\mu = B_\mu^a t_a, \quad \dots (1)$$

where t_a are generators of H . To avoid unnecessary factors of i , the t_a are taken to be anti-Hermitian and the coupling derivative is

$$D_\mu = \partial_\mu - B_\mu \quad \dots (2)$$

where the minimal coupling has been absorbed into B_μ . Each matter field belongs to some unitary representation of H and the potential acts on it according to this representation of the generator t_a . When one specializes again to the case $H = U(1)$, one has to reintroduce the factor i_g to make contact with the old notation. Topologically, the most important difference between a non-Abelian gauge theory and a set of Abelian (QED type) gauge fields is the compactness of the non-Abelian gauge group H . Thus in QCD , because $SU(3)$ is compact, the color electric charges defined with respect to any maximal Abelian subgroup are quantized. It implies that we can write down gauge field configurations that asymptotically look like magnetic monopole of any chosen Abelian direction. The spherically symmetric monopoles have the magnetic field.

$$\vec{B}^a = \frac{1}{2} Q^a \frac{\hat{r}}{r^2} \quad \dots (3)$$

where Q^a is a generator of H . They can be considered as $U(1)$ monopoles where $U(1)$ is the subgroup of H generated by Q^a .

In a realistic theory with electromagnetic and quark matter fields, Q^a may be diagonalized by a global gauge transformation and thus the solution of quantization condition

$$\exp(2\pi Q) = 1, \quad \dots (4)$$

may be written as

$$Q = i(mQ_e + nQ_{3c} + n'Q'_c) \quad \dots (5)$$

where Q_e is the electric $U(1)$ generator normalized to unity, Q_{3c} acts on the color states of the quark fields as the diagonal matrix – dia g ($-1/3, -1/3, 2/3$) and Q'_c acts as $(1, -1, 0)$. Moreover, the integers m and n have to satisfy additional condition

$$m + n = 0 \pmod{3} \quad \dots (6)$$

Taking into account the existence of quarks we find that for monopole with ordinary magnetic charge only, m must be multiple of 3. The monopoles with $m = 1$ are possible but they must have a color magnetic field in addition.

For non-Abelian H , the spherically symmetric ansatz (3) can only be valid for a limited range of distances. The confinement of color electric charge corresponds to the screening of color magnetic charge^[7]. In particular, for distances beyond 1 Fm the energy of the color magnetic field drops exponentially and not as r^{-2} as one would obtain from equation (3). This means that beyond 1 Fm one can neglect the difference between realistic monopoles and Dirac ones. Thus there are monopole field configurations in any non-Abelian gauge theory. To prove the phase structure of the theory, we can add a scalar field (*i.e.*, Higg's field) in the adjoint representation so long as this does not change the nature of flow of the coupling constant with energy. For asymptotically free theories, the low energy behavior is dominated by the Abelian monopoles of zero mass which are almost point-like. The interaction of point-like monopoles with gluons and charged particles can be studied as a dual analog of point-like charged particle interactions. It leads to condensation of monopole^[8]. Topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles which undergo condensation. This condensation leads to confinement^[9]. The scalar fields (*i.e.* Higg's fields) have all decoupled by now and hence this field ϕ plays a role of a regulator only. This theory also has massless gluons denoted by A_μ , charged massive gluons W_μ and monopoles which are coupled minimally to massless B_μ and electrically charged particle W_μ . These Abelian monopoles play the key role in the dual superconductor model^[10-15] of the QCD vacuum. Thus spontaneously broken non-Abelian gauge theories support a classical solution which is asymptotically equivalent to a monopole magnetic field and also leads to charge quantization.

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