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STUDY OF DUSTY VISCO-ELASTIC FLUID (KUVSHINISKI TYPE) USING LIGHTHILL TECHNIQUE

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In the present article a theoretical study of the motion of laminar flow of a dusty visco-elastic fluid (Kuvshiniski type) containing uniformly small solid particles between two infinitely extended parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its own plane is carried out. The expressions for velocity fields of fluid and dust particles are obtained using Light hill technique. It is seen that the nature of velocity profiles of dusty fluid and that of dust particles are identical. The interesting feature of the study is that at a certain level, the velocity profiles of dusty fluid and that of dust particles do not depend on elastic coefficient (α).The expression for skin friction at the lower plate due to dusty fluid and the expression for volume flow of dusty visco-elastic fluid discharged per unit breadth of the plate are also obtained in this study.

KEYWORDS : Visco-elastic fluid, laminar flow, elastic element, harmonic oscillation, Lighthill technique.

INTRODUCTION

The study of non-Newtonian fluids has drawn special attention under a wide range of geometrical, rheological and dynamical conditions. A few examples are the flow of nuclear fuel slurries, the flow of liquid metals and alloys such as the flow of gallium metal at ordinary temperatures , the flow of plasma, the flow of mercury amalgams, handling of biological fluids, the flow of blood, a Bingham fluid with some thixotropic behaviours, coating of paper, petroleum production, plastic extrusion, molten paper pulp, emulsion, paints, lubrication with heavy oils and greases, aqueous solutions of polyacrylamide and polyisobutylene, etc, as important raw materials and chemical products in a large variety of industrial processes. The subject of Rheology is of great technological importance as in many branches of modern industry, the problem arises of designing apparatus to transport or to process substances which cannot be governed by the classical stress-strain velocity relations. Visco-Elastic fluids are particular cases of non-Newtonian fluids that exhibit appreciable elastic behaviour and stress-strain velocity relations are time-dependent. Many common liquids such as oils, certain paints, polymer solutions, some organic liquids, and many new materials of industrial importance exhibit both viscous and elastic properties. In this article the velocity profiles of fluid and dust particles for different values of elastic coefficient are graphically discussed.

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Saffman has expressed model equations describing the influence of dust particles on the motion of fluids. Several authors using equations of Saffman have investigated several dusty gas flow problems in different situations. Kapur[5] investigated the problem of two immiscible viscous liquids between two fixed parallel plates under a certain pressure gradient. The flow of visco-elastic Maxwell liquid down an inclined plane was investigated by Bagchi and Maiti[1]. The unsteady flow of two immiscible visco-elastic conducting liquids between two inclined parallel plates has been studied by Lahiri and Ganguly[6]. Mandal[7] *et al* have considered unsteady flow of dusty visco-elastic (kuvshiniski type) liquid between two oscillating plates. Ghosh and Debnath[2] focused on unsteady hydromagnetic flows of a dusty viscous fluid between two oscillating plates. Johari[4] *et al* have studied the MHD flow of a dusty Visco-elastic (kuvshiniski type) liquid past in an inclined plane. Ghosh, Debnath[3] investigated the hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates. Singhp[10] *et al* have studied the MHD flow of a dusty visco-elastic liquid past on an inclined plane. Prakash, Kumar and Dwivedi[8] discussed MHD free convection flow of a viscoelastic (Kuvshiniski type) dusty gas through a semi-infinite plate moving with velocity decreasing exponentially with time. Krishna Ch. Nandy[11] have discussed the unsteady flow of dusty visco-elastic liquid between two parallel plates (Kuvshinisky type). Raju Kundu[12] *et.al.* studied the flow of dusty visco-elastic fluid between two parallel plates.

MATHEMATICAL FORMULATION OF THE PROBLEM AND ITS SOLUTION:

Let us suppose that the dusty visco-elastic fluid fills the region between two horizontal infinite parallel flat plates at a distance h apart. The lower plate is kept at rest and the upper one begins to perform harmonic oscillations with a frequency in its own plane. The physical model is shown in figure-1. The present analysis takes a co-ordinate system such that the *x*-axis coincides with the lower fixed plate and the *z*-axis is perpendicular to it.

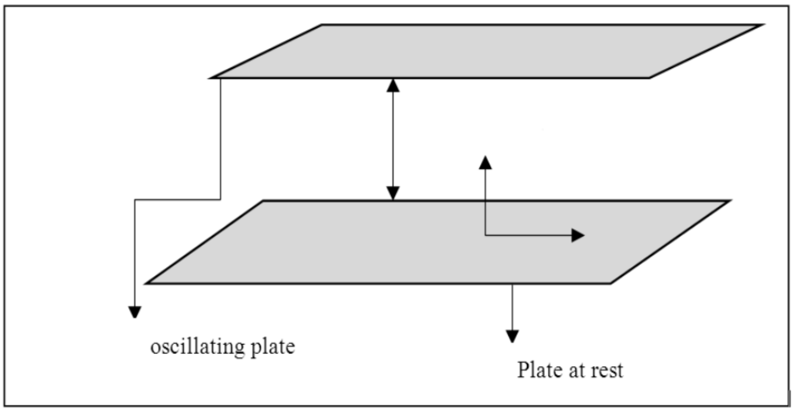

Fig. 1

The dust particles are assumed to be spherical in shape and uniform in size and the number density of dust particles is taken as constant throughout the flow and it be ρ_0. Since the plates are infinite, the velocity will depend on various parameter. For the constitutive equation we adopt Kuvshiniski type fluid, given by

$$P_{ij} = -p\delta_{ij} + p'_{ij} \quad \dots (1)$$

$$\left(1 + \lambda_0 \frac{D}{Dt}\right) p'_{ij} = 2\mu\, e_{ij} \quad \dots (2)$$

$$\frac{D}{Dt} p'_{ij} = \frac{\partial p'_{ij}}{\partial t} + u_m \frac{\partial p'_{ij}}{\partial x_m} \quad \dots (3)$$

$$2e_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \quad \dots (4)$$

where P_{ij} is stress tensor and p'_{ij} the deviatoric stress tensor, $\frac{D}{Dt}$ is the convective time derivative following a fluid element and u_i is the velocity of the fluid particle. Here λ_0 and μ denote the elastic coefficient and viscosity of the fluid. Using the above equations, we get the equation of motion of dusty visco-elastic fluid (dropping dashes)

$$\left(1 + \alpha \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = R \frac{\partial^2 u}{\partial z^2} + \frac{f}{\tau}\left(1 + \alpha \frac{\partial}{\partial t}\right)(v - u) \quad \dots (5)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u - v) \quad \dots (6)$$

Here $u' = \frac{u}{h\omega}$; $v' = \frac{v}{h\omega}$; $t' = \omega t$; $z' = \frac{z}{h}$; $\alpha = \lambda_0 \omega$; $R = \frac{h^2\omega}{\gamma}$

where, v is the velocity of dusty particle.

f= mass concentration = $\frac{mB_0}{\rho}$

τ= relaxation time = $\frac{m\omega}{K}$

The relevant initial and boundary conditions in non-dimensional form are $t \leq 0$

$$u = \frac{\partial u}{\partial t} = 0 \;\; for\ all\ \ z \quad \dots (7)$$

$$t > 0$$

$$u = a\, e^{i\beta t} \quad at\ \ z = 1$$

$$u = 0 \quad at\ \ z = 0 \quad \dots (8)$$

To solve the equations (5) and (6) we adopted the Lighthill technique and write

$$u = u_1(z) + \lambda_2 u_2(z) e^{i\omega t}\ , \qquad \lambda_2 = \text{constant} \neq 0 \quad \dots (9)$$

$$v = v_1(z) + \epsilon_2 v_2(z) e^{i\omega t}\ , \qquad \epsilon_2 = \text{constant} \neq 0 \quad \dots (10)$$

Equations (5) and (6) takes the form

$$\omega\lambda_2 u_2 (i - \alpha\omega)\, e^{i\omega t} = R \frac{d^2 u_1}{dz^2} + \frac{f}{\tau}(v_1 - u_1)$$
$$+ \left\{ R\lambda_2 \frac{d^2 u_2}{dz^2} + \frac{f}{\tau}(\epsilon_2 v_2 - \lambda_2 u_2) + \alpha i\omega(\epsilon_2 v_2 - \lambda_2 u_2) \right\} e^{i\omega t} \quad \dots (11)$$

and $$\epsilon_2 v_2 i\omega e^{i\omega t} = \frac{1}{\tau}\{u_1 - v_1 + (\lambda_2 u_2 - \epsilon_2 v_2)e^{i\omega t}\} \quad \text{... (12)}$$

The relevant initial and boundary conditions are

when $t \le 0$

$u_1 = u_2 = \frac{du_1}{dt} = \frac{du_2}{dt} = 0$ for all z (u_1 and u_2 are the function of z only)

For $t > 0$

$$u_1 = \frac{a\sin(\omega-\beta)t}{\sin\omega t} \quad and \quad u_2 = \frac{a\sin\beta t}{\lambda_2\sin\omega t} \quad at \ \ z = 1\} \quad \text{...(13)}$$

$$u_1 = u_2 = 0 \qquad at \ \ z = 0$$

Separating the real and imaginary parts of the equation (11) we get

Real part :

$$R\frac{d^2u_1}{dz^2} + \frac{f}{\tau}(v_1 - u_1) + \left\{R\lambda_2\frac{d^2u_2}{dz^2} + \frac{f}{\tau}(\epsilon_2 v_2 - \lambda_2 u_2)\right\}\cos\omega t$$
$$- \frac{f}{\tau}\alpha\omega(\epsilon_2 v_2 - \lambda_2 u_2)\sin\omega t + \omega\lambda_2 u_2(\omega\alpha\cos\omega t + \sin\omega t) = 0$$

... (14)

Imaginary part :

$$\left\{R\lambda_2\frac{d^2u_2}{dz^2} + \frac{f}{\tau}(\epsilon_2 v_2 - \lambda_2 u_2)\right\}\sin\omega t + \frac{f}{\tau}\alpha\omega(\epsilon_2 v_2 - \lambda_2 u_2)\cos\omega t + \omega^2\alpha\lambda_2 u_2\sin\omega t$$
$$-\omega\lambda_2 u_2\cos\omega t = 0 \quad \text{... (15)}$$

Separating the real and imaginary parts of the equation (12)

Real part :

$$-\omega\tau\epsilon_2 v_2\sin\omega t = (u_1 - v_1) + \lambda_2 u_2\cos\omega t - \epsilon_2 v_2\cos\omega t \quad \text{... (16)}$$

Imaginary part :

$$v_2 = \frac{\lambda_2 u_2\sin\omega t}{\epsilon_2(\tau\omega\cos\omega t + \sin\omega t)} \quad \text{... (17)}$$

From equation (15) and (17) we get

$$\frac{d^2u_2}{dz^2} - k^2u_2 = 0 \quad \text{... (18)}$$

where $$k^2 = \frac{\omega}{R}\left[\frac{f\cos\omega t\,(1 + \alpha\cot\omega t)}{\omega\tau\cos\omega t + \sin\omega t} + \cot\omega t - \omega\alpha\right]$$

Using boundary conditions, the solution of equation (18) is

$$u_2 = \frac{a\sin\beta t}{\lambda_2\sin\omega t}\left(\frac{\sinh kz}{\sinh k}\right) \quad \text{... (19)}$$

Therefore

$$v_2 = \frac{a\sin\beta t}{\epsilon_2(\tau\omega\cos\omega t + \sin\omega t)}\left(\frac{\sinh kz}{\sinh k}\right) \quad \text{... (20)}$$

From equation (14) we get

$$\frac{d^2u_1}{dz^2} + M\sinh kz = 0 \quad \text{... (21)}$$

where, $$M = \frac{a\sin\beta t}{R\sinh k}\left[\frac{cos^2\omega t(Rk^2\omega\tau + \omega^2\alpha\tau) + \sin\omega t\cos\omega\tau\ \ (Rk^2 + \omega^2\tau + \omega\alpha + \omega^2 f\alpha\, sin\,\omega t)\ + (1+f)\omega sin^2\omega t}{\sin\omega t\,(\omega\tau\cos\omega t + \sin\omega t)}\right]$$

Applying boundary conditions, the solution of equation (21) is

$$u_1 = -\frac{M}{k^2}\sinh kz + z\left(\frac{a\sin(\omega-\beta)t}{\sin\omega t} + \frac{M}{k^2}\sinh k\right) \quad \text{... (22)}$$

Therefore, we get from equation (16)

$$v_1 = -\frac{M}{k^2}\sinh kz + z\left(\frac{a\sin(\omega-\beta)\,t}{\sin\omega t} + \frac{M}{k^2}\sinh k\right) + \frac{a\omega\tau\sin\beta t}{\sin\omega t\,(\omega\tau\cos\omega t + \sin\omega t)}\left(\frac{\sinh kz}{\sinh k}\right) \quad \text{... (23)}$$

The velocity profile of dusty fluid is

$$u = -\frac{M}{k^2}\sinh kz + z\left(\frac{a\sin(\omega-\beta)t}{\sin\omega t} + \frac{M}{k^2}\sinh k\right) + \frac{a\sin\beta t}{\sin\omega t}\left(\frac{\sinh kz}{\sinh k}\right)e^{i\omega t},\quad t > 0 \ \text{... (24)}$$

The real part of the velocity profile of dusty fluid

$$u_r = -\frac{M}{k^2}\sinh kz + z\left(\frac{a\sin(\omega-\beta)t}{\sin\omega t} + \frac{M}{k^2}\sinh k\right) + \frac{a\sin\beta t}{\sin\omega t}\left(\frac{\sinh kz}{\sinh k}\right)\cos\omega t\ ,t\ > 0\ \text{... (25)}$$

The velocity profile of dust particles is

$$v = -\frac{M}{k^2}sinh\,kz + z\left(\frac{a\,sin(\omega-\beta)\,t}{sin\,\omega t} + \frac{M}{k^2}sinh\,k\right) + \frac{a\omega\tau\sin\beta t}{\sin\omega t\,(\omega\tau\cos\omega t + \sin\omega t)}\left(\frac{\sinh kz}{\sinh k}\right) + \frac{a\sin\beta t}{\omega\tau\cos\omega t + \sin\omega t}\left(\frac{\sinh kz}{\sinh k}\right)e^{i\omega t} \ \text{... (26)}$$

The real part of the velocity profile of dust particles is

$$v_r = -\frac{M}{k^2}\sinh kz + z\left(\frac{a\sin(\omega-\beta)\,t}{\sin\omega t} + \frac{M}{k^2}\sinh k\right) + \frac{a\omega\tau\sin\beta t}{\sin\omega t\,(\omega\tau\cos\omega t + \sin\omega t)}\left(\frac{\sinh kz}{\sinh k}\right) + \frac{a\sin\beta t}{\omega\tau\cos\omega t + \sin\omega t}\left(\frac{\sinh kz}{\sinh k}\right)\cos\omega t\ \text{... (28)}$$

The dimensionless shearing stress i.e., skin friction (τ_p) at the lower plate due to dusty visco-elastic fluid is

$$\tau_p = \left[\left(1 - \alpha \frac{\partial}{\partial t}\right)\frac{\partial u_r}{\partial z}\right]_{z=0}$$

Therefore,

$$\tau_p = -\frac{M}{k} + \frac{a\sin(\omega-\beta)t}{\sin\omega t} + \frac{M}{k^2}\sinh k + \frac{ak\sin\beta t\cos\omega t}{\sin\omega t\sinh k} - \alpha\left\{\frac{a(\omega-\beta)\cos(\omega-\beta)t}{\sin\omega t} - \frac{a\omega\sin(\omega-\beta)t\cos\omega t}{sin^2\omega t} + \frac{ak}{\sinh k}(\beta\cot\omega t\cos\beta t - \omega\sin\beta t\,\mathrm{cosec}^2\omega t)\right\}$$

The volume flow of dusty visco-elastic fluid discharged per unit breadth of the plate is given by

$$\varphi = 2\int_0^1 u_r dz$$

$$\varphi = 2\left[\frac{1}{k}(1-\cosh k)\left\{\frac{M}{k^2} - \frac{a\sin\beta t\cos\omega t}{\sin\omega t\sinh k}\right\} + \frac{1}{2}\left\{\frac{a\sin(\omega-\beta)t}{\sin\omega t} + \frac{M}{k^2}\sinh k\right\}\right]$$

RESULTS AND DISCUSSIONS:

The velocity profiles (real part) forvisco-elastic fluid are plotted in fig. (2) having $a = 1,\ R = 0.01,\ f = 0.01\ and\ \beta = \omega = \frac{\pi}{24}$ for different values of elastic coefficient. It is seen from the fig. (1) that the real part of velocity of dusty fluid (u_r) increases with increase ofparameter z. After reaching a maximum value the real part of velocity decreases gradually. At a certain level, all velocity profiles of real partmeet at a point. At that point, the real part of velocity of dusty fluid (u_r) does not depend on elastic coefficient.

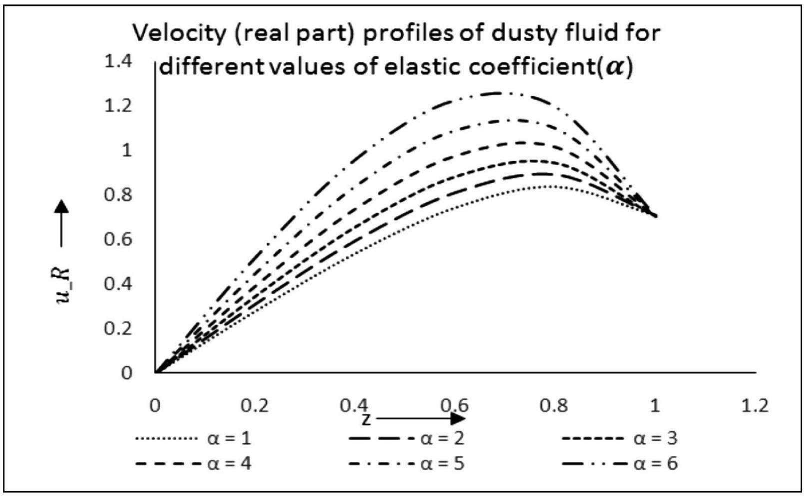

Fig. 2.

Fig. (2) represents the velocity profiles(real part) for dust particles having a = 1, R = 0.01, and f = 0.01 for different values of elastic coefficient. It is seen that the real part of velocity of dust particles (v_r) increases with increase of parameter z. After reaching a maximum value, the real part of velocity (v_r) decreases gradually and at a certain level, all velocity profiles meet at a point. At that point, the real part of velocity of dust particles (v_r) does not depend on elastic coefficient.

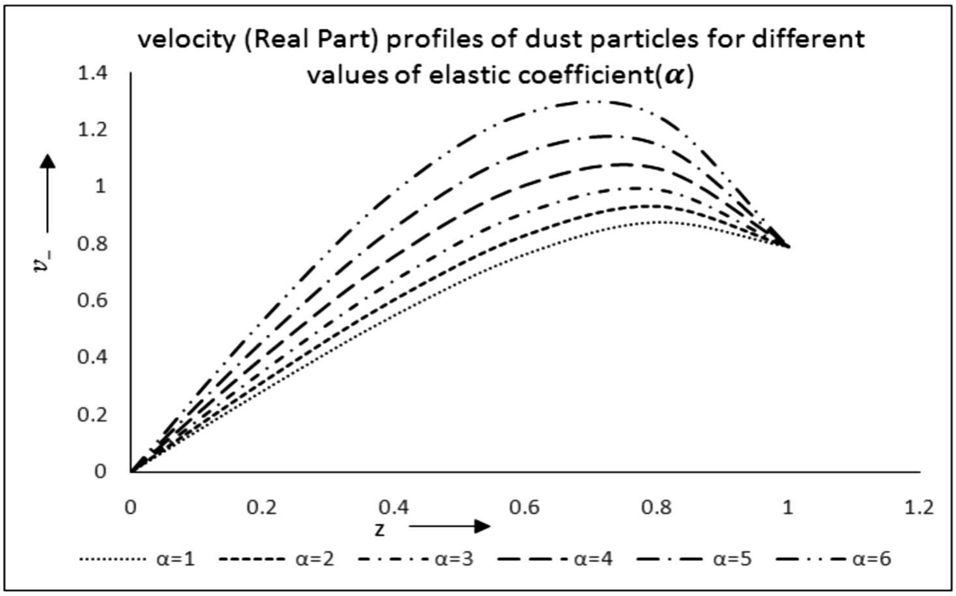

Fig. 3

The conclusions of the study:

1. It is clear from the above figures that the nature of velocity profiles of dusty fluid as well as that of dust particles are identical.

2. At a certain level, the velocity profiles of dusty fluid and that of dust particles do not depend on elastic coefficient.

Highlights:

1. The most interesting feature of the study is that the velocity profile of dusty fluid does not affect velocity profiles of dust particles. The velocity profiles of dusty fluid and that of dust particles are identical.

2. At a certain level, the velocity profiles of dusty fluid and that of dust particles do not depend on elastic coefficient.

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