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FLOW OF VISCOUS STRATIFIED FLUID PAST A POROUS BED DUE TO PRESSURE GRADIENT

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The flow of viscous stratified fluid in a channel with the porous bed has been studied in this article. To study the effects of the stratification factor and slip parameter on the flow,we divide the entire flow region into two zones. Zone-1 relates to the region between the impermeable upper plate and the lower one being the porous bed where the flow is governed by Navier Stokes equations. Zone-2 is the porous region where the flow is governed by the modified Darcy's law. In this paper, we have investigated the effects of the stratification factor (η) and porosity factor (σ) on the velocity profile of the flow producedby pressure gradient. In this article we have applied Lighthill's technique for obtaining the velocity profile of the stratified viscous flow.

KEYWORDS : Viscous stratified fluid, stratification factor, slip parameter, porosity factor, Navier Stokes Equation, Darcy's Law, Oscillating Pressure gradient, Lighthill's technique.

INTRODUCTION

The study of the flow of the viscous fluid past a porous medium without stratification has been studied by Beavers and Joseph [1], Beavers *et al* [2], and Rudraiah *et al* (1973). The study of stratified fluid is of great importance in the field of the petroleum industry because the density in petroleum oil varies with temperature. Channabasappa and Ranganna[3] considered the flow of viscous stratified fluid past a porous bed with the anticipation that stratification may provide a technique for studying the pore size in a porous medium. In this paper, he has shown that slip velocity is proportional to the pressure gradient. Gupta and Sharma[4] discussed the stratified viscous flow of variable viscosity between a porous bed and moving impermeable plate. Hari Kishan and Sharma [5] discussed the stratified viscous flow of variable viscosity between a porous bed and moving impermeable plate under the action of a body force. Das, D. and Nandy, K. [6] studied the motion under gravity of a viscous fluid through a pipe. Das, D.K. and Nandy, K.C. [8] discussed the unsteady laminar stratified flow over a porous bed. Karmakar, S., Biswas, P.K. and Nandy, K.C. [9] discussed the stratified flow of viscous fluid between a porous bed and an impermeable plate.

Biswas[10] et.al studied the stratified fluid of variable viscosity past a porous bed under the action of pressure gradient.

In this note, we have studied the effects of stratification factor (η) and porosity factor (σ) on the flow of viscous stratified fluid under the presence of an oscillating pressure gradient. We have studied the effects of η and σ on average velocity for small Reynolds number. We have also studied the effects of η and σ on the velocity profile for small Reynolds number.

THE MATHEMATICAL FORMULATION OF THE PROBLEM

The study the problem, we divide the entire flow region into two zones which are shown in fig.1. In zone-1, from the impermeable upper oscillating plate up to the interface, the flow is called the free flow which is governed by the usual Navier Stokes equations. In the other zone below the interface, the flow is governed by the Darcy's law.

Fig. 1

The basic equations for zone-1 are

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \qquad \dots(1)$$

$$\mu = \mu_0 e^{-\beta y} \;, \qquad \rho = \rho_0 e^{-\beta y} \qquad \dots(2)$$

$$\frac{\partial p}{\partial y} = -g\rho \qquad \dots(3)$$

The basic equations for zone-2 are

$$Q = Q_0\, e^{\beta y} \qquad \dots(4)$$

$$Q_0 = -\frac{K}{\mu}\frac{\partial p}{\partial x} \qquad \dots(5)$$

The relevant boundary conditions are

$$u = u_0 e^{i\omega t} \qquad at\ y = h \qquad for\ \ t > 0 \qquad \dots(6)$$

$$\frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{K}}(u_B - Q_0) \qquad \text{at } \ y = 0 \qquad \text{for } \ \ t > 0 \qquad \dots(7)$$

Using the dimensionless quantities

$$u' = \frac{u}{u_m} t = \frac{ht'}{u_m} x' = \frac{x}{h} y' = \frac{y}{h} p' = \frac{p}{\rho_0 u_m^2} u_0' = \frac{u_0}{u_m} V' = \frac{u_B}{u_m}$$

equation (1) takes the form (Dropping all primes)

$$\frac{\partial u}{\partial t} + \frac{\eta}{R}\frac{\partial u}{\partial y} - \frac{1}{R}\frac{\partial^2 u}{\partial y^2} = -e^{\eta y}\frac{\partial p}{\partial x} \quad \dots (8)$$

The relevant boundary conditions become

$$u = u_0 e^{i\omega t} \quad at \quad y = 1 \quad \text{for all } t \quad \dots (9)$$

and

$$\frac{\partial u}{\partial y} = \sigma\alpha\left(V + \frac{R}{\sigma^2}\frac{\partial p}{\partial x}\right) \quad at \quad y = 0 \quad \text{for all } t \quad \dots (10)$$

Light hill has investigated a new technique for solving the above equation by considering the velocity as the sum of two parts, one time-dependent and another time-independent. As such we can write

$$u = u_1(y) + \lambda u_2(y) e^{i\omega t} \quad \dots (11)$$

$$V = V_1(y) + \lambda V_2(y) e^{i\omega t} \quad \dots (12)$$

where λ is constant

Taking $\quad -\frac{\partial p}{\partial x} = A e^{i\omega t} \quad \dots (13)$

where A is constant.

Using equations (11), (12) and (13), equations (8) & (9) get the form

$$\frac{\eta}{R}\frac{\partial u_1(y)}{\partial y} - \frac{1}{R}\frac{\partial^2 u_1(y)}{\partial y^2} + \lambda u_2(y) i\omega\, e^{i\omega t} + \frac{\eta\lambda}{R}\frac{\partial u_2(y)}{\partial y} e^{i\omega} - \frac{\lambda}{R}\frac{\partial^2 u_2(y)}{\partial y^2} e^{i\omega t}$$
$$= A e^{\eta y} e^{i\omega t} \quad \dots (14)$$

$$\frac{\partial u_1(y)}{\partial y} + \lambda\frac{\partial u_2(y)}{\partial y} e^{i\omega t} = \sigma\alpha\left\{V_1(y) + \lambda V_2(y) e^{i\omega t} - \frac{R}{\sigma^2} A e^{i\omega t}\right\} \; at \; y$$
$$= 0 \quad \text{for all } t \quad \dots (15)$$

Resolving equation (14) & (15) into time-dependent and time-independent parts we get,

$$\lambda u_2(y) i\omega + \frac{\eta\lambda}{R}\frac{\partial u_2(y)}{\partial y} - \frac{\lambda}{R}\frac{\partial^2 u_2(y)}{\partial y^2} = A e^{\eta y} \quad \dots (16)$$

$$\frac{\partial^2 u_1(y)}{\partial y^2} - \eta\frac{\partial u_1(y)}{\partial y} = 0 \quad \dots (17)$$

$$\frac{\partial u_2(y)}{\partial y} = \sigma\alpha V_2(y) - \frac{A\alpha R}{\lambda\sigma} \quad at\ y = 0 \quad \text{for all } t \quad \dots (18)$$

$$\frac{\partial u_1(y)}{\partial y} = \sigma\alpha V_1(y) \quad at \quad y = 0 \quad \text{for all } t \quad \dots (19)$$

After using Lighthill's technique, equation (9) takes the form

$$u_1(y) = 0 \quad \text{at } y = 1 \quad \text{for all } t \quad \dots (20)$$

$$u_2(y) = \frac{u_0}{\lambda} \quad \text{at} \quad y = 1 \quad \text{for all } t \quad \dots (21)$$

From equation (16) we can write

$$\frac{\partial^2 u_2(y)}{\partial y^2} - \eta\frac{\partial u_2(y)}{\partial y} - i\omega R u_2(y) = \frac{AR}{\lambda} e^{\eta y}$$

After applying boundary conditions, the solution of the above equation is

$$u_2(y) = e^{\delta y}\left(\frac{(\delta-n)\left(\frac{u_0 e^{-\delta}}{\lambda}+\frac{iAe^{\delta}}{\omega\lambda}\right)-e^{-n}\left(\alpha\sigma V_2-\frac{AR\alpha}{\sigma\lambda}+\frac{i2A\delta}{\omega\lambda}\right)}{2\delta\sinh n-2n\cosh n}e^{ny} - \frac{(\delta+n)\left(\frac{u_0 e^{-\delta}}{\lambda}+\frac{iAe^{\delta}}{\omega\lambda}\right)-e^{-n}\left(\alpha\sigma V_2-\frac{AR\alpha}{\sigma\lambda}+\frac{i2A\delta}{\omega\lambda}\right)}{2\delta\sinh n-2n\cosh n}e^{-ny}\right) - \frac{iA}{\omega\lambda}e^{2\delta y}$$

where $2n=\sqrt{\eta^2+4K'}$, $K'=i\omega R$ *and* $2\delta=\eta$

The solution of the equation (17) will be

$$u_1(y)=\frac{\alpha\sigma V_1\left(-e^{2\delta}+e^{2\delta y}\right)}{2\delta}$$

Using the value of $u_1(y)$ and $u_2(y)$ in equation (11) we get,

$$u=\frac{\alpha\sigma V_1\left(-e^{2\delta}+e^{2\delta y}\right)}{2\delta} + \left[\frac{\left\{(\delta-n)e^{(\delta+n)y}-(\delta+n)e^{(\delta-n)y}\right\}\left(u_0e^{-\delta}+\frac{iAe^{\delta}}{\omega}\right)}{2\delta\sinh n-2n\cosh n} - \frac{e^{-n}\left(\alpha\sigma\lambda V_2-\frac{AR\alpha}{\sigma}+\frac{i2A\delta}{\omega}\right)\left(e^{(\delta+n)y}-e^{(\delta-n)y}\right)}{2\delta\sinh n-2n\cosh n} - \frac{iA}{\omega}e^{2\delta y}\right]e^{i\omega t} \quad \dots (22)$$

The real part of velocity u is

$$u_r=\frac{\alpha\sigma V_1\left(-e^{2\delta}+e^{2\delta y}\right)}{2\delta} + \left[\frac{\left\{(\delta-n)e^{(\delta+n)y}-(\delta+n)e^{(\delta-n)y}\right\}u_0e^{-\delta}-e^{-n}\left(\alpha\sigma\lambda V_2-\frac{AR\alpha}{\sigma}\right)\left(e^{(\delta+n)y}-e^{(\delta-n)y}\right)}{2\delta\sinh n-2n\cosh n}\right]\cos\omega t - \left[\frac{\frac{Ae^{\delta}}{\omega}\left\{(\delta-n)e^{(\delta+n)y}-(\delta+n)e^{(\delta-n)y}\right\}-\frac{2A\delta e^{-n}}{\omega}\left\{e^{(\delta+n)y}-e^{(\delta-n)y}\right\}}{2\delta\sinh n-2n\cosh n}-\frac{A}{\omega}e^{2\delta y}\right]\sin\omega t$$

If $\bar{u}$ denotes the dimensionless average velocity, then

$$\bar{u}=\int_0^1 u_r\,dy$$

$$\bar{u}=-\frac{\alpha\sigma V_1}{2\delta}e^{2\delta}+\frac{\alpha\sigma V_1}{4\delta^2}\left(e^{2\delta}-1\right) + \left[\frac{\left\{(\delta-n)^2\left(e^{\delta+n}-1\right)-(\delta+n)^2\left(e^{\delta-n}-1\right)\right\}u_0e^{-\delta}-e^{-n}\left(\alpha\sigma\lambda V_2-\frac{AR\alpha}{\sigma}\right)\left\{(\delta-n)e^{\delta+n}-(\delta+n)e^{\delta-n}+2n\right\}}{(\delta^2-n^2)(2\delta\sinh n-2n\cosh n)}\right]\cos\omega t$$

$$-\left[\frac{Ae^{\delta}\{(\delta-n)^2(e^{\delta+n}-1)-(\delta+n)^2(e^{\delta-n}-1)\}-2A\delta e^{-n}\left\{\begin{matrix}(\delta-n)(e^{\delta+n}-1)\\-(\delta+n)(e^{\delta-n}-1)\end{matrix}\right\}}{\omega(\delta^2-n^2)(2\delta\sinh n-2n\cosh n)}\right.$$

$$\left.-\frac{A}{2\delta\omega}(e^{2\delta}-1)\right]\sin\omega t$$

RESULTS AND DISCUSSION

The average velocities and the velocity profiles are shown in tables and figures below. In table-1, the average velocities ($\bar{u}$) of the flow generated by oscillating pressure gradient with respect to porosity factor (σ) for the various values of stratification factor (η) at time t = 0 are displayed. It shows that average velocity decreases as increase in porosity factor (σ) for the fixed value of the stratification factor (η).

The average velocities ($\bar{u}$) of the flow generated by the oscillating pressure gradient with respect to stratification factor (η) for the various values of porosity factor (σ) at time t = 0 are shown in table-2. It shows that an increase in the stratification factor (η) leads to a decrease in average velocity for a fixed value of the porosity factor (σ). Hence stratification is not favorable to the average velocity produced by the oscillating pressure gradient.

Table 1

For η = 0.2

σ	5	10	15	20	25
$\bar{u}$	0.190660	0.134260	0.07686	0.01992	– 0.03748

For η = 0.4

σ	5	10	15	20	25
$\bar{u}$	0.183050	0.117770	0.052484	– 0.012813	– 0.078092

For η = 0.6

σ	5	10	15	20	25
$\bar{u}$	0.173856	0.098731	0.023604	– 0.051614	– 0.126638

For η = 0.8

σ	5	10	15	20	25
$\bar{u}$	0.162790	0.076224	– 0.017290	– 0.096889	– 0.183444

Table – 2

For σ = 5

η	0.2	0.4	0.6	0.8
$\bar{u}$	0.190660	0.183050	0.173856	0.162790

For σ = 10

η	0.2	0.4	0.6	0.8
$\bar{u}$	0.134260	0.117770	0.098731	0.076224

For σ = 15

η	0.2	0.4	0.6	0.8
$\bar{u}$	0.076860	0.052484	0.023604	– 0.017290

For σ = 20

η	0.2	0.4	0.6	0.8
$\bar{u}$	0.019920	– 0.012813	– 0.051614	– 0.096889

For σ = 25

η	0.2	0.4	0.6	0.8
$\bar{u}$	– 0.037480	– 0.078092	– 0.126638	– 0.183444

The velocity profile of stratified viscous fluid in presence of oscillating pressure gradient for different values of stratification factor (η) and porosity factor (σ) are plotted in figures 2, 3, 4 and 5. The figures (Fig 2, 3, 4 & 5) show that for a fixed value of η, the velocity profile increases with increase in height (y) from the permeable surface. It is also seen from the figures that velocity is negative up to a certain height for increasing σ. This is the backflow of the fluid. If the value of σ is increased, backflow increases. At a certain height from the permeable surface, backflow disappears and above the height, velocity increases with height and attains a maximum value at the upper permeable plate.

Fig. 2

Fig. 3

Fig. 4

Fig. 5

NOMENCLATURE

μ_0	Coefficient of viscosity	ρ_0	Density at the interface *y* = 0
β	Stratification factor, $\beta > 0$	$\frac{\partial p}{\partial x}$	Pressure gradient
α	Slip parameter	K	Permeability coefficient
Q	Darcy Velocity	u_B	Slip velocity at nominal surface *y* = 0
η	Nondimensional stratification factor	σ	Porosity factor
R	Reynolds number		
V	Dimensionless slip velocity at nomial surface		
u_m	Maximum velocity		

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