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# NEW FORMS OF SOME NEUROSOPHIC COMPACT SPACES VIA GRILLS

**RUNU DHAR[1], SUMA PAUL[2] & SUPRIYA PAUL[3]**

*Department of Mathematics, Maharaja Bir Bikram University, Agartala-799004, Tripura, India.*
*E-mail: [1]runu.dhar@gmail.com, [2]suma40069@gmail.com & [3]sp988329@gmail.com*
*Corresponding author: Runu Dhar[1]*
*E-mail: [1]runu.dhar@gmail.com*

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The purpose of this paper is to introduce some new type of compactness in terms of grill a $G$ in a neutrosophic topological space. We call them neutrosophic ρ$G$ - compact space and neutrosophic σ$G$ - compact space. We investigate some of their basic properties and characterization theorems. We also study effects of various kinds of functions on them in neutrosophic topological space.

**Key Words :** Grill, neutrosophic $G$- compact, neutrosophic $Gg$- closed, neutrosophic ρ$G$- compact, neutrosophic σ$G$ - compact, neutrosophic set, neutrosophic topological space.

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## INTRODUCTION

Human beings are facing real life problems due to uncertainty. Zadeh [17] introduced the notion of fuzzy set to solve such real life problems due to uncertanity. Thereafter, Atanassov [1] introduced the notion of intuitionistic fuzzy set by adding non-membership value along with the membership value. As it was not sufficient to solve all real life problems due to uncertainty on decision making under uncertainty, Smarandache [14] introduced the notion of neutrosophic set (in brief, NS) where each element had three associated defining functions, namely the membership function ($T$), the non-membership function ($F$) and the indeterminacy function ($I$) defined on the universe of discourse $X$. These three functions are completely independent. Smarandache [15] further investigated on the neutrosophic theory. After the development of the notion of neutrosophic set theory, we can easily take decision to solve the real life problems on decision making. Thereafter, the notion of neutrosophic topological space (in brief, NTS) was first introduced by Salama and Alblowi [11], followed by Salama and Alblowi [12].

Choquet [2] initiated the brilliant notion of a grill. Subsequently, it turned out to be a very convenient tool for various topological and neutrosophic topological investigations. It is found from the literature that in many situations, grills are more effective than certain similar concepts like nets and filters. The notion of compactness in topological space via grills was introduced by Roy and Mukherjee [10]. Pal *et. al.* [8] introduced the notion of grill in neutrosophic topological space and neutrosophic minimal space. Pal and Dhar [7] introduced the notion of compactness in neutrosophic minimal space. Gupta and Gaur [6] introduced ρ*G*-compact space and σ*G*-compact space in topological space. Further different researchers [3, 4, 9, 13, 16] investigated in neutrosophic topological space. Following their works, we have motivated to introduce and investigate basic properties and results compactness via grills in neutrosophic topological space. Throughout this paper, this shall be denoted by neutrosophic *G*-compactness. We shall also focus on to construct new types of neutrosophic spaces through grills. The paper indicates different section as follows. The next section briefly focuses some known definitions and results related to neutrosophic set and neutrosophic topological space. In section 3, we introduce the notion of neutrosophic ρ*G*-compact space and investigate several basic properties of this space. Section 4 unfolds the notion of neutrosophic σ*G* -compact space and its basic properties in neutrosophic topological space.

## PRELIMINARIES

In this section, we recall some basic concepts and results which are relevant for this article.

**Definition 2.1.** [2] A collection $G$ of nonempty subsets of a set $X$ is called a grill if

(i) $A \in G$ and $A \subseteq B \subseteq X$ implies that $B \in G$ and

(ii) $A \cup B \in G$ $(A, B \subseteq X)$ implies that $A \in G$ or $B \in G$.

**Definition 2.2.** [10] Let $G$ be a grill on a topological space $(X, \tau)$. A cover $\{U_\alpha : \alpha \in \Lambda\}$ of $X$ is said to be a $G$ - cover if there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $X \setminus \cup_{\alpha\in\Lambda 0} U_\alpha \notin G$.

**Definition 2.3.** [11] Let $X$ be an universal set. A neutrosophic set $A$ in $X$ is a set contains triplet having truthness, falseness and indeterminacy membership values that can be characterized independently, denoted by $T_A$, $F_A$, $I_A$ in [0,1]. The neutrosophic set is denoted as follows:

$$A = \{(x, T_A(x), F_A(x), I_A(x)) : x\in X$$

and $T_A(x), F_A(x), I_A(x) \in [0,1]\}$ with the condition

$$0 \leq T_A(x) + F_A(x) + I_A(x) \leq 3.$$

The null and full NSs on a nonempty set $X$ are denoted by $0_N$ and $1_N$, defined as follows:

**Definition 2.4.** [11] The neutrosophic sets $0_N$ and $1_N$ in $X$ are represented as follows:

(*i*) $0_N = \{<x, 0, 0, 1>: x \in X\}$.

(*ii*) $0_N = \{<x, 0, 1, 1 >: x \in X\}$.

(*iii*) $0_N = \{<x, 0, 1, 0 >: x \in X\}$.

(*iv*) $0_N = \{<x, 0, 0, 0 >: x \in X\}$.

(*v*) $1_N = \{<x, 1, 0, 0 >: x \in X\}$.

($vi$) $1_N = \{<x, 1, 0, 1>: x \in X\}$.

($vii$) $1_N = \{<x, 1, 1, 0>: x \in X\}$.

($viii$) $1_N = \{<x, 1, 1, 1>: x \in X\}$.

Clearly**,** $0_N \subseteq 1_N$. We have, for any neutrosophic set $A$, $0_N \subseteq A \subseteq 1_N$.

***Definition 2.5.*** [11] Let $X$ be a non-empty set and $T$ be the collection of neutrosophic subsets of $X$. Then $T$ is said to be a neutrosophic topology (in brief, NT) on $X$ if the following properties holds:

($i$) $0_N$, $1_N \in T$.

($ii$) $U_1, U_2 \in \mathrm{T} \Rightarrow U_1 \cap \mathrm{U}_2 \in T$.

($iii$) $\cup_{i \in \Delta} u_i \in T$, for every $\{u_i: i \in \Delta\} \subseteq T$.

Then ($X$, $T$) is called a neutrosophic topological space (in brief, NTS) over $X$. The members of $T$ are called neutrosophic open sets (in brief, NOS). A neutrosophic set $D$ is called neutrosophic closed set (in brief, NCS) if and only if $D^c$ is a neutrosophic open set.

**Definition 2.6.** [11] Let ($X$, $T$) be a NTS and $U$ be a NS in $X$. Then the neutrosophic interior (in short, $N_{int}$) and neutrosophic closure (in short, $N_{cl}$) of $U$ are defined by

$N_{int}(U) = \cup \{E : E \text{ is a } NOS \text{ in } X \text{ and } E \subseteq U\}$,

$N_{cl}(U) = \cap \{F : F \text{ is a } NCS \text{ in } X \text{ and } U \subseteq F\}$.

**Remark 2.7.** [9] Clearly $N_{int}(U)$ is the largest neutrosophic open set over $X$ which is contained in $U$ and $N_{cl}(U)$ is the smallest neutrosophic closed set over $X$ which contains $U$.

**Proposition 2.8.** [11] For any NS $B$ in ($X$, $T$), we have

($i$) $N_{int}(B^c) = (N_{cl}(B))^c$.

($ii$) $N_{cl}(B^c) = (N_{int}(B))^c$.

**Definition 2.9.** [11] Let $X$ be a neutrosophic topological space. A subcollection $G$ (not containing $0_N$) of P($X$) is called a grill on $X$ if $G$ satisfies the following conditions:

($i$) $A \in G$ and $A \subseteq B$ implies $B \in G$.

($ii$) $A$, $B \subseteq X$ and $A \cup B \in G$ implies that $A \in G$ or $B \in G$.

**Definition 2.10.** [5] Let $G$ be a grill on a neutrosophic topological space ($X$, $T$). A cover $\{U_\alpha : \alpha \in \Lambda\}$ of $X$ is said to be a neutrosophic $G$-cover if there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $X \setminus \bigcup_{\alpha \in \Lambda_0} U_\alpha \notin G$.

**Definition 2.11.** [5] Let $G$ be a grill on a neutrosophic topological space ($X$, $T$). Then ($X$, $T$) is said to be neutrosophic compact with respect to the grill $G$ or simply neutrosophic $G$-compact if every open cover of $X$ is a neutrosophic $G$-cover.

## NEUTROSOPHIC $\rho G$-COMPACT SPACES

In this section, we introduce the notion of $\rho G$ - compact space and investigate some of its basic properties in neutrosophic topological space.

**Definition 3.1.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G* and *A* be a neutrosophic subset of *X*. Then *A* is said to be neutrosophic ρ*G*-compact if for every family $\{V_\alpha\}_{\alpha\in\Lambda}$ of neutrosophic open subsets of *X*, if $A - \cup_{\alpha\in\Lambda} V_\alpha \notin G$, then there exists finite subset $\Lambda_0$ of $\Lambda$ such that $A - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$. The grill neutrosophic topological space (*X*, *T*, *G*) is said to be neutrosophic ρ*G*-compact if *X* is neutrosophic ρ*G*- compact.

It is obvious that if (*X*, *T*, *G*) is neutrosophic ρ*G*-compact then (*X*, *T*, *G*) is neutrosophic *G*-compact.

**Definition 3.2.** A neutrosophic subset *A* of a grill neutrosophic topological space (*X*, *T*, *G*) is said to be neutrosophic *Gg*-closed if for every $U \in T$, if $A - U \notin G$, then $\mathrm{cl}(A) \subseteq U$.

It is easy to check if *A* is neutrosophic *Gg*-closed then *A* is neutrosophic *g*-closed.

**Theorem 3.3.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G* and *B* be a base for *T*. Then ( *X*, *T*, *G* ) is neutrosophic ρ*G*-compact if and only if all family $\{V_\alpha\}_{\alpha\in\Lambda}$ of neutrosophic open sets in *B*, if $X - \cup_{\alpha\in\Lambda} V_\alpha \notin G$, then there exists finite subset $\Lambda_0$ of $\Lambda$ such that $X - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$.

**Proof.** Necessary part is obvious.

For sufficiency, let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of non - empty neutrosophic open subsets of *X* such that $X - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. For all $\alpha \in \Lambda$, there exists a family $\{W_{\alpha\beta} \in \Lambda_\alpha\}$ of elements in *B* such that $V_\alpha = \bigcup_{\beta\in\Lambda_\alpha} W_{\alpha\beta}$. Given that $X - \cup_{\alpha\in\Lambda} \bigcup_{\beta\in\Lambda_\alpha} W_{\alpha\beta} \notin G$ and (*X*, *T*, *G*) is neutrosophic ρ*G* - compact, there exists $W_{\alpha_1\beta_1}$, $W_{\alpha_2\beta_2}$, ... ..., $W_{\alpha_n\beta_n}$ such that $X - \bigcup_{i=1}^{n} W_{\alpha_i\beta_i} \notin G$. But $X - \bigcup_{i=1}^{n} V_{\alpha_i} \subseteq X - \bigcup_{i=1}^{n} W_{\alpha_i\beta_i}$ and so $X - \bigcup_{i=1}^{n} V_{\alpha_i} \notin G$.

**Theorem 3.4.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G*. Then the following are equivalent:

(i) (*X*, *T*, *G*) is a neutrosophic ρ*G* - compact.

(ii) (*X*, $T_G$, *G*) is a neutrosophic ρ*G* - compact.

(iii) For any family $\{F_\alpha\}_{\alpha\in\Lambda}$of neutrosophicclosd subsets of X, if $\bigcap_{\alpha\in\Lambda} F_\alpha \notin G$, then there exists a finite subset $\Lambda_0$ of Λ, such that $\bigcap_{\alpha\in\Lambda_0} F_\alpha \notin G$.

**Proof.** (i) ⇔(iii). This is easy to prove.

(ii) ⇔ (i). Obvious, since $T_G$ is finer than *T*.

(i) ⇔ (ii). We know that $B = \{U - A : U \in T \text{ and } A \notin G\}$ is a base for the neutrosophic topology $T_G$. Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of non-empty neutrosophic open subsets in *B* such that $X - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. For each $\alpha \in \Lambda$, there exists $W_\alpha \in T$ and $A_\alpha \notin G$ such that $V_\alpha = W_\alpha - A_\alpha$. Since $X - \cup_{\alpha\in\Lambda} W_\alpha \notin G$, then there exists a finite subset $\Lambda_0$ of Λ such that $X - \cup_{\alpha\in\Lambda_0} W_\alpha \notin G$. Now $(X - \cup_{\alpha\in\Lambda_0} V_\alpha) \subseteq (X - \cup_{\alpha\in\Lambda_0} W_\alpha) \cup \left(\cup_{\alpha\in\Lambda_0} A_\alpha\right) \notin G$ and so $X - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$.

Therefore (*X*, $T_G$, *G*) is a neutrosophic ρ*G* - compact.

**Theorem 3.5.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G* and $A \subseteq X$ be neutrosophic *Gg* - closed set. Then *A* is neutrosophic ρ*G* - compact.

**Proof.** Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of neutrosophic open subsets in *A* such that $A - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Since *A* is $Gg$ - closed, $cl(A) \subseteq \cup_{\alpha\in\Lambda} V_\alpha$. Then $X = (X - cl(A))) \cup \cup_{\alpha\in\Lambda} V_\alpha$ and so $X - [(X - cl(A)) \cup \cup_{\alpha\in\Lambda} V_\alpha] = 0_N \notin G$. Given that *X* is neutrosophic ρ*G* - compact. So there exists a finite subset $\Lambda_0$ of Λ such that $X - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$. Then $X - [(X - cl(A)) \cup \cup_{\alpha\in\Lambda_0} V_\alpha] \notin G$. But $X - [(X - cl(A)) \cup \cup_{\alpha\in\Lambda_0} V_\alpha] = \text{cl(A)} - \cup_{\alpha\in\Lambda_0} V_\alpha$ and since $A - \cup_{\alpha\in\Lambda_0} V_\alpha \subseteq \text{cl(A)} - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$, we have $A - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$ implies A is neutrosophic ρ*G* - compact.

**Theorem 3.6.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G*. If *A* and *B* are two neutrosophic ρ*G*-compact subsets of the space (*X*, *T*, *G*), then $A \cup B$ is neutrosophic ρ*G*-compact.

**Proof.** Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of neutrosophc open subsets of *X* such that $(A\cup B) - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Since $A - \cup_{\alpha\in\Lambda} V_\alpha \subseteq (A\cup B) - \cup_{\alpha\in\Lambda} V_\alpha$ and $B - \cup_{\alpha\in\Lambda} V_\alpha \subseteq (A\cup B) - \cup_{\alpha\in\Lambda} V_\alpha$, therefore $A - \cup_{\alpha\in\Lambda} V_\alpha \notin G$ and $B - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Hence there exist finite subsets $\Lambda_1$ and $\Lambda_2$ of Λ with $A - \cup_{\alpha\in\Lambda_1} V_\alpha \notin G$ and $B - \cup_{\alpha\in\Lambda_2} V_\alpha \notin G$. This implies that $A - \cup_{\alpha\in\Lambda_1\cup\Lambda_2} V_\alpha \notin G$ and $B - \cup_{\alpha\in\Lambda_1\cup\Lambda_2} V_\alpha \notin G$ and so $(A \cup B) - \cup_{\alpha\in\Lambda_1\cup\Lambda_2} V_\alpha \notin G$.

**Theorem 3.7.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G* and $A \subseteq X$. Suppose that for all $U \in T$, if $A - U \notin G$, then there exist $B \subseteq X$ such that *B* is neutrosophic ρ*G*-compact, $A \subseteq B$and $B - U \notin G$. Then *A* is neutrosophic ρ*G*-compact.

**Proof.** Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of neutrosophc open subsets of *X* such that $A - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. There exists $B \subseteq X$ such that *B* is neutrosophic ρ*G* - compact, $A \subseteq B$ and $B - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Then there exists a finite subset $\Lambda_0$ of Λ such that $B - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$. Since $A - \cup_{\alpha\in\Lambda_0} V_\alpha \subseteq B - \cup_{\alpha\in\Lambda_0} V_\alpha$, we have $A - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$.

**Theorem 3.8.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G* and $A \subseteq B \subseteq X$, $B \subseteq cl(A)$ and *A* be neutrosophic *Gg*-closed. Then *A* is neutrosophic ρ*G*-compact if and only if *B* is neutrosophic ρ*G* - compact.

**Proof.** Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of neutrosophc open subsets of *X* such that $B - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Then $A - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Given that *A* is neutrosophic ρ*G* - compact. So there exists a finite subset $\Lambda_0$ of Λ such that $A - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$. Since *A* is neutrosophic *Gg*-closed, $cl(A) \subseteq \cup_{\alpha\in\Lambda_0} V_\alpha$ and so $cl(A) - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$. This implies $A - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$.

Conversely, let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of neutrosophc open subsets of *X* such that $A - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Given that *A* is neutrosophic *Gg* - closed, so $cl(A) - \cup_{\alpha\in\Lambda} V_\alpha = 0_N \notin G$. This implies $B - \cup_{\alpha\in\Lambda} V_\alpha \notin G$. Since *B* is neutrosophic ρ*G*-compact, there exists a finite subset $\Lambda_0$ of Λ such that $B - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$. Hence $A - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$.

**Theorem 3.9.** Let (*X*, *T*, *G*) be a neutrosophic topological space with a grill *G* and (*X*, *T*) be Hausdorff. If *A* is neutrosophic ρ*G*-compact subset of *X*, then *A* is neutrosophic closed in $(X, T_G)$.

**Proof.** Let *A* be neutrosophic ρ*G*-compact subset of a Hausdorff grill neutrosophic topological space (*X*, *T*, *G*). Let $x \notin A$. Then for each $y \in A$, there exist there exist neighbourhoods $U_x$ and $V_y$ of $x$ and $y$ respectively such that $U_x \cap V_y = 0_N$. Note that $x \notin cl(V_y)$. Now $\{V_y : y \in A\}$ is a neutrosophic $T$ - open cover of $A$. Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of neutrosophc open subsets of $X$. Given that $A$ is neutrosophic ρ$G$-compact. So there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $A - \cup_{\alpha\in\Lambda_0} V_\alpha \notin G$.As $x \notin$ cl($V_y$), so for each $y$ implies $x \notin \cup_{y\in\Lambda_0} cl(V_y) = cl(\cup_{y\in\Lambda_0} V_y)$. Let $U = X - cl(\cup_{y\in\Lambda_0} V_y)$ and $J = A - cl(\cup_{y\in\Lambda_0} V_y) \subseteq A - (\cup_{y\in\Lambda_0} V_y) = G_1$ (say) with $G_1 \notin G$. Then $U - J \in T_G(x)$ and $(U - J) \cap A = 0_N \Rightarrow x \notin 0_N(A)$. Hence $0_N(A)$ $A$, so $A$ is neutrosophic closed in $(X, T_G)$.

**Theorem 3.10.** Let $(X, T, G)$ be a neutrosophic topological space with respect to a grill $G$ and $(X, T, G)$ is neutrosophic ρ$G$-compact. Let $(X, T)$ *and* $(Y, T_1)$ be two neutrosophic topological spaces. If $f : (X, T) \to (Y, T_1)$ is a neutrosophic continuous function and $H = \{B \subseteq Y : f^{-1}(B) \in G\}$, then the following hold:

(i) $H$ is a grill on $Y$.

(ii) $(Y, T_1, H)$ is neutrosophic ρ$G$ - compact.

**Proof.** (i) Suppose that $A \subseteq B \subseteq Y$ and $A \in H$. Since $f^{-1}(A) \subseteq f^{-1}(B) \in G$, then $f^{-1}(B) \in G$ and so $B \in H$. Now if $A \notin H$ and $B \notin H$ then $f^{-1}(A) \notin G$ and $f^{-1}(A) \notin G$ and then $f^{-1}(A \cup B) = f^{-1}(A) \cup (f^{-1}(B) \notin G$. This implies $A \cup B \notin G$.

(ii) Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of non - empty neutrosophic open subsets of $Y$ such that $Y - \bigcup_{\alpha\in\Lambda} V_\alpha \notin H$. Since $X - \bigcup_{\alpha\in\Lambda} f^{-1}(V_\alpha) = f^{-1}(Y - \bigcup_{\alpha\in\Lambda} V_\alpha) \notin G$. Given that $(X, T, G)$ is neutrosophic ρ$G$ - compact. Hence there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $f^{-1}(Y - \bigcup_{\alpha\in\Lambda_0} V_\alpha) = X - \bigcup_{\alpha\in\Lambda_0} f^{-1}(V_\alpha) \in G$ . Thus $Y - \bigcup_{\alpha\in\Lambda_0} V_\alpha \in H$.

**Theorem 3.11.** Let $(X, T, G)$ be a neutrosophic topological space with respect to a grill $G$ such that $(X, T, G)$ is neutrosophic ρ$G$ - compact. Let $(X, T)$ *and* $(Y, T_1)$ be two neutrosophic topological spaces. If $f : (X, T) \to (Y, T_1)$ is a neutrosophic continuous bijective function, then $(Y, T_1, f(G))$ is neutrosophic ρ$G$ - compact.

**Proof.** Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of non - empty neutrosophic open subsets of $Y$ such that $Y - \bigcup_{\alpha\in\Lambda} V_\alpha \in f(G)$. There exists $H \in G$ with $Y - \bigcup_{\alpha\in\Lambda} V_\alpha = f(H)$. Then $H = f^{-1}(f(H))$ and $X - \bigcup_{\alpha\in\Lambda} f^{-1}(V_\alpha) \notin G$. Given that $(X, T, G)$ is neutrosophic σ$G$ - compact. Therefore there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $f^{-1}(Y - \bigcup_{\alpha\in\Lambda_0} V_\alpha) = X - \bigcup_{\alpha\in\Lambda_0} f^{-1}(V_\alpha) \notin G$ . Since $f$ is bijective and $K$ is a grill on $Y$, then the set $f^{-1}(K) = \{f^{-1}(J) : J \in K\}$ is a grill on $X$.

**Theorem 3.12.** Let $(Y, T_1, H)$ be neutrosophic ρ$H$ - compact and $(X, T)$ and $(Y, T_1)$ be two neutrosophic topological spaces. Let $f : (X, T) \to (Y, T_1)$ be a neutrosophic continuous function. Then $(X, T, f^{-1}(H))$ is neutrosophic ρ$f^{-1}(H)$ - compact.

**Proof.** Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of non - empty neutrosophic open subsets of $X$ such that $X - \bigcup_{\alpha\in\Lambda} V_\alpha \notin f^{-1}(H)$. There exists $J \in H$ with $X - \bigcup_{\alpha\in\Lambda} V_\alpha = f^{-1}(J)$. Then $Y - \bigcup_{\alpha\in\Lambda} f(V_\alpha) = f(f^{-1}(J)) = J \in H$. Given that $(Y, T_1, H)$ is neutrosophic ρ$H$ - compact. Therefore there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $f(X - \bigcup_{\alpha\in\Lambda_0} V_\alpha) = Y - f(\bigcup_{\alpha\in\Lambda_0} V_\alpha) \notin H$. This implies that $X - \bigcup_{\alpha\in\Lambda_0} V_\alpha \notin f^{-1}(H)$.

## NEUTROSOPHIC $\sigma G$-COMPACT SPACE

In this section, we introduce the notion of $\sigma G$ - compact space and investigate some basic properties in neutrosophic topological space.

**Definition 4.1.** Let $(X, T, G)$ be a neutrosophic topological space with respect to a grill $G$ and $A \subseteq X$. Then $A$ is said to be neutrosophic $\sigma G$-compact space if for every family $\{V_\alpha\}_{\alpha \in \Lambda}$ of neutrosophic open subsets of $X$ if $A - \bigcup_{\alpha \in \Lambda} V_\alpha \notin G$, then there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $A \subseteq \bigcup_{\alpha \in \Lambda_0} V_\alpha$. The space $(X, T, G)$ is said to be neutrosophic $\sigma G$-compact space if $X$ is neutrosophic $\sigma G$-compact space.

We note that if $(X, T, G)$ is a neutrosophic topological grill space and $(X, T_G, G)$ is neutrosophic $\sigma G$-compact, then $(X, T, G)$ is neutrosophic $\sigma G$-compact. Also $(X, T, G)$ is neutrosophic $\sigma G$-compact if and only if for any family $\{F_\alpha\}_{\alpha \in \Lambda}$ of neutrosophic closed subsets of $X$, if $\bigcap_{\alpha \in \Lambda} F_\alpha \notin G$, then there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $\bigcap_{\alpha \in \Lambda_0} F_\alpha = 0_N$.

It is clear that if $(X, T, G)$ is neutrosophic $\sigma G$-compact then $(X, T, G)$ is neutrosophic $\rho G$-compact and $(X, T)$ is neutrosophic compact.

**Theorem 4.2.** Let $(X, T, G)$ be a neutrosophic topological space with respect to a grill $G$ and $B$ a base for $T$. Then $(X, T, G)$ is neutrosophic $\sigma G$-compact if and only if for every family $\{V_\alpha\}_{\alpha \in \Lambda}$ of neutrosophic open sets in $B$ , if $X - \bigcup_{\alpha \in \Lambda} V_\alpha \notin G$, then there exists, finite subset $\Lambda_0$ of $\Lambda$ such that $X = \bigcup_{\alpha \in \Lambda_0} V_\alpha$.

**Proof.** Necessary part is easy to proof. Conversely, let $\{V_\alpha\}_{\alpha \in \Lambda}$ be a family of non - empty neutrosophic open subsets of $X$ such that $X - \bigcup_{\alpha \in \Lambda} V_\alpha \notin G$. For all $\alpha \in \Lambda$, there exists a family $\{W_{\alpha\beta} : \beta \in \Lambda_\alpha\}$ of elements in $B$ such that $V_\alpha = \bigcup_{\beta \in \Lambda} W_{\alpha\beta}$. Given that $X - \bigcup_{\alpha \in \Lambda} \bigcup_{\beta \in \Lambda_\alpha} W_{\alpha\beta} \notin G$ and $(X, T, G)$ is neutrosophic $\sigma G$-compact, there exist $W_{\alpha_1\beta_1}$, $W_{\alpha_2\beta_2}$,………, $W_{\alpha_r\beta_r}$ such that $X = \bigcup_{i=1}^{r} W_{\alpha_i\beta_i}$. But $X = \bigcup_{i=1}^{r} W_{\alpha_i\beta_i} \subseteq \bigcup_{i=1}^{r} V_{\alpha_i} \subseteq X$ and so $X = \bigcup_{i=1}^{r} V_{\alpha_i}$.

Now we investigate the behavior of subspaces of a neutrosophic $\sigma G$-compact space.

**Theorem 4.3.** Let $(X, T, G)$ be a neutrosophic topological space with respect to a grill $G$. Let $(X, T, G)$ be a neutrosophic $\sigma G$-compact space and $A \subseteq X$ be $Gg$-closed, then $A$ is neutrosophic $\sigma G$-compact.

**Proof.** Let $\{V_\alpha\}_{\alpha \in \Lambda}$ be a family of non - empty neutrosophic open subsets of $X$ such that $A - \bigcup_{\alpha \in \Lambda} V_\alpha \notin G$. Since $A$ is neutrosophic $Gg$ - closed, $cl(A) \subseteq \bigcup_{\alpha \in \Lambda} V_\alpha$. Then $X = (X - cl(A)) \cup (\bigcup_{\alpha \in \Lambda} V_\alpha)$ and so $X - [(X - cl(A)) \cup (\bigcup_{\alpha \in \Lambda} V_\alpha)] = 0_N \notin G$. Given that $X$ is neutrosophic $\sigma G$-compact. Then there exists, finite subset $\Lambda_0$ of $\Lambda$ such that $X = (X - cl(A)) \cup (\bigcup_{\alpha \in \Lambda_0} V_\alpha)$. Then $A = A \cap [(X - cl(A)) \cup (\bigcup_{\alpha \in \Lambda_0} V_\alpha)] = [A \cap \bigcup_{\alpha \in \Lambda_0} V_\alpha] \subseteq \bigcup_{\alpha \in \Lambda_0} V_\alpha$.

**Theorem 4.4.** Let $(X, T, G)$ be a neutrosophic topological space with respect to a grill $G$. If $A$ and $B$ are neutrosophic $\sigma G$-compact subsets of the space $(X, T, G)$, $A \cup B$ is neutrosophic $\sigma G$-compact.

**Proof.** This proof is similar to that of Theorem 2.8.

**Theorem 4.5.** Let (*X*, *T*, *G*) be a neutrosophic topological space with respect to a grill *G*. If *A* and *B* are neutrosophic σ*G*-compact subsets of the space (*X*, *T*, *G*), A ∪ *B* is neutrosophic σ*G*-compact.

**Proof.** This proof is similar to that of Theorem 3.6.

**Theorem 4.6.** Let (*X*, *T*, *G*) be a neutrosophic topological space with respect to a grill *G* and $A \subseteq B \subseteq X$ and $B \subseteq cl(A)$. Then the following hold:

(i) If *A* is neutrosophic *g*-closed and neutrosophic σ*G*-compact, then *B* is neutrosophic σ*G*-compact.

(ii) If *A* is neutrosophic *Gg*-closed and *B* is neutrosophic σ*G*-compact, then *A* is neutrosophic σ*G*-compact.

**Proof.** (i) Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of non - empty neutrosophic open subsets of *X* such that $B - \bigcup_{\alpha\in\Lambda} V_\alpha \notin G$. Then $A - \bigcup_{\alpha\in\Lambda} V_\alpha \notin G$. Given that *A* is neutrosophic σ*G*-compact. So there exists a finite subset $\Lambda_0$ of Λ such that $A \subseteq \bigcup_{\alpha\in\Lambda_0} V_\alpha$. Since *A* is neutrosophic *g*-closed, $cl(A) \subseteq \bigcup_{\alpha\in\Lambda_0} V_\alpha$ and this implies $B \subseteq \bigcup_{\alpha\in\Lambda_0} V_\alpha$.

(ii) Let $\{V_\alpha\}\alpha \in \Lambda$ be a family of non-empty neutrosophic open subsets of *X* such that $A - \bigcup_{\alpha\in\Lambda} V_\alpha \notin G$. Given that *A* is neutrosophic *Gg*-closed. So $cl(A) - \bigcup_{\alpha\in\Lambda} V_\alpha = 0_N \notin G$. This implies $B - \bigcup_{\alpha\in\Lambda} V_\alpha \notin G$. Since *B* is neutrosophic σ*G*-compact, therefore there exists a finite subset $\Lambda_0$ of Λ such that $B \subseteq \bigcup_{\alpha\in\Lambda_0} V_\alpha$. Hence $A \subseteq \bigcup_{\alpha\in\Lambda_0} V_\alpha$.

**Theorem 4.7.** Let (*X*, *T*, *G*) be a neutrosophic topological space with respect to a grill *G* such that (*X*, *T*) is Hausdorff. If *A* is neutrosophic σ*G*-compact subset of *X*, then *A* is neutrosophic closed in $(X, T_G)$.

**Proof.** This is an easy consequence of Theorem 3.9.

**Theorem 4.8.** Let (*X*, *T*, *G*) be a neutrosophic topological space with respect to a grill *G* such that (*X*, *T*, *G*) is neutrosophic σ*G*-compact. If $f : (X, T) \to (Y, T_1)$ is a neutrosophic continuous subjective function and if $H = \{B \subseteq Y : f^{-1}(B) \notin G\}$, then $(Y, T_1, H)$ is neutrosophic σ*H*-compact.

**Proof.** In Theorem 3.10, we prove that *H* is a grill on *Y*. Let $\{V_\alpha\}_{\alpha\in\Lambda}$be a family of non - empty neutrosophic open subsets of *Y* such that $Y - \bigcup_{\alpha\in\Lambda} V_\alpha \notin G$. Since $X - \bigcup_{\alpha\in\Lambda} f^{-1}(V_\alpha) = f^{-1}(Y - \bigcup_{\alpha\in\Lambda} V_\alpha) \notin G$ and (*X*, *T*, *G*) is neutrosophic σG - compact, therefore there exists a finite subset $\Lambda_0$ of Λ such that $X = \bigcup_{\alpha\in\Lambda_0} f^{-1}(V_\alpha)$. Given that *f* is surjective. So we have $Y = \bigcup_{\alpha\in\Lambda_0} V_\alpha$.

**Theorem 4.9.** Let (*X*, *T*, *G*) be a neutrosophic topological space with respect to a grill *G* such that (*X*, *T*, *G*) is neutrosophic σ*G* - compact. Let (*X*, *T*) *and* $(Y, T_1)$ be two neutrosophic topological spaces. If $f : (X, T) \to (Y, T_1)$ is a neutrosophic continuous bijective, then $(Y, T_1, f(G))$ is neutrosophic σ*f*(*G*) - compact.

**Proof.** Let $\{V_\alpha\}\alpha \in \Lambda$ be a family of non - empty neutrosophic open subsets of *Y* such that $Y - \bigcup_{\alpha\in\Lambda} V_\alpha \notin f(G)$. There exists $H \in G$ with $Y - \bigcup_{\alpha\in\Lambda} V_\alpha \neq f(H)$. Then $H = f^{-1}(f(H))$ and $X - \bigcup_{\alpha\in\Lambda} f^{-1}(V_\alpha) \notin G$. Given that (*X*, *T*, *G*) is neutrosophic σ*G* - compact. Therefore there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $X = \bigcup_{\alpha\in\Lambda_0} f^{-1}(V_\alpha)$. Since $f$ is bijective, so we have $Y = \bigcup_{\alpha\in\Lambda_0} V_\alpha$.

**Theorem 4.10.** Let $(X, T)$ and $(Y, T_1)$ be two neutrosophic topological spaces and $f : (X, T) \to (Y, T_1)$ be a neutrosophic bijective and open function. Also $(Y, T_1, H)$ is neutrosophic $\sigma H$ - compact space. Then $(X, T, f^{-1}(H))$ is neutrosophic $\sigma f^{-1}$(H)-compact.

**Proof.** Let $\{V_\alpha\}_{\alpha\in\Lambda}$ be a family of non-empty neutrosophic open subsets of $X$ such that $X - \bigcup_{\alpha\in\Lambda} V_\alpha \notin f^{-1}(H)$. There exists $K \in H$ with $X - \bigcup_{\alpha\in\Lambda} V_\alpha = f^{-1}(K)$. Then $Y - \bigcup_{\alpha\in\Lambda} f(V_\alpha) = f(f^{-1}(K)) = K \notin H$. Given that $(Y, T_1, H)$ is neutrosophic $\sigma H$ - compact. Therefore there exists a finite subset $\Lambda_0$ of $\Lambda$ such that $Y = \bigcup_{\alpha\in\Lambda_0} f(V_\alpha)$. This implies that $X = \bigcup_{\alpha\in\Lambda_0} V_\alpha$.

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