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FUZZY S_β - OPEN SETS AND FUZZY S_β - OPERATIONS

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In this paper we have obtained the Lagrange Space with a Special (γ, β)-Metric (1.1), where γ is a cubic metric and β is a 1-form metric. We have also calculated the fundamental tensor, its inverse, Euler-Lagrange equations, semispray coefficient, the canonical nonlinear connections and some important properties for this Lagrange space.

In this paper a new class of fuzzy sets, called fuzzy S_β – open sets is to be introduced. Some of their basic properties are also to be investigated in fuzzy topological spaces. The notion of fuzzy S_β – operations is also to be introduced. Some of its basic properties are also to be studied in fuzzy topological spaces.

Key words: Fuzzy set, fuzzy β - open set, fuzzy topological space, fuzzy S_β – open set and fuzzy S_β – operations.

Subject classification. 54A40

Introduction

Zadeh [15] introduced the notion of fuzzy sets in 1965. Thereafter realizing the potentiality, researchers investigated on fuzzy sets in different aspects and successfully applied it for further investigations in all the branches of science and technology. The notion of fuzzy topology was introduced by Chang [4] in 1968. Azad [3] introduced the concepts of fuzzy semiopen sets, fuzzy semicontinuous mappings and weakly fuzzy continuity in fuzzy topological spaces. S. S. Thakur and S. Singh [12] introduced the concepts of fuzzy semi preopen sets and fuzzy semi precontinuity in fuzzy topological spaces. Abd El- Monsef *et al* [1] introduced the concepts of β -open sets and β - continuous functions in general topology and Alla [2] introduced these concepts in fuzzy setting. Khalaf and Ahmed [6] introduced and studied a new class of semiopen sets, called S_β - open sets, they then introduced and investigated S_β - continuous functions in general topological spaces. Besides these, different researchers [5, 7, 8, 9, 11, 13, 14] have contributed a lots to the fuzzy set theory. The findings from those works lead to this paper with the aims to define fuzzy S_β - open sets and fuzzy S_β – operations and to study their basic properties in fuzzy setting. In section 2, the different concepts of known fuzzy sets and fuzzy mappings would be mentioned as ready reference. In section 3, the concept of fuzzy S_β – open sets would be introduced and some of their basic properties would be investigated in fuzzy topological spaces. In section 4, fuzzy S_β – operators would be introduced and studied.

PRELIMINARIES

IIn this section, some preliminary results and definitions have been mentioned as ready reference.

Definition 2.1. [15] Let A and B be two fuzzy sets in a crisp set X and the membership functions of them be μ_A and μ_B respectively. Then

(i) A is equal to B, *i.e.*, $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, for all $x \in X$,

(ii) A is called a subset of B if and only if $\mu_A(x) \leq \mu_B(x)$, for all $x \in X$,

(iii) the Union of two fuzzy sets A and B is denoted by $A \vee B$ and its membership function is given by $\mu_{A \vee B} = \max(\mu_A, \mu_B)$,

(iv) the Intersection of two fuzzy sets A and B is denoted by $A \wedge B$ and its membership function is given by $\mu_{A \wedge B} = \min(\mu_A, \mu_B)$,

(v) the Complement of a fuzzy set A is defined as the negation of the specified membership function. Symbolically it can be written as $\mu^c{}_A = 1 - \mu_A$.

Definition 2.2. [10] A fuzzy point x_p in X is a fuzzy set in X defined by

$$x_p(y) = p \ (0 < p \leq 1), \text{ for } y = x$$
$$= 0 \qquad , \text{ for } y \neq x \ (y \in X),$$

x and p are respectively the support and the value of x_p.

A fuzzy point x_p is said to belong to a fuzzy set A of X if and only if $p \leq A(x)$. A fuzzy set A in X is the union of all fuzzy points which belong to A.

Definition 2.3. [15] Suppose τ is a family of fuzzy subsets in X which satisfies the following axioms :

(i) $0_X, 1_X \in \tau$.

(ii) If $A, B \in \tau$, then $A \wedge B \in \tau$.

(iii) If $A_j \in \tau$ for all j from the index set J, $\vee_{j \in J} A_j \in \tau$.

Then τ is called a fuzzy topology for X and the pair (X, τ) is called a fuzzy topological space. The elements of τ are called fuzzy open subsets. The complement of each member in τ is defined as a fuzzy closed set in X (with respect to τ) or simply a fuzzy closed set in X.

Throughout the paper, the spaces X and Y always represent fuzzy topological spaces (X, τ) and (Y, σ) respectively.

Definition 2.4. A fuzzy set A in a fuzzy topological space (fts, in short) X is called

(i) [3] fuzzy semiopen set if $A \leq \text{clint}A$,

(ii) [11] fuzzy preopen set if $A \leq \text{intcl}A$,

(iii) [2] fuzzy β - open set if $A \leq \text{clintcl}A$, equivalently, if there exists a fuzzy preopen set B such that $B \leq A \leq B$.

From definition it follows that each fuzzy semiopen and fuzzy preopen set implies fuzzy β - open set.

Definition 2.5. [3, 5]. Let A be a fuzzy set in a fts X, the fuzzy preclosure (resp. fuzzy semiclosure, fuzzy preinterior and fuzzy semiinterior) of A denoted by pcl$A$ (resp. scl$A$, pint$A$ and sint$A$) are defined as follows :

$\text{pcl}A\ (\text{scl}A) = \wedge\{B : A \leq B, B \text{ is fuzzy preclosed (fuzzy semiclosed)}\}$,

$\text{pint}A\ (\text{sint}A) = \vee\{B : A \geq B, B \text{ is fuzzy preopen (fuzzy semiopen)}\}$.

Definition 2.6. [2] Let A be a fuzzy set in a fts X. The fuzzy β - closure (βcl) and β - interior (β int) of A are defined as follows :

$\beta\text{cl}A = \wedge\{B : A \leq B, B \text{ is fuzzy } \beta \text{ - closed}\}$,

$\beta\text{int}A = \vee\{B : A \geq B, B \text{ is fuzzy } \beta \text{ - open}\}$.

It is obvious that $\beta\text{cl}(A)^c = (\beta\text{int}A)^c$ and $\beta\text{int}(A)^c = (\beta\text{cl}A)^c$.

Definition 2.7. [2] A function $f : X \to Y$ is said to be fuzzy β - continuous (resp. M β - continuous) if the inverse image of every fuzzy open (resp. fuzzy β - open) set in Y is fuzzy β - open (resp. fuzzy β - open) set in X.

FUZZY S_β - OPEN SETS

In this section, the concept of fuzzy S_β - open sets in fuzzy topological spaces would be introduced and some basic properties of this set in fuzzy setting would be investigated.

Definition 3.1. A fuzzy semiopen subset A of a fuzzy topological space (X, τ) is said to be fuzzy S_β - open if for each fuzzy point $x_p \in A$ there exists a fuzzy β - closed set F such that $x_p \in F \leq A$. A fuzzy subset B of a fuzzy topological space X is fuzzy S_β - closed if its complement is fuzzy S_β - open.

The family of all fuzzy S_β - open subsets of X is denoted by $S_\beta O(X)$.

Proposition 3.2. A fuzzy subset A of a fuzzy topological space (X, τ) is fuzzy S_β – open set if and only if A is fuzzy semiopen and it is a union of fuzzy β - closed sets.

Proof. Obvious.

Theorem 3.3. The union of an arbitrary collection of fuzzy S_β – open sets in a fuzzy topological space (X, τ) is also fuzzy S_β – open set.

Proof. Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy S_β – open sets in a fuzzy topological space (X, τ). It is required to show that $\cup_{\alpha \in \Lambda} A_\alpha$ is a fuzzy S_β – open set. The union of an arbitrary collection of fuzzy semiopen sets is fuzzy semiopen. Suppose that $x_p \in \cup_{\alpha \in \Lambda} A_\alpha$. This implies that there exists $\alpha_0 \in \Lambda$ such that $x_p \in A_{\alpha_0}$ and since A_{α_0} is a fuzzy S_β – open set, so there exists a fuzzy β - closed set F in X such that $x_p \in F \leq A_{\alpha_0} \leq \cup_{\alpha \in \Lambda} A_\alpha$. Therefore $\cup_{\alpha \in \Lambda} A_\alpha$ is a fuzzy S_β – open set.

Theorem 3.4. The intersection of an arbitrary collection of fuzzy S_β – closed sets in a fuzzy topological space (X, τ) is also fuzzy S_β – closed set.

Proof. Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy S_β – open sets in a fuzzy topological space (X, τ). Then $\cup_{\alpha \in \Lambda} A_\alpha$ is a fuzzy S_β – open set. Therefore $(\cup_{\alpha \in \Lambda} A_\alpha)^c$ is a fuzzy S_β – closed set. Hence $\cap_{\alpha \in \Lambda} A_\alpha^c$ is a fuzzy S_β – closed set.

Remark 3.5. The intersection of two fuzzy S_β – open sets is not a fuzzy S_β – open set.

Example 3.6. Let X = {a, b, c, d}, τ = {0_x, 1_x, {(a, 0.4), (b, 0.7), (c, 0.5), (d, 0.6)}, {(a, 0.8), (b, 0), (c, 0.3), (d, 0.4)}}. Then one can verify that A = {(a, 0.4), (b, 0.1), (c, 0), (d, 0.7)} and B = {(a, 0.2), (b, 0.3), (c, 0.3), (d, 0.6)} are fuzzy S_β – open sets in X. But A ^ B = {(a, 0.2), (b, 0.1), (c, 0), (d, 0.6)} is not fuzzy S_β – open set.

Proposition 3.7. A fuzzy subset G in the fuzzy topological space (X, τ) is fuzzy S_β – open if and only if for each fuzzy point $x_p \in G$ there exists a fuzzy S_β – open set H such that $x_p \in H \leq G$.

Proof. Let G be a fuzzy S_β – open set in X. Then for each fuzzy point $x_p \in G$, we have G is a fuzzy S_β – open set containing x_p such that $x_p \in G \leq G$.

Conversely, suppose that for each fuzzy point $x_p \in G$, there exists a fuzzy S_β – open set H such that $x_p \in H \leq G$. Then G is a union of fuzzy S_β – open sets. Hence by Theorem 3.3, G is fuzzy S_β – open.

Proposition 3.8. A fuzzy subset A of a fuzzy topological space (X, τ) is fuzzy regular β – open if A is a member of $S_\beta O(X)$.

Proof. If $A \in S_\beta O(X)$, then A is fuzzy semiopen and for each $x_p \in A$, there exists a fuzzy β – closed set F such that $x_p \in F \leq A$. Therefore $x_p \in F = \beta clF \leq A$. So we get $x_p \in \beta clF \leq A$. Since A ∈ SO(X), then A ∈ βO(X) and $x_p \in \beta clF \leq A$, it follows that A is fuzzy regular βO(X).

Proposition 3.9.

(i) Every fuzzy S_β – open set is fuzzy regular β – open set.
(ii) Every fuzzy regular closed set is fuzzy S_β – open set.

(iii) Every fuzzy regular open set is fuzzy S_β – closed set.

Proof. Obvious.

Proposition 3.10. If a fuzzy topological space (X, τ) is fuzzy T_1 – space, $S_\beta O(X) = SO(X)$.

Proof. Since every fuzzy closed set is fuzzy β – closed and every fuzzy singleton set is fuzzy closed. Hence $S_\beta O(X) = SO(X)$.

Proposition 3.11. If the family of all fuzzy semiopen subsets of a fuzzy topological space is a fuzzy topology on (X, τ) then the family of all fuzzy $S_\beta O(X)$ is also a fuzzy topology on (X, τ).

Proof. Obvious.

Proposition 3.12. Let (X, τ) be a fuzzy topological space and if X is extremally disconnected then $S_\beta O(X)$ forms a fuzzy topology on X.

Proof. Obvious.

FUZZY S_β - OPEN SETS

Definition 4.1. A fuzzy subset N of a fuzzy topological space (X, τ) is called fuzzy S_β – neighbourhood of a fuzzy subset A of X, if there exists a fuzzy S_β – open set U such that $A \leq U \leq N$. When A is equal to a fuzzy point x_p, we say that N is a fuzzy S_β – neighbourhood of x_p.

Definition 4.2. A fuzzy point x_p is said to be a fuzzy S_β – interior point of A if there exists a fuzzy S_β – open set U containing x_p such that $x_p \in U \leq A$. The set of all fuzzy S_β – interior points of A is said to be fuzzy S_β – interior of A and it is denoted by S_β in A.

Proposition 4.3. Let A be any fuzzy subset of a fuzzy topological space (X, τ). If a fuzzy point x_p is in the fuzzy S_β – interior of A, then there exists a fuzzy semiclosed set F of X containing x_p such that $F \leq A$.

Proof. Suppose that $x_p \in S_\beta$ int A. Then there exists a fuzzy S_β – open set U of X containing x_p such that $U \leq A$. Since U is a fuzzy S_β – open set, so there exists a fuzzy β – closed set F containing x_p such that $F \leq U \leq A$. Hence $x_p \in F \leq A$.

Proposition 4.4. For any fuzzy subset A of a fuzzy topological space (X, τ), the following statements are true :

(i) The fuzzy S_β – interior of A is the union of all fuzzy S_β – open sets contained in A.
(ii) $S_\beta$intA is the largest fuzzy S_β – open set contained in A.
(iii) A is fuzzy S_β – open set if and only if $A = S_\beta$ int A.

Proposition 4.5. If A and B are any two fuzzy subsets of a fuzzy topological space (X, τ), then :

(i) $S_\beta int(0_X) = 0_X$ and $S_\beta int(1_X) = 1_X$.
(ii) $S_\beta intA \leq A$.
(iii) If $A \leq B$, $S_\beta intA \leq S_\beta intB$.
(iv) $S_\beta intA \vee S_\beta int B \leq S_\beta int (A \vee B)$.
(v) $S_\beta int(A \wedge B) \leq S_\beta int A \wedge S_\beta int B$.
(vi) A is fuzzy S_β – open set at x_p in X if and only if x_p is a fuzzy point of $S_\beta intA$.

Proof. Straight forward.

Definition 4.6. Intersection of all fuzzy S_β – closed sets containing F is called the fuzzy S_β – closure of F and is denoted by $S_\beta clF$.

Corollary 4.7. Let A be a fuzzy set of a fuzzy topological space (X, τ). A fuzzy point x_p in X is in fuzzy S_β – closure of A if and only if $A \wedge U \neq 0_X$, for every fuzzy S_β – open set U containing x_p.

Proof. Obvious.

Proposition 4.8. Let A be any fuzzy subset of a fuzzy topological space (X, τ). If $A \wedge F \neq 0_X$ for every fuzzy β – closed set F of X containing fuzzy point x_p, then the fuzzy point x_p is in the fuzzy S_β – closure of A.

Proof. Suppose that U is any fuzzy S_β – open set containing fuzzy point x_p. Then there exists fuzzy β – closed set F such that $x_p \in F \leq U$. So by hypothesis $A \wedge F \neq 0_X$ which implies that $A \wedge U \neq 0_X$ for every fuzzy S_β – open set U containing x_p. Therefore, by Corollary 4.7., $x_p \in S_\beta clA$.

Proposition 4.9. For any fuzzy subset F of a fuzzy topological space (X, τ), the following statements are true :

(i) $S_\beta clF$ is the intersection of all fuzzy S_β – closed sets in X containing F.
(ii) $S_\beta clF$ is the smallest fuzzy S_β – closed set containing F.
(iii) F is fuzzy S_β – closed set if and only if $F = S_\beta clF$.

Proof. Obvious.

Proposition 4.10. Let A be any fuzzy subset of a fuzzy topological space (X, τ). If a fuzzy point x_p is in the fuzzy S_β – closure of A, then $A \wedge F \neq 0_X$ for every fuzzy β – closed set F of X containing fuzzy point x_p.

Proof. Suppose that $x_p \in S_\beta clA$. Then by Corollary 4.7., $A \wedge U \neq 0_X$ for every fuzzy S_β – open set U containing x_p. Since U is fuzzy S_β – open set, so there exists a fuzzy β – closed set F containing x_p such that $x_p \in F \leq A$. Hence $A \wedge F \neq 0_X$.

Theorem 4.11. If F and E are any two fuzzy subsets of a fuzzy topological space (X, τ), then

(i) $S_\beta cl(0_X) = 0_X$ and $S_\beta cl(1_X) = 1_X$.

(ii) For any fuzzy subset F of X, $F \leq S_\beta(clF)$.
(iii) If $F \leq E$, then $S_\beta clF \leq S_\beta clE$.
(iv) $S_\beta(clF) \vee S_\beta clE \leq S_\beta cl(F \vee E)$.
(v) $S_\beta cl(F \wedge E) \leq S_\beta clF \wedge S_\beta clE$.

Proof. Obvious.

Theorem 4.12. For any fuzzy subset A of a fuzzy topological space (X, τ), the following statements are true :

(i) $1_X - S_\beta clA = S_\beta int(1_X - A)$.
(ii) $1_X - S_\beta intA = S_\beta cl(1_X - A)$.
(iii) $S_\beta intA = 1_X - S_\beta cl(1_X - A)$.

Proof. Obvious.

Definition 4.13. Let A be a fuzzy subset of a fuzzy topological space (X, τ). A fuzzy point x_p of X is said to be fuzzy S_β – limit point of A if for each fuzzy S_β – open set U containing x_p, $U \wedge (A - x_p) \neq 0_X$. The set of all fuzzy S_β – limit point of A is called fuzzy S_β – derived set of A and is denoted by $S_\beta D(A)$.

Proposition 4.14. Let A be any fuzzy subset of a fuzzy topological space (X, τ). If $F \wedge (A - x_p) \neq 0_X$, for every fuzzy β – closed set F containing fuzzy point x_p, then $x_p \in S_\beta D(A)$.

Proof. Let U be any fuzzy S_β – open set U containing x_p. Then there exists fuzzy β – closed set F such that $x_p \in F \leq U$. By hypothesis, we have $F \wedge (A - x_p) \neq 0_X$. Hence $U \wedge (A - x_p) \neq 0_X$. Therefore, a fuzzy point $x_p \in S_\beta D(A)$.

Proposition 4.15. If a fuzzy subset A of a fuzzy topological space (X, τ) is fuzzy S_β – closed, then A contains the set of all its fuzzy S_β – limit point.

Proof. Suppose that A is fuzzy S_β – closed set. Then 1_X - A is fuzzy S_β – open set. Thus A is fuzzy S_β – closed set if and only if each fuzzy point of 1_X - A has fuzzy S_β – neighbourhood contained in 1_X – A, *i.e.*, if and only if no fuzzy point of 1_X - A is fuzzy S_β – limit point of A or equivalently that A contains each of its fuzzy S_β – limit points.

Proposition 4.16. Let F and E be two fuzzy subsets of a fuzzy topological space (X, τ). If $F \leq E$, then $S_\beta D(F) \leq S_\beta D(E)$.

Proof. Obvious.

Theorem 4.17. Let A and B be two fuzzy subsets of a fuzzy topological space (X, τ). Then we have the following properties :

(i) $S_\beta D(0_X) = 0_X$.
(ii) $x_p \in S_\beta D(A)$ implies $x_p \in S_\beta D(1_X - A)$.
(iii) $S_\beta D(A) \vee S_\beta D(B) \leq S_\beta D(A \vee B)$.
(iv) $S_\beta D(A \wedge B) \leq S_\beta D(A) \wedge S_\beta D(B)$.

(v) If A is fuzzy S_β – closed, then $S_\beta D(A) \leq A$.

Proof. Obvious.

Theorem 4.18. Let (X, τ) be a fuzzy topological space and A be a fuzzy subset of (X, τ). The the followings hold :

(i) $A \vee S_\beta D(A)$ is fuzzy S_β – closed.

(ii) $S_\beta D(S_\beta D(A))$ - $A \leq S_\beta D(A)$.

(iii) $S_\beta D(A \vee S_\beta D(A)) \leq A \vee S_\beta D(A)$.

Proof.

(i) Let $x_p \in A \vee S_\beta D(A)$. Then $x_p \notin A$ and $x_p \notin D(A)$. This implies that there exists a fuzzy S_β – open set N_x in X which contains no point of A other than x_p. But $x_p \notin A$, so N_x contains no point of A which implies that $N_x \leq 1_X$ - A. Again N_x is a fuzzy S_β – open set. It is a neighbourhood of each of its fuzzy points. But N_x does not contain any point of A, no point of N_x can be fuzzy S_β – limit point of A. Therefore no point of N_x can belong to $S_\beta D(A)$. This implies that $N_x \leq 1_X - S_\beta D(A)$. Hence it follows that $x_p \in N_x \leq (1_X - A) \wedge (1_X - S_\beta D(A) \leq 1_X - (A \wedge S_\beta D(A))$. Therefore $A \vee S_\beta D(A)$ is fuzzy S_β – closed.

(ii) If $x_p \in S_\beta D S_\beta D(A) - A$ and U is a fuzzy S_β – open set containing x_p, then $U \wedge S_\beta D(A) - x_p \neq 0_X$. Let $y_p \in (U \wedge S_\beta D(A) - x_p)$. Then $y_p \in U$ and $y_p \in S_\beta D(A)$. So $U \wedge (A - y_p) \neq 0_Y$. Let $z_p \in (U \wedge (A - y_p))$. Then $z_p \neq x_p$ for $z_p \in A$ and $x_p \notin A$. Hence $U \wedge (A - x_p) \neq 0_X$. Therefore, $x_p \in S_\beta D(A)$.

(iii) Let $x_p \in S_\beta D(A \vee S_\beta D(A))$. If If $x_p \in A$, the result is obvious. Let $x_p \in S_\beta D(A \vee S_\beta D(A)) - A$. Then for fuzzy S_β – open set U containing x_p, $U \wedge (A \vee S_\beta D(A)) - x_p)) \neq 0_X$, thus $U \wedge (A - x_p) \neq 0_X$ or $U \wedge S_\beta D(A)(x_p) \neq 0_X$. Now it follows similarly from (ii) that $U \wedge (A - x_p) \neq 0_X$. Hence $x_p \in S_\beta D(A)$. Therefore $S_\beta D(A \vee S_\beta D(A)) \leq A \vee S_\beta D(A)$.

Theorem 4.19. Let A be a fuzzy subset of a fuzzy topological space (X, τ). Then $S_\beta clA = A \vee S_\beta D(A)$.

Proof. Since $S_\beta D(A) \leq S_\beta clA$ and $A \leq S_\beta clA$, we have $A \vee S_\beta D(A) \leq S_\beta clA$. Again since $S_\beta clA$ is the smallest fuzzy S_β – closed set containing A, but by the proposition 3.2., $A \vee S_\beta D(A)$ is fuzzy S_β – closed. Hence $S_\beta clA \leq A \vee S_\beta D(A)$. Thus $S_\beta clA = A \vee S_\beta D(A)$.

Theorem 4.20. Let A be a fuzzy subset of a fuzzy topological space (X, τ). Then $S_\beta intA = A - S_\beta D(1_X - A)$.

Proof. Obvious.

Definition 4.21. Let A be a fuzzy subset of a fuzzy topological space (X, τ). Then $S_\beta clA - S_\beta intA$ is called fuzzy S_β – boundary of A and is denoted by $S_\beta Bd(A)$.

Proposition 4.22. For any fuzzy subset A of a fuzzy topological space (X, τ), the following statements are true :

(i) $S_\beta clA = S_\beta intA \vee S_\beta Bd(A)$.

(ii) $S_\beta intA \wedge S_\beta Bd(A) = 0_X$.

(iii) $S_\beta Bd(A) = S_\beta clA \wedge S_\beta cl(1_X - A)$.

(iv) $S_\beta Bd(A)$ is fuzzy S_β – closed.

Proof. Obvious.

Proposition 4.23. For any fuzzy subset A of a fuzzy topological space (X, τ), the following statements are true :

(i) $S_\beta Bd(A) = S_\beta Bd(1_X - A)$.

(ii) $A \in$ FSO(X) if and only if $S_\beta Bd(A) \leq (1_X - A)$, that is $A \wedge S_\beta Bd(A) = 0_X$.

(iii) $A \leq S_\beta C(X)$ if and only if $S_\beta Bd(A) \leq A$.

(iv) $S_\beta Bd(S_\beta(Bd(A))) \leq S_\beta Bd(A)$.

(v) $S_\beta Bd(S_\beta intA) \leq S_\beta Bd(A)$.

(vi) $S_\beta Bd(S_\beta clA) \leq S_\beta Bd(A)$.

(vii) $S_\beta intA = A - S_\beta Bd(A)$.

Proof. Obvious.

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