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TO STUDY OF SOME MULTIVALENT FUNCTIONS DEFINED BY DERIVATIVE OPERATOR OF RUSCHEWEYH TYPE

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By means of certain extended derivative operator of Ruscheweyh type, we introduce and investigate two new subclasses $S^p_{n,m}(\lambda, b, \delta)$ and $G^p_{n,m}(\lambda, b, \delta)$ of p-valently analytic functions of complex order. The various results obtained here for each of these subclasses included coefficient estimate, distortion theorem, radius of starlikeness, convexity and closure theorem, weighted mean, arithmetic mean and linear combination of regular function.

Keywords & Phrases: - Multivalent function, weighted mean, arithmetic mean, linear combination of regular function.

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INTRODUCTION

Let $A(n)$ denote the class of functions $f(z)$ normalized by

$$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k , \qquad \dots (1)$$

$$a_k \geq 0 \quad \text{and} \quad n, p \in \mathbb{N} = \{1,2,3, \dots.\}$$

which are analytic and p-valent in the open unit disc $U = \{z : z \in \mathbb{C} and \mid z \mid < 1\}$

We introduce here an extended linear derivative operator of Ruscheweyhtype :

$D^{\lambda,p} : A(1) \to A(1)$which is defined by

$$D^{\lambda,p}(f(z)) = z^p - \sum_{k=n+p}^{\infty} \binom{\lambda + k - 1}{k - p} a_k z^k$$

where $\lambda > -p , f \in A(n)$ ... (2)

In particular when $\lambda = n , n \in \mathbb{N}$, it is easily observed from (2) that

$$D^{n,p}(f(z)) = \frac{z^p (z^{n-p} f(z))^{(n)}}{n!}$$

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$$n \in \mathbb{N} \cup \{0\} . p \in \mathbb{N}$$

So that

$$D^{1,p}(f(z)) = (1-p)f(z) + zf'(z)$$

$$D^{2,p}(f(z)) = \frac{(1-p)(2-p)}{2!} f(z) + (2-p)zf'(z) + \frac{z^2}{2!} f''(z)$$

And so on

$$(D^{\lambda,p}(f(z)))^{(m)} = \binom{p}{m} z^{p-m} - \sum_{k=n+p}^{\infty} \binom{k}{m} \binom{\lambda+k-1}{k-p} a_k z^{k-m}$$

where $$\binom{k}{m} = \frac{k(k-1)(k-2) \ldots \ldots (k-m+1)}{m!}$$

By using the operator $D^{\lambda,p}(f(z))$, we introduce new subclass $S_{n,m}^{p}(\lambda, b, \delta)$ of p-valently analytic function $f(z)$ satisfying the following inequality

$$\left| \frac{1}{b} \left(\frac{\delta z \left(D^{\lambda,p}(f(z))\right)^{(m+1)} + \lambda z^2 \left(D^{\lambda,p}(f(z))\right)^{(m+2)}}{\lambda z \left(D^{\lambda,p}(f(z))\right)^{(m+1)} + (\delta - \lambda)\left(D^{\lambda,p}(f(z))\right)^{(m)}} - (p-m) \right) \right| < 1$$

$p \in \mathbb{N}$, $m \in \mathbb{N} \, U \{0\} z \in U$, $p > \max(m, -\lambda)$, $b \in \mathbb{C} \, U \{0\}$, $\lambda \geq 0 , 0 < \delta \leq 1$

Furthermore a function $f(z)$ is said to belong to the class $G_{n,m}^{p}(\lambda, b, \delta)$ if and only if

$$zf'(z) \in S_{n,m}^{p}(\lambda, b, \delta)$$

The object of the present paper is to investigate the various properties and characteristics of analytic p-valent functions belonging tothe subclasses $S_{n,m}^{p}(\lambda, b, \delta)$ and $G_{n,m}^{p}(\lambda, b, \delta)$ which we have defined here. Apart from deriving a set of coefficient bounds for each of these function classes, we establish distortion theorem, radius of starlikeness, convexity and closure theorem.

PRELIMINARY RESULTS

THEOREM 1:- A function $f(z) \in A(n)$ and defined by

$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$, $a_k \geq 0$ *and* $p \in \mathbb{N}$, is in $S_{n,m}^{p}(\lambda, b, \delta)$ if and only if

$$\sum_{k=n+p}^{\infty} \binom{\lambda+k-1}{k-p} \binom{k}{m} [\lambda(k-m-1)+\delta)][k-p+|b|] a_k \leq |b| \binom{p}{m} [\lambda(p-m-1)+\delta]$$

COROLLARY 1.1:- $f(z) \in S_{n,m}^{p}(\lambda, b, \delta)$ then

$$a_k \leq \frac{|b| \binom{p}{m} [\lambda(p-m-1)+\delta]}{\binom{k}{m} \binom{\lambda+k-1}{k-p} [\lambda(k-m-1)+\delta][k-p+|b|]}$$

COROLLARY 1.2:- for $p = 1, m = 0$ we have

$$a_k \leq \frac{|b|\delta}{\binom{\lambda+k-1}{k-1}[\lambda(k-1)+\delta][k-1+|b|]}, \quad k \geq n+p$$

COROLLARY 1.3:- for $p = 1, m = 1$ we have

$$a_k \leq \frac{|b|[\delta-\lambda]}{k\binom{\lambda+k-1}{k-p}[\lambda k+\delta][k-1+|b|]}$$

COROLLARY 1.4:- for $p = 1, m = 1, \lambda = 1$ we have

$$a_k \leq \frac{|b|[\delta-1]}{k^2[k+\delta][k-1+|b|]}$$

THEOREM 2:- A function $f(z) \in A(n)$ and defined by

$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$, $a_k \geq 0$ $\quad and\ p \in \mathbb{N}$, is in $G_{n,m}^p(\lambda, b, \delta)$ if and only if

$$\sum_{k=n+p}^{\infty}\binom{\lambda+k-1}{k-p}\binom{k}{m}k[\lambda(k-m-1)+\delta)][k-p+|b|] \leq |b|\binom{p}{m}p[\lambda(p-m-1)+\delta]$$

THEOREM 3:- If $f(z) \in S_{n,m}^p(\lambda, b, \delta)$ then

$$|\boldsymbol{z}|^{\boldsymbol{p}} - |\boldsymbol{z}|^{\boldsymbol{n+p}}\frac{|b|\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]} \leq |f(z)|$$

$$\leq |\boldsymbol{z}|^{\boldsymbol{p}} + |\boldsymbol{z}|^{\boldsymbol{n+p}}\frac{|b|\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]}$$

With equality hold for

$$f(z) = z^p - z^{n+p}\frac{|b|\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]}$$

THEOREM 4:- If $f(z) \in G_{n,m}^p(\lambda, b, \delta)$ then

$$|\boldsymbol{z}|^{\boldsymbol{p}} - |\boldsymbol{z}|^{\boldsymbol{n+p}}\frac{|b|p\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}(n+p)[\lambda(n+p-m-1)+\delta][n+|b|]} \leq |f(z)|$$

$$\leq |\boldsymbol{z}|^{\boldsymbol{p}} + |\boldsymbol{z}|^{\boldsymbol{n+p}}\frac{|b|p\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}(n+p)[\lambda(n+p-m-1)+\delta][n+|b|]}$$

With equality hold for

$$f(z) = z^p - z^{n+p}\frac{|b|p\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}(n+p)[\lambda(n+p-m-1)+\delta][n+|b|]}$$

THEOREM 5:- If $f(z) \in S_{n,m}^p(\lambda, b, \delta)$ then

$$p|z|^{p-1} - |z|^{n+p-1} \frac{|b|\binom{p}{m}(n+p)[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]} \leq |f'(z)|$$

$$\leq p|z|^{p-1} + |z|^{n+p-1} \frac{|b|\binom{p}{m}(n+p)[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]}$$

With equality hold for

$$f(z) = z^p - z^{n+p} \frac{|b|\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]}$$

THEOREM 6:- If $f(z) \in G_{n,m}^{p}(\lambda, b, \delta)$ then

$$p|z|^{p-1} - |z|^{n+p-1} \frac{|b|\binom{p}{m}p[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]} \leq |f'(z)|$$

$$\leq p|z|^{p-1} + |z|^{n+p-1} \frac{|b|\binom{p}{m}p[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}[\lambda(n+p-m-1)+\delta][n+|b|]}$$

With equality hold for

$$f(z) = z^p - z^{n+p} \frac{|b|p\binom{p}{m}[\lambda(p-m-1)+\delta]}{\binom{n+p}{m}\binom{\lambda+n+p-1}{n}(n+p)[\lambda(n+p-m-1)+\delta][n+|b|]}$$

THEOREM 7:- If $f(z) \in S_{n,m}^{p}(\lambda, b, \delta)$, then f is close to convex of order $\propto$ in $|z| < r_1(p, n, m, \lambda, b, \delta, \propto)$ where

$$r_1(p, n, m, \lambda, b, \delta, \propto) = \inf_k \left(\left(\binom{\lambda+k-1}{k-p} \binom{k}{m} \frac{(p-\propto)[\lambda(k-m-1)+\delta)][k-p+|b|]}{k|b|\binom{p}{m}[\lambda(p-m-1)+\delta]} \right)^{\frac{1}{k-p}} \right)$$

THEOREM 8:- If $f(z) \in S_{n,m}^{p}(\lambda, b, \delta)$, then f is starlike of order $\propto$ in

$|z| < r_2(p, n, m, \lambda, b, \delta, \propto)$ where

$$r_2(p, n, m, \lambda, b, \delta, \propto) = \inf_k \left(\left(\binom{\lambda+k-1}{k-p} \binom{k}{m} \frac{(p-\propto)[\lambda(k-m-1)+\delta)][k-p+|b|]}{(k-\propto)|b|\binom{p}{m}[\lambda(p-m-1)+\delta]} \right)^{\frac{1}{k-p}} \right)$$

THEOREM 9:- If $f(z) \in S_{n,m}^{p}(\lambda, b, \delta)$, then f is convex of order $\propto$ in $|z| < r_3(p, n, m, \lambda, b, \delta, \propto)$ where

$r_3(p, n, m, \lambda, b, \delta, \propto)$

$$= \begin{matrix} inf \\ k \end{matrix} \left(\left(\binom{\lambda + k - 1}{k - p} \binom{k}{m} \frac{p(p-\propto)[\lambda(k-m-1)+\delta)][k-p+|b|]}{k(k-\propto)|b|\binom{p}{m}[\lambda(p-m-1)+\delta]} \right)^{\frac{1}{k-p}} \right)$$

THEOREM 10 : Let $f_1(z) = z^p$

and $$f_k(z) = z^p - \binom{\lambda + k - 1}{k - p} \binom{k}{m} \frac{[\lambda(k-m-1)+\delta)][k-p+|b|]}{|b|\binom{p}{m}[\lambda(p-m-1)+\delta]} z^k$$

for $k \geq n + p$

Then $f(z) \in S^p_{n,m}(\lambda, b, \delta)$ if and only if $f(z)$ can be expressed in the form

$$f(z) = \lambda_1 f_1(z) + \sum_{k=n+p}^{\infty} \lambda_k f_k(z)$$

where $$\lambda_k \geq 0 \text{ and } \lambda_1 + \sum_{k=n+p}^{\infty} \lambda_k = 1$$

WEIGHTED MEAN, ARITHMETIC MEAN AND LINEAR COMBINATION

Following the earlier works by W.G.Asthan, H.D. Mustafa and E.K. Mouajeeb [2] weighted mean , arithmetic mean and linear combination of regular function .

Definition 2.1 : Let f , $g \in \mathcal{B}(A, B, \delta)$then the weighted mean w_{fg} of f and g is defined as

$$w_{fg} = \frac{1}{2}[(1-t)f(z) + (1+t)g(z)] \ , 0 < t < 1$$

Definition 2.2 : Let $f_i(z) = z^p - \sum_{k=1+p}^{\infty} a_{i,k} z^k$, $i = 1,2,3, \ldots, m$ be the functions in the class $\mathcal{B}(A, B, \delta)$ then the arithmetic mean of f_i ($i = 1,2,3, \ldots, m$) is defined by

$$g(z) = \frac{1}{m} \sum_{i=1}^{m} f_i(z)$$

Definition 2.3 : Let $f_i(z) = z^p - \sum_{k=1+p}^{\infty} a_{i,k} z^k$, $i = 1,2,3, \ldots, m$ be the functions in the class $\mathcal{B}(A, B, \delta)$then the linear combination of f_i ($i = 1,2,3, \ldots, m$) is defined by

$$G(z) = \sum_{i=1}^{m} k_i f_i(z) , \quad \text{where} \quad \sum_{i=1}^{m} k_i = 1$$

THEOREM 2.1 Let $f, g \in S^p_{n,m}(\lambda, b, \delta)$. Then the weighted mean w_{fg} of f and g is also in the class $S^p_{n,m}(\lambda, b, \delta)$

PROOF: By Definition 3.1 , we have

$$w_{fg} = \frac{1}{2}[(1-t)f(z) + (1+t)g(z)]$$

$$= \frac{1}{2}\left[(1-t)\left(z^p - \sum_{k=n+p}^{\infty} a_k z^k\right) + (1+t)\left(z^p - \sum_{k=n+p}^{\infty} b_k z^k\right)\right]$$

$$= z^p - \sum_{k=n+p}^{\infty} \frac{1}{2}[(1-t)a_k + (1+t)b_k] z^k \qquad \ldots (3.1)$$

Since $g \in S_{n,m}^{p}(\lambda, b, \delta)$so by **THEOREM 1** we have

$$\sum_{k=n+p}^{\infty} \binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]a_k \leq |b|\binom{p}{m}[\lambda(p-m-1)+\delta]$$

and

$$\sum_{k=n+p}^{\infty} \binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]b_k \leq |b|\binom{p}{m}[\lambda(p-m-1)+\delta]$$

Therefore

$$\sum_{k=n+p}^{\infty} \binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p]+|b|] \left[\frac{1}{2}[(1-t)a_k + (1+t)b_k]\right]$$

$$= \frac{1}{2}(1-t)\sum_{k=n+p}^{\infty} \binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]a_k$$

$$+\frac{1}{2}(1+t)\sum_{k=n+p}^{\infty} \binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]b_k$$

$$\leq \frac{1}{2}(1-t)\left[|b|\binom{p}{m}[\lambda(p-m-1)+\delta]\right]$$

$$+\frac{1}{2}(1+t)\left[|b|\binom{p}{m}[\lambda(p-m-1)+\delta]\right]$$

$$= |b|\binom{p}{m}[\lambda(p-m-1)+\delta]$$

Therefore

$$w_{fg} \in S_{n,m}^{p}(\lambda, b, \delta)$$

Hence the proof of theorem is completed.

THEOREM 3.2 Let $f_i(z) = z^p - \sum_{k=n+p}^{\infty} a_{i,k} z^k$, $i = 1,2,3,\dots,m$ be the functions in the class $S_{n,m}^{p}(\lambda, b, \delta)$ then the arithmetic mean of f_i ($i = 1,2,3,\dots,m$) is defined by

$g(z) = \frac{1}{m}\sum_{i=1}^{m} f_i(z)$ is also in the class $S_{n,m}^{p}(\lambda, b, \delta)$

PROOF : Since $f_i(z) = z^p - \sum_{k=n+p}^{\infty} a_{i,k} z^k$, $i = 1, 2, 3, \dots, m$

Therefore

$$g(z) = \frac{1}{m}\sum_{i=1}^{m} f_i(z)$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left(z^p - \sum_{k=n+p}^{\infty} a_{i,k} z^k\right)$$

$$= z^p - \sum_{k=1+p}^{\infty}\left(\frac{1}{m}\sum_{i=1}^{m} a_{i,k}\right) z^k$$

We have $f_i(z) = z^p - \sum_{k=n+p}^{\infty} a_{i,k} z^k$, $i = 1,2,3,\dots,m$ are in the class $S_{n,m}^{p}(\lambda, b, \delta)$

So by **THEOREM 1** we have

$$\sum_{k=1+p}^{\infty}\binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]\left(\frac{1}{m}\sum_{i=1}^{m} a_{i,k}\right)$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left(\sum_{k=1+p}^{\infty}\binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]\, a_{i,k}\right)$$

$$\leq \frac{1}{m}\sum_{i=1}^{m}|b|\binom{p}{m}[\lambda(p-m-1)+\delta] = |b|\binom{p}{m}[\lambda(p-m-1)+\delta]$$

Hence the proof of theorem is completed.

THEOREM 3.3 Let $f_i(z) = z^p - \sum_{k=n+p}^{\infty} a_{i,k} z^k$, $i = 1,2,3,\dots,m$ be the functions in the class $S_{n,m}^{p}(\lambda, b, \delta)$ then the linear combination of f_i ($i = 1,2,3,\dots,m$) is defined by

$G(z) = \sum_{i=1}^{m} k_i f_i(z)$, where $\sum_{i=1}^{m} k_i = 1$ is also in the class $S_{n,m}^{p}(\lambda, b, \delta)$

PROOF: Let $f_i(z) = z^p - \sum_{k=n+p}^{\infty} a_{i,k} z^k$, $i = 1,2,3,\dots,m$ be the functions in the class $S_{n,m}^{p}(\lambda, b, \delta)$

so by **THEOREM 1** we have

$$\sum_{k=1+p}^{\infty}\binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]a_{i,k}$$

$$\leq |b|\binom{p}{m}[\lambda(p-m-1)+\delta]$$

$$G(z) = \sum_{i=1}^{m} k_i f_i(z)$$

$$G(z) = \sum_{i=1}^{m} k_i \left(z^p - \sum_{k=n+p}^{\infty} a_{i,k}\, z^k \right)$$

$$G(z) = z^p - \sum_{k=n+p}^{\infty} \left(\sum_{i=1}^{m} k_i a_{i,k} \right) z^k$$

So by **THEOREM 1** we have

$$\sum_{k=n+p}^{\infty} \binom{\lambda+k-1}{k-p} \binom{k}{m} [\lambda(k-m-1)+\delta)][k-p+|b|] \left(\sum_{i=1}^{m} k_i a_{i,k} \right)$$

$$= \sum_{i=1}^{m} k_i \left(\sum_{k=1+p}^{\infty} \binom{\lambda+k-1}{k-p} \binom{k}{m} [\lambda(k-m-1)+\delta)][k-p+|b|]\, a_{i,k} \right)$$

$$\leq \sum_{i=1}^{m} k_i |b| \binom{p}{m} [\lambda(p-m-1)+\delta] = |b| \binom{p}{m} [\lambda(p-m-1)+\delta]$$

Hence the proof of theorem is completed.

APPLICATION OF FRACTION CALCULUS AND OTHER POPERTIES

Various operators of fractional calculus have been studied in the literature rather extensively. Now we recall the following definitions.

DEFINITION 3.1 The integral operator studied by Bernardi is

$$L_c[f] = \frac{p+c}{z^c} \int_0^z f(x) x^{c-1} dx$$

THEOREM 3.1: If $f \in S^p_{n,m}(\lambda, b, \delta)$ then $L_c[f]$ is also in the class $S^p_{n,m}(\lambda, b, \delta)$

PROOF: Let $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k\, z^k$ then

$$L_c[f] = \frac{p+c}{z^c} \int_0^z \left(x^p - \sum_{k=n+p}^{\infty} a_k\, x^k \right) x^{c-1} dx$$

$$= \frac{p+c}{z^c} \left[\left(\frac{1}{c+p} x^{c+p} - \sum_{k=n+p}^{\infty} \frac{1}{k+c} a_k\, x^{k+c} \right)_0^z \right]$$

$$= z^p - \sum_{k=n+p}^{\infty} \frac{p+c}{k+c} a_k\, z^k$$

Since $c > -p$, $k \geq p + n$ then $\frac{p+c}{k+c} \leq n$ so we have

$$\sum_{k=n+p}^{\infty} \frac{\binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]}{|b|\binom{p}{m}[\lambda(p-m-1)+\delta]}\left(\frac{p+c}{k+c}\right)a_k$$

$$\leq \sum_{k=1+p}^{\infty} \frac{\binom{\lambda+k-1}{k-p}\binom{k}{m}[\lambda(k-m-1)+\delta)][k-p+|b|]}{|b|\binom{p}{m}[\lambda(p-m-1)+\delta]} a_k < 1$$

Therefore $L_c[f]$ is also in the class $S^p_{n,m}(\lambda, b, \delta)$

Similarly we can prove

THEOREM 3.3: Let $f \in S^p_{n,m}(\lambda, b, \delta)$then for every $\chi \geq 0$ then the function

$$L_\chi(z) = (1-\chi)f(z) + p\chi \int_0^z \frac{f(y)}{y}\, dy$$

is also in the class $S^p_{n,m}(\lambda, b, \delta)$

THEOREM 3.4: Let $f \in S^p_{n,m}(\lambda, b, \delta)$ then for every $\chi \geq 0$ then the function

$$M_\chi(z) = (1-\chi)z^p + p\chi \int_0^z \frac{f(y)}{y}\, dy$$

is also in the class $S^p_{n,m}(\lambda, b, \delta)$

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