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RAYLEIGH-TAYLOR CONVECTION IN A RIVLIN-ERICKSEN DUSTY PLASMA IN PRESENCE OF VARYING MAGNETIC FIELD SATURATING IN A POROUS MEDIUM

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In the present paper, Rayleigh-Taylor instability of visco-elastic (Rivlin-Ericksen) dusty plasma in the presence of magnetic field through a porous medium is considered. Following the linearized perturbation theory and normal mode technique, the dispersion relation is obtained. The system is found to be always stable for $\alpha_1 > \alpha_2$ and unstable for $\alpha_1 < \alpha_2$ under certain conditions. The case of exponentially varying density, viscosity, visco-elasticity, magnetic field and particles number density is also considered. For $\beta < 0$, the system is found to be stable always. For $\beta < 0$ the non- oscillatory modes are always stable and estimate of *n* for the growth rate of oscillatory stable modes are given by $|n|^2 > \frac{D}{B}$.

Keywords: Rayleigh-Taylor instability, Rivlin-Ericksen dusty plasma, Magnetic field, Porous medium.

INTRODUCTION

Chandrasekhar [1] has discussed the theoretical and experimental results on the onset of thermal instability in a fluid layer under varying assumption of hydrodynamics. Further Rayleigh-Taylor instability of two viscous superposed conducting fluid in presence of a uniform horizontal magnetic field has been considered by Bahtia [2]. Sisodia and Gupta [3] and Sriratava and Singh [4] have studied the unsteady flow of a dusty Visco-elastic (Rivlin-Ericksen) fluid through channel of different cross-section in the presence of the time-dependent pressure gradient.

While discussing the problem of Rayleigh-Taylor instability of viscous; visco-elastic fluids through porous medium Sharma and Kumar [5] found that as in both Newtonian viscous, visco-elastic fluids, the system is stable for a potentially stable case and unstable for potential unstable case. Kumar [6] have studied the problem of Rayleigh-Taylor instability of Rivlin-Eriksen elastico-viscous fluid in the presence of suspended particles through porous medium. Kumar [7] has studied the stability of superposed viscous, visco-elastic (Rivlin-Ericksen) fluid in the presence of suspended particles through porous medium. It is found that the presence of magnetic field stabilizes for a certain wave number, whereas the system is unstable for all wave number in the absence of magnetic field for the potentially unstable configuration.

With the growing importance of non-Newtonian fluids in the modern technology and industries, the investigations of such fluids are desirable. Rivlin-Ericksen [8] is an important class of visco-elastic fluids. Khan and Bhatia [9] have considered the problem of stability of two superposed visco-elastic fluids in the presence of a horizontal magnetic field and found that elasticity has stabilizing effect and viscosity has a destabilizing effect on the growth rate of unstable mode of disturbances. Kumar and Lal [10] have studied the stability of two superposed Rivlin-Ericksen viscous, visco-elastic fluids and found that both kinematic viscosity and kinematic visco-elasticity have stabilizing effect. The problem of instability of two rotating visco-elastic (Rivlin-Ericksen) superposed fluids with suspended particles in porous medium has been discussed by Kumar, Lal and Sharma [11]. They have found that system is stable for potentially stable configuration and unstable for potentially unstable configuration. Sengupta and Basak [12] have studied the stability of two superposed visco-elastic (Maxwell) fluids in a verticals magnetic field. They have found that both viscosity and visco-elasticity of fluid have stabilizing influence while medium permeability has mostly destabilizing effect on the growth rate of unstable mode of disturbances.

Rivlin-Ericksen visco-elastic fluid plays a significant role industrial application. In view of the fact that hydro magnetic stability of stratified Rivlin-Ericksen visco-elastic in the presence of suspended particles through porous medium may find application in modern technology. This topic has been studied by several researchers. However, hydrodynamic stability of stratified Rivlin-Ericksen visco-elastic fluid in presence of suspended particles through porous medium seems uninvestigated so far.

CONSTITUTIVE EQUATIONS

Consider a static state of a fluid in which an incompressible, visco-elastic (Rivlin-Ericksen) fluid layer of variable density is arranged in horizontal strata. The particles of the fluid are assumed to be non-conducting. This fluid particle layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and medium permeability k_1. The pressure p and density ρ are functions of the vertical co-ordinate z only. The fluid is under the action of gravity $\mathbf{g}$ (0, 0, –g) and the variable horizontal magnetic field $\mathbf{H}$ (H(z), 0, 0).

Let ρ, p, μ, μ', and $\mathbf{q}$ (u, v, w) denote respectively the density, pressure, viscosity, viscoelasticity and the velocity of the hydromagnetic fluid, $\mathbf{q}_d(\bar{x}, t)$ and N $(\bar{x}, t)$ denote the velocity and number density of particles, respectively $\boldsymbol{x} = (x, y, z)$, $K = 6\pi\mu\eta$, where η is the particle radius, is a constant. Then the equations of motion and continuity for the (Rivlin–Ericksen) fluid are,

$$\frac{\rho}{\in}\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\in}(\boldsymbol{q}.\nabla\boldsymbol{q}\right] = -\nabla p + \mathbf{g}\rho - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\boldsymbol{q} \times \frac{\mu_e}{4\pi}[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}] + \frac{KN}{\in}(\boldsymbol{q}_d - \boldsymbol{q}), \quad \text{... (1)}$$

$$\nabla.\boldsymbol{q} = 0, \quad \text{... (2)}$$

$$\in\frac{\partial\rho}{\partial t} + (\boldsymbol{q}.\nabla)\rho = 0, \quad \text{... (3)}$$

$$\in\frac{\partial\boldsymbol{H}}{\partial t} = (\boldsymbol{H}.\nabla)\boldsymbol{q} - (\boldsymbol{q}.\nabla)\boldsymbol{H} \quad \text{... (4)}$$

$$\nabla.\boldsymbol{H} = 0. \quad \text{... (5)}$$

The equations of motion for continuity for the particles are,

$$mN\left[\frac{\partial\boldsymbol{q}_d}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}_d.\nabla)\boldsymbol{q}_d\right] = KN(\boldsymbol{q} - \boldsymbol{q}_d) \quad \text{... (6)}$$

$$\in\frac{\partial N}{\partial t} + \nabla(N.\boldsymbol{q}_d) = 0. \quad \text{... (7)}$$

where mN is the mass of the particles per unit volume.

BASIC STATE AND PERTURBATION EQUATIONS

The time independent solution of equation (1) to (7), whose stability we wish to examine is that of an incompressible, visco-elastic (Rivlin–Ericksen) fluid of variable density arranged in a horizontal strata in a homogeneous isotropic porous medium. In the undisturbed state, the fluid is at rest and magnetic field acts in the vertical direction (z-direction), therefore the basic state of which we wish to examine the stability is characterized by,

$$\mathbf{q}\ (0, 0, 0),\ \mathbf{q}_d = (0, 0, 0),\ \mathbf{H} = (H(z), 0, 0),\ \rho = \rho(z),\ p = p(z) \quad \text{... (8)}$$

To examine the character of equilibrium the system is slightly perturbed. Here, we assume that the small disturbances are the functions of space and time variables. Hence, the perturbed flow may be represented as:

$$\mathbf{q} = (0, 0, 0) + (u, v, w),\ \mathbf{q}_d = (0, 0, 0) + (l, r, s),\ \mathbf{h} = (H(z), 0, 0) + (h_x, h_y, h_z),\ \rho = \rho(z) + \delta\rho,\ p = p(z) + \delta p \quad \text{... (9)}$$

where **q**(u, v, w), $\mathbf{q}_d$(l, r, s), **h** (h_x, h_y, h_z), $\delta\rho$, δp denote respectively the perturbations in fluid velocity **q**(0, 0, 0), particles velocity $\mathbf{q}_d$ (0, 0, 0), magnetic field **H**(H, 0, 0), density ρ and pressure p. Using equation (9) into governing equation (1) to (7) and linearizing them, we have

$$\frac{\rho}{\in}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla\delta p + g\delta p + \frac{\mu_e}{4\pi}\{\nabla\times\boldsymbol{h})\times\boldsymbol{H} + (\nabla\times\boldsymbol{H})\times\boldsymbol{h}] - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\boldsymbol{q} + \frac{kN}{\in}(\boldsymbol{q}_d - \boldsymbol{q}), \quad \text{....(10)}$$

$$\nabla.\boldsymbol{q} = 0, \quad \text{... (11)}$$

$$\in\frac{\partial \boldsymbol{h}}{\partial t} = (\boldsymbol{H}.\nabla)\boldsymbol{q} - (\boldsymbol{q}.\nabla)\boldsymbol{H}, \quad \text{...(12)}$$

$$\nabla.\boldsymbol{h} = 0 \quad \text{...(13)}$$

$$\left[\frac{m}{k}\frac{\partial}{\partial t} + 1\right]\boldsymbol{q}_d = \boldsymbol{q}. \quad \text{...(14)}$$

In addition to equations (1) to (4), we have the equation

$$\in\frac{\partial}{\partial t}\delta\rho = -w.D\rho \quad \text{... (15)}$$

which ensure that the density of every particle remains unchanged as it travel with its motion.

Applying the normal mode technique to analyze the perturbation we seek solutions whose dependence on x, y and t is given by,

$$\exp\left\{i\left(k_x x + k_y y\right) + nt\right\} \quad \text{... (16)}$$

where k_x and k_y are the horizontal components of the wave number, $k = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number and n is the growth rate, which is, in general, a complex constant.

In the considered form of the perturbations in equation (16), equations (10) to (15) gives,

$$\left[n'\frac{1}{k_1}(v + v'n)\right]\rho v = -ik_x\delta p + \frac{\mu_e}{4\pi}h_z(DH)\,, \quad \text{... (17)}$$

$$\left[n'\frac{1}{k_1}(v + v'n)\right]\rho v = -ik_y\delta p + \frac{\mu_e H}{4\pi}h_z(ik_x h_y - ik_y h_x), \quad \text{... (18)}$$

$$\left[n' + \frac{1}{k_i}(v + v'n)\right]W = -D\delta p - g\delta\rho + \frac{\mu_e H}{4\pi}\left[ik_x h_z - Dh_x - h_x\frac{DH}{H}\right], \quad \text{... (19)}$$

$$ik_x u + ik_y v + D w = 0, \quad \text{... (20)}$$

$$\in nh_x = ik_x Hu - w(DH), \quad \text{... (21)}$$

$$\in nh_y = ik_x Hv, \quad \text{... (22)}$$

$$\in nh_z = ik_x Hw, \quad \text{... (23)}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0 \quad \text{... (24)}$$

$$\in n\delta\rho = -w(D\rho). \quad \text{...(25)}$$

where $n' = \dfrac{n}{\in}\left[1 + \dfrac{mNK/\rho}{mn+k}\right], v = \dfrac{\mu}{\rho}$ and $D \equiv \dfrac{d}{dz}$.

Multiplying equations (17) by ik_x and (18) by ik_y and then adding, we get

$$\left[n' + \frac{1}{k_1}(v + v'n)\right]\rho Dw = k^2\delta p - \frac{\mu_e}{4\pi} ik_x h_x (DH) + \frac{\mu_e^2}{4\pi} \frac{H}{\in n} k_x k_y \varsigma + \frac{\mu_e H}{4\pi \in n}(DH)w. \quad \text{... (26)}$$

Multiplying equation (19) by k^2, we get

$$\left[n' + \frac{1}{k_1}(v + v'n)\right]\rho k^2 w = -Dk^2\delta p + \frac{gk^2(D\rho)w}{\in n} + \frac{\mu_e Hk^2}{4\pi}\left[ik_x h_z - Dh_x - h_x \frac{DH}{H}\right] \quad \text{... (27)}$$

Now, on subtracting equation (27) from equation (26), we obtain

$$n'\left[D(\rho Dw) - \rho k^2 w\right] + \frac{1}{k_1}\{D(\rho(v' + v'n)Dw) - \rho k^2(v + v'n)w\} + \frac{gk^2(D\rho)w}{\in n}$$

$$+ \frac{\mu_e H^2 k_x^2}{4\pi \in n}(D^2 - k^2)w + \frac{\mu_e k_x^2}{4\pi \in n} D(H^2 Dw) = 0 \quad \text{... (28)}$$

where $n' = \dfrac{n}{\in}\left[1 + \dfrac{mNK/\rho}{mn+K}\right], v = \dfrac{\mu}{\rho}$ and $D \equiv \dfrac{d}{dz}$.

ANALYTICAL DISCUSSION

(a) Two Uniform Fluids Separated by a Horizontal Boundary

Consider the case when two superposed fluids of uniform density ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2 uniform visco-elasticities μ'_1 and μ'_2, magnetic fields H_1 and H_2 separated by a horizontal boundary at $z = 0$, subscript 1 and 2 distinguish the lower and upper fluids, respectively. Then each region of constant ρ, μ, μ' and H, equation (28) reduces to

$$(D^2 - k^2)\, w = 0. \quad \text{... (29)}$$

The general solution of equation (29) is

$$w = Be^{kz} + Be^{-kz}. \qquad ...(30)$$

where A and B are arbitrary constants.

The boundary conditions to be satisfied are:

The velocity w should vanish when $z \to -\infty$ (for the lower fluid) on and $z \to -\infty$ for

(i) upper fluid.

(ii) $w(z)$ is continuous at $z = 0$.

(iii) The pressure should be continuous across the interface.

Applying the boundary conditions (i) and (ii) to (30), we have

$$w_1 = Ae^{kz} \qquad (z < 0) \qquad ...(31)$$

and

$$w_2 = Ae^{-kz} \qquad (z > 0) \qquad ...(32)$$

The same constant A being chosen to ensure the continuity at $z = 0$. Thus continuity of pressure implies that

$$\Delta_0(n'\rho Dw) + \frac{1}{k_1}\Delta_0(\rho(v + v'n)Dw) + \frac{gk^2}{\in n}\Delta_0(\rho)w_0 + \frac{k_x^2}{4\pi \in n}\Delta_0(H^2 Dw) = 0. \qquad ...(33)$$

Applying the conditions (31) and (32), to the solutions of (33), we obtain

$$n'_2\,\rho_2 Dw_2 - n'_1\,\rho_1 Dw_1 + \frac{1}{k_1}[\rho_2(v_2 + v'_2\,n)Dw_2 - \rho_1(v_1 + v'_1\,n)Dw_1]$$

$$+\frac{\mu_e k_x^2}{4\pi \in n}[H_2^2 Dw_2 - H_1^2 Dw_1] + \frac{gk^2}{\in n}(\rho_2 - \rho_2)w_0 = 0 \qquad ...(34)$$

On using $n' = \frac{n}{\in}\left[1 + \frac{mNk}{\rho(mn+k)}\right]$, equation (34) becomes

$$n^3\left[1 + \frac{\in}{k_1}(\alpha_2 v_2 + \alpha_1 v'_1)\right] + n^2\left[\frac{2NK}{\rho_1 + \rho_2} + \frac{\in}{k_1}(v_2\alpha_2 + v_1\alpha_1) + \frac{K}{m}\left\{1 + \frac{\in}{k_1}(\alpha_2 v'_2 + \alpha_1 v'_1)\right\}\right]$$

$$+n\left[\frac{\in K}{mk_1}(\alpha_2 v_2 + \alpha_{11} v_1) + 2k_x^2 V_A^2 - gk(\alpha_2 - \alpha_1)\right] + \left[\frac{K}{m}(2k_x^2 V_A^2 - gk(\alpha_2 - \alpha_1))\right] = 0$$

...(35)

where $\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}, v_{1.2} = \frac{\mu_{1,2}}{\rho_{1,2}}$ and $V_A^2 = \frac{\mu_e H_1^2}{4\pi(\rho_1 + \rho_2)} = \frac{\mu_e H_2^2}{4\pi(\rho_1 + \rho_2)}$

Theorem–1: The system is stable under the condition $\alpha_1 > \alpha_2$.

Proof: If $\alpha_1 > \alpha_2$, then for the potentially stable case, $\alpha_1 > \alpha_2$ and equation (35) does not involve any change of sign and so does not allow any positives roots. Therefore, the system is stable.

Theorem–2: The system is stable under the condition $\alpha_1<\alpha_2$, provided $2k_x^2 V_A^2 > gk(\alpha_2-\alpha_1)$.

Proof: If $\alpha_1 > \alpha_2$, then for the potentially stable case, $\alpha_1 < \alpha_2$ and if $2k_x^2 V_A^2 > gk(\alpha_2-\alpha_1)$, then equation (35) does not admit any change of sign and therefore does not allow any positive value of *n*. Therefore, the system is unstable.

Theorem–3: The system is unstable under the conditions $\alpha_1 < \alpha_2$, provided that if $2k_x^2 V_A^2 < gk(\alpha_2-\alpha_1)$.

Proof: If $2k_x^2 V_A^2 < gk(\alpha_2-\alpha_1)$, then the constant term in the equation (35) is negative, therefore allow one change of sign and so has at most one positive root. The occurrence of a positive root implies that the system is unstable.

(a) The Case of Exponentially Varying Density, Viscosity, Visco-elasticity, Magnetic Field and Particles Number Density

Let us assume that

$$\rho = \rho_0 e^{\beta z},\ N = N_0 e^{\beta z},\ \mu = \mu_0 e^{\beta z},\ H^2(z) = H_0^2 e^{\beta z} \text{ and } \mu' = \mu_0 e^{\beta z} \qquad \text{... (36)}$$

where ρ_0, N_0, μ_0, μ'_0, m_0, H_0 and β all are constants. Equation (36) shows that the coefficient of kinematic visco-elasticity v' and the Alfven velocity V_A are constant everywhere. Substituting the values of ρ, μ, μ', $H^2(z)$ and N in equation (36), we obtain

$$\left[\frac{n}{\in}V + \frac{N_0 k/\rho_0}{\in n + \frac{K}{m}} + \frac{(v_0 + v'_0\, n)}{k_1} + \frac{k_x^2 V_A^2}{\in n}\right](D^2 - k^2)w + \frac{g\beta k^2 w}{\in n} = 0. \qquad \text{...(37)}$$

where $$v_0 = \frac{\mu_0}{\rho_0},\ v'_0 = \frac{\mu'_0}{\rho_0},\ K = 6\pi\mu\eta.$$

Consider the case of two free boundaries. The boundary conditions for the case of two free surfaces are

$$w = 0,\ D^2 w = 0 \text{ at } z = 0 \text{ and } z = d. \qquad \text{... (38)}$$

The proper solution of equation (37) satisfying conditions (38) is given by

$$w = A\sin\frac{m\pi z}{d}, \qquad \text{... (39)}$$

where *A* is a constant and *m* is any integer. Using equation (39), (37) gives

$$n^3[1 + F\in] + n^2\left[\frac{K}{m}(1 + F\in) + \frac{N_0 K}{\rho_0} + \frac{\in v_0}{k_1}\right] + n\left[\frac{\in v_0 K}{m k_1} + k_x^2 V_A^2 - \frac{g\beta k^2}{L}\right] + \frac{K}{m}\left[k_x^2 V_A^2 - \frac{g\beta k^2}{L}\right] = 0, \qquad \text{.... (40)}$$

where, $F = \frac{v'_0}{k_1}$ and $L = \left(\frac{m\pi}{d}\right)^2 + k^2.$

Theorem–4: For β < 0 the system is always stable.

Proof: For the stable stratification (β < 0), equation (40) does not admit any positive value of *n* and so the system is stable for disturbances of all wave numbers.

Theorem–5: For β > 0 the system is stable or unstable according as $k_x^2 V_A^2 > or < \frac{g\beta k^2}{L}$.

Proof: Let β > 0 and $k_x^2 V_A^2 > \frac{g\beta K^2}{L}$, then equation (40) does not allow any positive value of *n* and so that the system is always stable for disturbances of all wave number. On the other hand, if $k_x^2 V_A^2 < \frac{g\beta k^2}{L}$ then the constant term in (40) is negative. Therefore allow one change of sign and so has one positive root. The occurrence of a positive root implies that the system is unstable.

In the absence of magnetic field, the system is clearly unstable for β > 0. However, the system can be completely stabilized by a magnetic field $V_A^2 > \frac{g\beta k^2}{k_x^2 L}$.

The discussion below is divided into two sections. Section-I deals with the oscillatory ($n_i \neq 0$) modes and section-2 deals with the non-oscillatory modes ($n_i = 0$).

Section – 1 Discussion of Oscillatory Modes

Equation (40) can be written as

$$An^3 + Bn^2 + Cn + D = 0, \quad ...(41)$$

where, $A = (1 + \in F), B = \frac{K}{m}(1 + F \in) + \frac{N_0 K}{\rho_0}; C = \frac{\in v_0 K}{m k_1} + k_x^2 V_A^2 - \frac{g\beta k^2}{L}$

and $D = \frac{K}{m}\left[V_A^2 k_x^2 - \frac{g\beta k^2}{L}\right]$

The real and imaginary parts of (41) after dividing by *n* are,

$$A\ (n_r^2 - n_i^2) + Bn_r + C + \frac{Dn_r}{|n|^2} = 0 \quad ...(42)$$

and

$$n_i\left[2An_r + B - \frac{D}{|n|^2}\right] = 0. \quad ...(43)$$

Theorem-6: For β < 0 the estimate of *n* of the growth rate of oscillatory stable mode are given by $|n|^2 > \frac{D}{B}$.

Proof: If β < 0 then the value of *D* is definite positive. *B* is also positive. Since the modes are oscillatory, *i.e.* $n_i \neq 0$, If n_r is negative (for stable mode), then for the consistency of (43), we must have. $|n|^2 > \frac{D}{B}$. Hence, for β < 0 the estimate of *n* for the growth rate of oscillatory stable modes are given by $|n|^2 > \frac{D}{B}$.

Theorem -7: β < 0 the estimate of *n* for the growth rate of oscillatory unstable modes are given by $|n|^2 < \frac{D}{B}$.

Proof: If β < 0 then the value of *D* is positive definite. Since the modes are oscillatory, *i.e.*, $n_i \neq 0$. If n_r is positive (for unstable mode), then for the consistency of (43), we must have $|n|^2 < \frac{D}{B}$. Hence, for β < 0. The estimate of a for the growth rate of oscillatory stable modes are given by $|n|^2 < \frac{D}{B}$.

Theorem -8: β > 0 and $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$ the estimate of *n* for the growth rate of oscillatory stable or unstable modes respectively are given by $|n|^2 > \frac{D}{B}$ or $|n|^2 < \frac{D}{B}$.

Proof: If β > 0 and $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$, the value of *D* is positive definite. *A*, *B* are already positive, since the modes are oscillatory, ($n_i \neq 0$) and stable (n_r negative) and n_r is positive (for unstable mode). Then for the consistency of (43) and under the conditions β > 0, and $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$, the estimate on *n* for the growth rate of oscillatory stable or unstable modes are given by $|n|^2 > \frac{D}{B}$ or $|n|^2 < \frac{D}{B}$.

Section- 2 Discussion of Non-oscillatory Modes

For non-oscillatory modes, we must have $n_i = 0$, then equation (41) becomes

$$A\ n_r^3 + Bn_r^2 + Cn_r + D = 0, \qquad \text{... (44)}$$

where $A = (1 + \in F)$; $B = \left[\frac{K}{m}(1 + F \in) + \frac{N_0 K}{\rho_0} + \frac{\in \nu_0}{k_1}\right]$;

$$C = \left[\frac{\in v_0 K}{k_1 m} + k_x^2 V_A^2 - \frac{g\beta k^2}{L}\right]; \;\; D = \left[K_x^2 V_A^2 - \frac{g\beta k^2}{L}\right]$$

Theorem–9: For β < 0, the non-oscillatory modes are always stable.

Proof: If β < 0, equation (44) does not involve any change of sign and therefore does not any positive roots. Therefore, the non-oscillatory modes are stable for all wave numbers according to the given condition.

Theorem–10: For β > 0, the non-oscillatory modes are stable under the condition $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$.

Proof: For β > 0 and $k_x^2 V_A^2 > \frac{g\beta k^2}{L}$, then equation (44) does not involve any change of sign and therefore does not allow any positive root. Therefore, the non-oscillatory modes are stable.

Theorem–11: For β < 0, then non-oscillatory modes are unstable if $k_x^2 V_A^2 < \frac{g\beta k^2}{L}$.

Proof: For β < 0, $k_x^2 V_A^2 < \frac{g\beta k^2}{L}$, then the value of *D* is definite negative.

Therefore equation (44) involves at least one change of sign, so (44) has at least one positive root, which implies the non-oscillatory modes are unstable.

Theorem – 12: For β > 0 and $k_x^2 V_A^2 < \frac{g\beta k^2}{L}$, there are wave propagating for a given wave number (Two damped and one amplified).

Proof: The roots of the equation (44) are $n_{r_1}, n_{r_2}, n_{r_3}$ then using the theory of equation, we get

$$n_{r_1}.n_{r_2}.n_{r_3} = -\frac{D}{A} > 0 \;\; \text{and} \;\; n_{r_1} + n_{r_2} + n_{r_3} = -\frac{B}{A} < 0$$

when β > 0 and $k_x^2 V_A^2 < \frac{g\beta k^2}{L}$, then *D* is definite negative. Also, *A* and *B* are positive definite, so that the product of the roots is positive and the sum of the roots is negative. Therefore the possibility that all the three non-oscillatory modes can be unstable, ruled out. It follow that two waves of propagation are damped and one is amplified for a given wave number.

CONCLUSION :

In the present paper, the Rayleigh-Taylor instability of visco-elastic, (Rivlin-Ericksen) dusty fluid in the presence of magnetic field through porous medium is considered. Following the linearized perturbation theory and normal mode analysis, the dispersion relation is obtained. The system is found to be always stable for $\alpha_1 > \alpha_2$ and unstable for $\alpha_1 < \alpha_2$ under certain conditions. The case of exponentially varying density, viscosity, visco-elasticity, magnetic field and particles number density is also considered. For $\beta < 0$, the system is found to be stable always. For $\beta < 0$ the non- oscillatory modes are always stable and estimate of n for the growth rate of oscillatory stable modes are given by $|n|^2 > \frac{D}{B}$.

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