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ON IRREDUCIBILITY OF AFFINE SPACE CURVES

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Our main aim here is to extend the assumptions of Torreli's theorem for any characteristic p for the curves are hyperelleptic Torrelis's theoremfor curves and surfaces are generally based in Riemann Roch theorem. Our main thrust is develop its lattice structure assuming symmetric product.

Key words : Variety, projective, biregular, Torreli's theorem hyperplane, canonical, multiplicity.

INTRODUCTION

Let X be any algebraic abstract variety defined over a field F which is algebraically closed. Let us denote the sheaf of local rings by $\theta(x).\,(x)^m = x \times x \ldots . \times x$ is the cartesian product on which the symmetric group G_q operates by permutation of the coordinate point where each operator is a biregular transformation. We choose here a suitable quotient space $(X)^{\Gamma} = (X)^q / \Gamma$, where $\Gamma \subset G_q$ equipped with natural ring structure. In particular, if any set of points of X is contained in the affine open subset of X, space $(X)^{\Gamma}$ possess a structure of algebraic variety satisfying the conditions:

(i) If X is projective, then $(X)^{\Gamma}$ is also projective

(ii) If X is locally normal, so is $(X)^{\Gamma}$

(iii) If $\Gamma = G_q, (X)^q = (X)^{\Gamma}$.

Theorem : The symmetric product $(X)^q$ of an algebraic non-singular curve X is non-singular.

Proof : Let us put $(A_1)+(A_2) \ldots \ldots + (A_q) = z$, a point of $(X)^q$ property $A_1 + \ldots + A_n = n_1 P_1 + n_2 P_2 + \ldots + n_k P_k \left(\sum n_i = q, p_i \neq p_j\right)$. The point Z is considered here on affine open set of type $(U)^{(q)}$ such that U is affine open set on X containing $P_1, P_2, \ldots P_k$. We further suppose that there is a regular function t on U such that (i) $t - (t)p_1$ is a uniformizing parameter on X

at P_1 for $1 \leq i \leq k(ii)(t)_{pi} \neq (t)_{pj}$ if $i \neq j$. We consider a regular function t_s induced on $(U)^{(q)}$ by the projection on the s-th factor by the function t on U. Putting $\varphi_1 = t_1 + t_2 \ldots + t_q$, $\ldots \varphi_q = t_1 . t_2 \ldots t_q$ to be rational regular function on (U) which is invariant under G_q.

Definition (Hyperelliptic) : Let M be a Riemann surface with genus ≥ 2. Then M is hyperelliptic if and only if there exists an integral divisor D on Mwith deg D=2, dim $L(D) \geq 2$.

Theorem : The rational map $\varphi : C' \to \Gamma$ is purely in separable.

Proof : Let C_1, C_2 be any two curves with same envelop Γ. C_1 and C_2 are birationally equivalent for which let us suppose that

$$[K(C_1) : K(\Gamma)] = p^{\alpha_1}$$

$$[K(C_2) : K(\Gamma)] = p^{\alpha_2}$$

Such that $K(C_1)p^{\alpha_1} = K(\Gamma)$ and $K(C_2)p^{\alpha_2} = K(\Gamma)$

where $$\alpha_1 \geq \alpha_2 \Rightarrow K(C_2) = K(C_1)p^{\alpha_1} - p^{\alpha_2}$$

We now show that $\alpha_1 = \alpha_2$ for which let us choose σ as the automorphism of the universal domain Ω gives by the exponentiation to the power $p^{\alpha_1 - \alpha_2}$ such that $K(C_1)^\sigma = K(C_2)$

If $f(x', y') = 0$ be equation of the curve C_1, then $f^\sigma(x'', y'') = 0$ is the equation of curve C_2.

Let us define $K(\Gamma)$ as the field given by

$$\frac{K(y'' - x'')\left(\frac{\partial f}{\partial x'}\right)}{\frac{\partial f}{\partial y}}, \frac{\frac{\partial f}{\partial x'}}{\frac{\partial f}{\partial y'}}$$

whose generators are computed at M'.

By an appropriate application automorphism σ, we obtain the required field

$$K^\sigma(y'' - x'')\left\{\left(\frac{\partial f}{\partial x'}\right) / \left(\frac{\partial f}{\partial y'}\right)^\sigma\right\}, \left\{\left(\frac{\partial f}{\partial x'}\right) / \left(\frac{\partial f}{\partial y'}\right)\right\}^\sigma$$

Since K is perfect $K^\sigma = K$

$$\Rightarrow \quad \left(\frac{\partial f}{\partial x'}\right)^\sigma = \frac{\partial f^\sigma}{\partial x''}\left(\frac{\partial f}{\partial x'}\right)^\sigma = \frac{\partial f^\sigma}{\partial y''}$$

$\Rightarrow$ σ carries the extension $K(C_1)/K(\Gamma)$ over the extension $K(C_2)/K(\Gamma)$.

$\Rightarrow$ $p^{\alpha_1} = p^{\alpha_2}$

Hence the theorem is proved.

Theorem (Hyperelliptic case) : For $p \neq 2$, Torreli's theorem is valid for hyperelliptic case. But in case $p = 2$, the rational function Q(x)=A(x) with A, B (x) with A, B in $K[x]$ such that prime divisor of B is of odd multiplicity with no common factor.

Proof : Let us consider a quadratic extension of the field K(x) generated by an equation of the type $y^2 - y = Q(x)$ where $Q(x) \in K(x)$ such that $q(x) \to Q(x) + {Q_0}^2(x)$ with $Q(x) \in$ K(x). It has been assumed here that (i) for a hyperelliptic curve X the model $(X)^{(g-1)}$ is intrinsically defined with respect to biregular transformations in terms of the field $K((X)^{(g-1)})$. (ii) The canonical system on $(X)^{(g-1)}$ has a fixed part in the subvariety which is the locus of the point $(P_1) + ({P_2}^{\sigma}) + .. (P_{g-1})$ where σ is the automorphism of X into itself. For $p = 2$, $\sigma(x, y) \to (x, y + 1)$.

We choose here a suitable canonical map j : $(X)^{(g-1)} \to {P^f}_{(g-1)}$ such that $f^{-1}(y)$ has the minimum number of distinct points $(y \epsilon P_{g-1})$

⇒ The curve C is rational normal and the obscular hyperplane at any point $P \in C$ is uniquely detemined and meets C only at P.

⇒ The locus of the points y is rational curve Γ of P_{g-1}

⇒ The set $f^{-1}(\Gamma)$ consists of the curves on which are the images on $(X)^{g-1}$ of the curve x under $F_n: Q \to h(Q) + (g - 1 - h)$

⇒ diagonal of $(X)^{g-1}$ does not be on the common part of the canonical system.

Let us further consider $K((Y)^{(g-1)}) = K((X)^{(g-1)})$ for some curve Y. We thus get a biregular map of $(Y)^{(g-1)}$ onto $(X)^{(g-1)}$ which shows that the diagonal Δ_0 of $(Y)^{(g-1)}$ is birationally equivalent with diagonal Δ_0 of $(X)^{(g-1)}$. Hence we finally conclude that

$$[K(Y): K({\Delta_0}')] = [K(X): K({\Delta_0}')] \Rightarrow K(Y) \approx K(X).$$

CONCLUSION

(i) A generic Riemann surface of genus g ≥ 3 is non-hyperelliptic.

(ii) A Riemann surface of genus at least two is hyperelliptic if and only if the number of Weierstrass points on it is equal to 2g+2.

(iii) Torreli's theorem and Abel's theorem in its Jacobian version assert that divisors of Grassmann manifold are linearly equivalent to each other.

(iv) The set of all Riemann surfaces of genus g ≥ 2 depends on 3g-3 parameters.

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