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GRAPH OF MUTUALLY DISJOINT ELEMENTS IN BCK - ALGEBRA

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Here we discuss graph of disjoint elements in BCH – algebra. Here we have developed some results connecting complete and hyper complete graphs with BCK - algebras.

Key words - Complete and hyper complete graph, Dijoint elements, BCK – algebra.

INTRODUCTION

Definition (1.1) :- BCH – algebra is a system (E, *, 0) having a non empty set E, a binary operation * and a fixed element $x, y, z \in E$ satisfied the conditions:

(i) $0 * x = 0$

(ii) $x * 0 = x$

(iii) $((x * y) * (x * z) * (z * y) = 0$

(iv) $(x * ((x * y)) * y = 0$

(v) $x * y = 0\ y * x \Rightarrow x = y$.

Definition (1.2) :-A pair $\{x, y\}$ of E is said to be mutually disjoint if

$$x * y = x \text{ and } y * x = y.$$

Recently some close relations between disjoint elements and BCK – algebras have been developed by Rashmi Rani and Puja.

We mention the results for easy reference.

Theorem (1.3):- Every finite set can be made into a BCK – algebra under a suitable binary operation such that every pair of elements the set is mutually disjoint.

Theorem (1.4):- (E, *, 0) be a finite BCK – algebra such that elements $0 \equiv a_o, a_1, \ldots.., a_{n-1}$ of E be mutually disjoint. Let b be an object not in E and let $E^1 = E \cup \{b\}$. Then there exist binary operations '0' and '•' in E such that (E^1 , •, 0) is a BCK – algebra and (E^1 , •, 0) is a BCH – algebra which is not a BCI – algebra.

EXAMPLE

Example (2.1):- Let S = (0, u, v, w} and a binary operation * be defined in S as

Table (2.1)

$*_1$	0	u	v	w
0	0	0	0	0
u	u	0	u	u
v	v	0	0	v
w	w	0	0	0

Example (2.2) :- V = {A, B, C, D, E, F, G, H} be the set of triplets with entries 0 and 1 such that

A = (0 0 0), B = (0 0 1), C = (0 1 0) , D = (0 1 1) , E = (1 0 1) , F = (1 1 0) , G = (1 1 0), H = (1 1 1)

We assume that binary operation '⊗' in V is extended by the binary operation * on S = {0, 1} given by

*	0	1
0	0	0
1	1	0

Then (V ,⊗, A) is a BCK – algebra where binary operation table is given by

Table (2.2)

⊗	A	B	C	D	E	F	G	H
A	A	A	A	A	A	A	A	A
B	B	A	B	A	B	A	B	A
C	C	C	A	A	C	C	A	A
D	D	C	B	A	D	C	B	A
E	E	E	E	E	A	A	A	A
F	F	E	F	E	B	A	B	A
G	G	G	E	E	C	C	A	A
H	H	G	F	E	D	C	B	A

The sets of distinct mutually disjoint elements are{A, B},

{A, C},{A, D}, {A, E}, {A, F}, {A, G},{A, H}, {A, B, G},

{A, C, F},{A, D, E}, {A, B, C,E}.

Example (2.3) :-Let V= {a_o, a_1, a_2, a_3, a_4,a_5} and let a binary operation ⊗ be defined in V by the following table

Table (2.3)

⊗	a_o	a_1	a_2	a_3	a_4	a_5
a_o	a_o	a_o	a_o	a_o	a_o	a_o
a_1	a_1	a_o	a_1	a_1	a_1	a_1
a_2	a_2	a_2	a_o	a_2	a_2	a_2
a_3	a_3	a_3	a_3	a_o	a_3	a_3
a_4	a_4	a_4	a_4	a_4	a_o	a_4
a_5	a_5	a_5	a_5	a_5	a_5	a_o

(V, ⊗, a_o) is a BCK – algebra by theorem (1.3).

Example (2.4) :-Let V= {a_o, a_1, a_2, a_3, a_4, a_5, a_6} and let a binary operation ⊗ be defined in V by the following table

Table (2.4)

⊗	a_o	a_1	a_2	a_3	a_4	a_5	a_6
a_o	a_o	a_o	a_o	a_o	a_o	a_o	a_o
a_1	a_1	a_o	a_1	a_1	a_1	a_1	a_1
a_2	a_2	a_2	a_o	a_2	a_2	a_2	a_2
a_3	a_3	a_3	a_3	a_o	a_3	a_3	a_3
a_4	a_4	a_4	a_4	a_4	a_o	a_4	a_4
a_5	a_5	a_5	a_5	a_5	a_5	a_o	a_5
a_6	a_6	a_6	a_6	a_6	a_6	a_6	a_o

Then (V, ⊗, a_o) is a BCK – algebra by theorem (1.4).

GRAPH OF DISJOINT ELEMENTS

Definition (3.1):-(a) A graph $G = (V, E)$ consists of a finite set V of points called vertices and set E of finite lines called edges. An edge connecting two points $x, y \in V$ is denoted by xy.

(b) A graph $G = (V, E)$ is said to be complete $x, y \in E(G)$. In other words, every vertex of is connected to every other vertex by an edges.

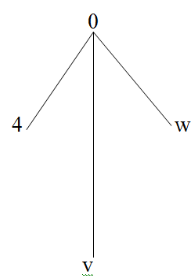
(c) A graph is said to be hyper complete if only one edge connecting any two distinct points of V is missing from a complete graph.

(d) A graph G is called a star graph if any one vertex of V is connected with all other vertices of V.

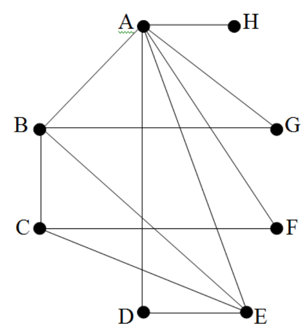
Definition (3.2) :- Let $(V, *, 0)$ be a BCK – algebra. A graph $G = (V, E)$ in V is called a graph of mutually disjoint elements if it is a simple graph in which two distinct elements u and v are connected by an edge uviff u and v are mutually disjoint.

Now we present some graphs of mutually disjoint elements for BCK – algebras discussed in above examples.

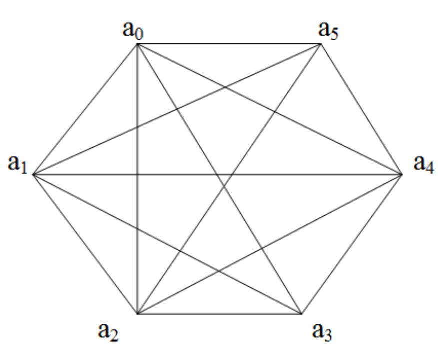
For tables 1, 2, 3 and 4 the required graphs are 1, 2, 3 and 4 respectively:



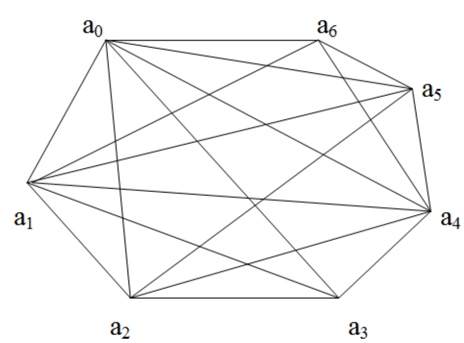
Graph 1



Graph 2



Graph 3



Graph 4

Graph 1 is a star graph which is a part of all graphs of mutually disjoint elements in all BCK – algebras, graph 2 is a simple, graph 3 is a complete graph and graph 4 is a hyper complete graphs in which edge joining a_3 and a_3 is missing.

SOME RESULTS

Now we have some results:

Theorem (4.1):- Given a complete graph with (n + 1) vertices there exists a BCK – algebra containing (n + 1) elements which are mutually disjoint and the corresponding graph of mutually disjoint elements coincides with the given graph.

Proof: Let a_0, a_1 ,a_2 ,……..a_n be the vertices of a complete graph $G = (V, E)$. We have $0 = a_0$ and define a binary operation * on V , $0 * a_i = 0$, $a_i * 0 = a_i$, $a_i * a_j = a_i$ for $i \neq j$ and $a_i * a_j = 0$ for $i = j$ where I, $j = 1, 2, \ldots, n$. Then $(V, *, 0)$ is a BCK – algebra of mutually disjoint elements by theorem ().

This means that the corresponding graph of mutually disjoint elements of V coincides with the given graph.

Theorem (4.2):-Given a hyper complete graph with (n + 1) vertices there exists a BCK – algebra. Containing $(n + 1)$ elements in which all pairs of disjoint elements, except one pair, are pairs of mutually disjoint elements such that the corresponding graph of mutually disjoint elements is the given graph.

Proof: Let $G = (V, E)$ be a hyper complete graph where

$V = \{a_0, a_1, a_2, \ldots\ldots a_n\}$. We assume that the pair $\{a_k, a_n\}$ is not connected by an edge. We take $0 = a_0$ and define a binary operation $\otimes$ on as follows:

$$0 \equiv a_0 \otimes a_i = a_0 , i = 1, 2, \ldots, n$$

$$a_i \otimes a_0 = a_i , i = 1, 2, \ldots, n$$

$$a_i \otimes a_i = 0$$

$$a_i \otimes a_j = a_i \text{ for } i \neq j , I, j = 1, 2, \ldots, n - 1.$$

For a fixed $k \neq 0$, $a_k \otimes a_n = a_n$, $a_n \otimes a_k = 0$

For $i = 1, 2, \ldots, k - 1, k + 1, \ldots, a_{n-1}$

$$a_i \otimes a_n = a_i, a_n \otimes a_i = a_n.$$

Under the above binary operation $(V, \otimes, a_0)$ is a BCK – algebra in which all pairs of distinct elements, except $\{a_k , a_n\}$ are mutually disjoint.

This means that the corresponding graph of mutually disjoint elements of V coincides with the given graph. (missing the edges a_k , a_n).

Using the results given in [ ] the above result can also be developed for graphs missing two or three edges.

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