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SOME PROPERTIES OF A LORENTZIAN PARA-SASAKIAN MANIFOLD

DR. R.C. KASHYAP

Assistant Professor, R.H. Govt. PG College, Kashipur (U.S. Nagar)

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I.P. Sasakian Manifold we have shown that the vanishing Bockhner curvature tensor yields *R* (*X*. *Y*. *T*. *T*)=0 and the decomposition if curvature tensor $R(X,Y)Z = \lambda(X,Y)FZ$ gives $\lambda(Z,T)=0$ and $\lambda(FX,FY) = -\lambda(X,Y)$ also $W(FX,FY,T)=0$.

PRELEMINARIES

A differentiable manifold of dimension n is said to be Lorentzian – Para Sasakian Manifold if it admits $a(1,-1)$ tensor field satisfying Tarafdar and Bhattacharya [9] for a vector field ξ, a covariant vector η and a Lorentzian metric g :

$$\eta(\xi) = -1 \quad \ldots(1.1)$$

$$\phi^2 = 1 + \eta \otimes \xi \quad \ldots(1.2)$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y) \quad \ldots(1.3)$$

$$(\nabla_X \phi)(Y) = [g(X,Y) + \eta(X)\eta(Y)] + [X + \eta(X)\xi]\eta(Y) \quad \ldots(1.4)$$

$$\phi(\xi) = 0, \ \eta(\phi(X)) = 0, \quad \text{rank } \phi = n-1 \quad \ldots(1.5)$$

If a L.P. Sasakian manifold *M* is η -Einstein, if Ricci-tensor S is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \quad \ldots(1.6)$$

where *X*, *Y* and *a*, *b* are some functions on *M*.

Tarafdar and Bhattacharya [9] has shown that

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) \quad \ldots(1.7)$$

$$R(\xi,X)Y = g(X,Y)\xi - g(X,Z)\eta(Y) - \eta(Y)X \quad \ldots(1.8)$$

$$R(\xi,X)\xi = X + \eta(X)\xi \quad \ldots(1.9)$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y \qquad \ldots(1.10)$$

$$S(X,\xi) = (\eta - 1)\eta(X) \qquad \ldots(1.11)$$

$$S(\phi X, \phi Y) = S(X) + (\eta - 1)\eta(X)\eta(Y) \qquad \ldots(1.12)$$

The conformal curvature tensor is given by :

$$C(X,Y)Z = R(X,Y)Z = \frac{1}{(n-2)}[g(Y,Z)\phi X - g(X,Z)\phi Y + S(Y,Z)X - S(X,Z)Y]$$

$$+\frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y] \qquad \ldots(1.13)$$

where $\qquad S(X,Y) = g(\phi X, Y)$

Let us define :

$$C(X,Y,Z,W) = g(C(X,Y)Z,W) \qquad \ldots(1.14)$$

then we get

$$C(X,Y,Z,W) = R(X,Y,Z,W) + \frac{1}{n-2}[g(Y,Z)S(X,W) - g(X,Z)S(Y,W)$$

$$+S(Y,Z)g(X,W) - S(X,Z)g(Y,W)]$$

$$+\frac{r}{(n-1)(n-2)}[g(Y,Z)g(X,W) - S(X,Z)g(Y,W)] \qquad \ldots(1.15)$$

Since

$$\underset{X,Y,Z}{G} C(X,Y,Z,W) = C(X,Y,Z,W) + C(Y,Z,X,W) + C(Z,X,Y,W) \qquad \ldots(1.16)$$

From (1.15) and (1.16), we obtain

$$\underset{X,Y,Z}{G} C(X,Y,Z,W) = \underset{X,Y,Z}{G} R(X,Y,Z)\, W$$

Theorem (1.1) : In a Lorentzian Para Sasakian manifold, the conformal curvature satisfies

$$\underset{X,Y,Z}{G} C(X,Y,Z,W) = \underset{X,Y,Z}{G} R(X,Y,Z,W)$$

Putting $W = \xi$ in (1.15), we have

$$C(X,Y,Z,\xi) = R(X,Y,Z,\xi) - \frac{1}{(n-2)}[g(Y,Z,)S(X,\xi) - g(X,Z)S(Y,\xi)$$

$$+S(Y,Z)\eta(X) - S(X,Z)\eta(Y)]$$

$$+\frac{r}{(n-1)(n-2)}[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)] \quad \ldots(1.17)$$

Keeping in view of (1.11) : (1.17) yields

$$C(X,Y,Z,\xi)=\frac{R(X,Y,Z,\xi)+[g(Y,Z]\eta(X)-g(X,Z)\eta(Y)].[R-(n-1)^2]}{(n-1)(n-2)}$$

$$+\frac{1}{n-2}[S(Y,Z)\eta(X)-S(X,Z)\eta(Y)] \quad \ldots(1.18)$$

From (1.17), we have

$$C(X,Y,Z,\xi)+C(Y,X,Z,\xi)=R(X,Y,Z,\xi)+R(Y,X,Z,\xi) \quad \ldots(1.19)$$

The Gauss equation on *M* is given by

$$\bar{R}(X,Y,Z,W)=R(X,Y,Z,W)+g(h(X,W),h(Y,Z)) \quad \ldots(1.20)$$

Taking cyclic sum on X,Y,Z , we get

$$\underset{X,Y,Z}{\bar{G}} R(X,Y,Z,W)=\underset{X,Y,Z}{G} R(X,Y,Z,W)+2[g(h(X,W),h(Y,Z)+g(h(Y,W),h(Z,X))$$

$$+g(h(Z,W),h(X,Y))] \quad \ldots(1.21)$$

Thus we have

Theorem (1.2) : In an almost Lorentzian – Para Sasakian Manifold, we have

$$\underset{X,Y,Z}{G} C(X,Y,Z)W=\underset{X,Y,Z}{G} R(X,Y,Z,W)$$

$$C(X,Y,Z,\xi)+C(Y,X,Z,\xi)=R(X,Y,Z,\xi)+R(Y,X,Z,\xi)$$

$$\bar{R}(X,Y,Z,W)=\underset{X,Y,Z}{G} R(X,Y,Z,W)+2[g(h(X,W),h(Y,Z)$$

$$+g(h(Y,W),h(X,Z))+g(h(Z,W),h(X,Y))]$$

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