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UNSTEDY MHD FLOW OF AN INCOMPRESSIBLE CONDUCTING FLUID THROUGH CYLINDRICAL POROUS DUCTS WITH PARABOLIC SECTION

DR. VISHVAS CHAND

School of Basic and Applied Science , Shobhit University Gangoh Saharanpur 247341 (U.P.) INDIA
Email: vishvas.chand@ shobhituniversity.ac.in

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In the present chapter we have investigated the unsteady flow of an incompressible conducting fluid through cylindrical porous ducts with parabolic section. The exact solution of the fluid equations for constant pressure distribution has been found. Some observations about the vorticity of the flow have been made.

Keywords: Porous medium, Magnetic field, Magnetic induction.

INTRODUCTION:

The subject of homogeneous flow through porous media has many technical and engineering applications in fields such as Petroleum industry and surface water hydrology. Muscut [18], Brydon and Dickey [3] have discussed the flow through porous media in connection with filtration. Ahamadi and Manavi [1] have derived the general equation of motion for the flow of a viscous fluid through a rigid porous medium and applied the results obtained to some basic flow problems. Mittal and Raina [13, 14] studied the vorticity of hydrodynamic and MHD flow of viscous incompressible fluid through porous media. Varshney and Sharma [25] investigated the theoretical analysis of steady viscous incompressible flow through porous medium in an inclined channel.

Narshima Murthy *et al* [19] have discussed the influences of magnetic field on the velocity of a conducting fluid in a porous media. Kumar *et al* [11] have given a theoretical analysis of an unsteady laminar flow of viscous incompressible and electrically conducting fluid through a porous medium in a channel in the presence a radial magnetic field and time dependent pressure gradient. Recently Mittal *et al* [15, 16, 17] investigated the vorticity of hydrodynamic and MHD flow of a steady viscous conducting fluid downs an inclined porous conducting plane with a bed of varying permeability. Shukla, P.K., Dhasmana V. and Bijalwan, M. [23] has recently studied the MHD flow of incompressible conducting fluid through cylindrical porous duct with parabolic section.

The purpose of this chapter is to discuss the influences of an incompressible conducting fluid through cylindrical porous duct with parabolic section while constant pressure is applied. In this case we have derived closed form solution of the governing equations and the effects of uniform applied magnetic field are indicated. Some observations have been made about the velocity and the vorticity of the flow.

NOMENCLATURE:

ρ	=	Density
$\bar{v}$	=	Velocity Vector
p	=	Pressure.
μ	=	Fluid Viscosity
k	=	Permeability
$\bar{J}$	=	Current density vector
$\bar{H}$	=	Magnetic and Induction
Ho	=	Magnetic Field

FORMULATION OF THE PROBLEM:

Let us consider the motion of an incompressible, viscous, electrically conducting fluid, Permeated by an applied magnetic field in an isotropic porous medium. The equations governing the motion are

$$\frac{\partial v}{\partial \rho} + \nabla(\rho \bar{v}) = 0 \qquad ....(1)$$

$$\rho \frac{\partial \bar{v}}{\partial t} = -\nabla \rho - \frac{\mu \bar{v}}{k} + \mu \nabla^2 \bar{v} + \bar{J} \times \bar{H} \qquad ...(2)$$

Let us consider the flow in a cylindrical porous tube with parabolic cross-section. The three impervious surface are given by

$$\psi = \text{ constant } - \text{ parabolic cylinder } y^2 + 2\psi x = \psi^2$$

$$\phi = \text{ constant } - \text{ parabolic cylinder } y^2 + 2\phi x = \phi^2 \qquad ...(3)$$

$$z = \text{ constant } - \text{ plane normal to the cylinder axis.}$$

Let ψ denotes the coordinate parallel to the flow direction, ϕ the coordinate perpendicular to the flow and z the coordinate perpendicular to ψ and ϕ coordinates. The applied magnetic field Ho is uniform and is transverse to the flow. Let us consider the uniform unsteady motion of an incompressible fluid through the cylindrical porous tube with parabolic cross-section. Let u_0 represents the suction velocity at the axis of the tube, then from equation of continuity $\frac{\partial v}{\partial z} = 0$ with the condition that at $z = 0, v = u_0$ everywhere. From the symmetry of the problem all physical variables will be functions of *z* only. Let the pressure *p* be constant. For simplicity we assume that R_m the magnetic Reynolds number is small, there by rendering Maxwell's equations redundant.

Now the equation of motion thus reduces to

$$\frac{\partial \overline{v}}{\partial t} - \frac{\mu \partial^2 v}{\rho \partial z^2} + \left(\frac{\mu}{k\rho} + \frac{\sigma H o^2}{\rho}\right) v = 0 \qquad ...(4)$$

Let $\frac{\mu}{\rho} = \alpha$ and

$$\left(\frac{\mu}{k\rho} + \frac{\sigma H o^2}{\rho}\right) = \beta \qquad ...(5)$$

Then the equation (4), becomes

$$\frac{\partial \overline{v}}{\partial t} - \alpha \frac{\mu \partial^2 v}{\partial z^2} + \beta v \qquad ...(6)$$

The boundary conditions of the problem are

$$v(z,0) = v_0$$

$$v_x(0,t) = 0$$

$$v(a,t) = 0 \qquad ...(7)$$

Applying Laplace transforms on each term of equation (4), we get

$$p\overline{v} - v(z,0) - \alpha \frac{\partial^2 \overline{v}}{\partial z^2} \beta \overline{v} = 0$$

By equation (7) this reduces to

$$\alpha \frac{\partial^2 \overline{v}}{\partial z^2} - (\beta + p)\overline{v} = \overline{v}_0 \qquad ...(8)$$

Also on applying Laplace transforms on conditions given by (7), we get

$$v_x(0,p) = 0$$

$$\overline{v}(a,p) = 0 \qquad ...(9)$$

The solution of equation (8) will be

$$\overline{v} = C \cosh\left(\sqrt{\frac{\beta + p}{\alpha}}\right) z + C_2 \sinh\left(\sqrt{\frac{\beta + p}{\alpha}}\right) z + \left(\frac{v_0}{\beta + p}\right) \qquad ...(10)$$

By applying condition (9), we get

$$C_1 = -\left(\frac{\frac{v_0}{(\beta+p)}}{\cosh\left(\sqrt{\frac{\beta+p}{\alpha}}\right)a} \right) \text{ and } C_2 = 0 \qquad ...(11)$$

Hence solution (10), becomes

$$\bar{v} = \frac{v_0}{(\beta+p)} - \left(\frac{\frac{v_0}{(\beta+p)}}{\cosh\left(\sqrt{\frac{\beta+p}{\alpha}}\right)a} \right) \cosh\left(\sqrt{\frac{\beta+p}{\alpha}}\right)z \qquad ...(12)$$

where, $\bar{v} = \int_0^\infty e^{-pt} v dt$

Taking Inverse Laplace transform of the equation (12), the velocity distribution is given by

$$v' = \exp\left\{-\frac{\mu}{\rho}\left(\frac{1}{k'}+M^2\right)\right\} - \frac{\mu}{\rho}\exp\left\{-\left(\frac{1}{k'}+M^2\right)t\right\}$$

$$\times \left[1 + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)}\exp\left\{\frac{(2n-1)^n \pi^2 \mu t}{4a^2\rho}\right\}.\cos\frac{(2n-1)}{2a}\pi z\right] \qquad ...(13)$$

$$k' = \frac{k}{a^2}, M = a\beta\sqrt{\frac{\sigma}{\rho\nu}}, v' = \frac{v}{v_0}$$

where.
$$\psi' = \frac{2\mu}{\rho}\exp\left\{-\left(\frac{1}{k'}+M^2\right)t\right\}.\sum_{n=1}^{\infty}(-1)^n \exp\left\{\frac{(2n-1)^n \pi^2 \mu t}{4a^2\rho}\right\}.\sin\frac{(2n-1)}{2a}\pi z$$

$$\psi' = \frac{a\psi}{v_0} \qquad ...(14)$$

NUMERICAL RESULTS AND DISCUSSION

Table-(1)

***k*′**=0.1, t=0.1

	Z	0.0	0.2	0.4	0.6	0.8	1.0
M=0	*v*′	-0.1403	0.0652	0.0097	0.0848	0.1592	0.2348
M=.5	*v*′	-0.1383	-0.0651	0.0081	0.0812	0.1544	0.2276
M=2	*v*′	-0.1095	-0.0593	-0.0089	0.0413	0.0916	0.1418
M=5	*v*′	-0.0212	0.0151	-0.0089	-0.0027	0.0034	0.0095

Table-(2)

$\boldsymbol{k'}$=0.2, t=0.1

	Z	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
M=0	v'	-0.1790	-0.0553	0.0683	01920	0.3157	0.4394
M=.5	v'	-0.1773	-0.0566	0.0639	0.1846	0.3053	0.4259
M=2	v'	-0.1483	-0.0654	-0.0175	0.1004	0.1833	0.2662
M=5	v'	-0.0324	0.0222	-0.0121	-0.0019	0.0082	0.0183

Table-(3)

$\boldsymbol{k'}$=0.1, t =0.1

	Z	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
M=0	v'	-0.2435×10^{-8}	-0.2434×10^{-8}	-0.2432×10^{-8}	-0.2431×10^{-8}	-0.2430×10^{-8}	-0.2428×10^{-8}
M=.5	v'	-0.1479×10^{-8}	-0.1478×10^{-8}	-0.1477×10^{-8}	-0.1476×10^{-8}	-0.1475×10^{-8}	-0.1474×10^{-8}
M=2	v'	-0.8272×10^{-12}	-0.8267×10^{-12}	-0.8262×10^{-12}	-0.8257×10^{-12}	-0.8252×10^{-12}	-0.8247×10^{-12}
M=5	v'	-0.4770×10^{-30}	-0.4767×10^{-30}	-0.4765×10^{-30}	-0.4762×10^{-30}	-0.475×10^{-30}	-0.4756×10^{-30}

Table-(4)

$\boldsymbol{k'}$=0.2, t=0.1

	Z	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
M=0	v'	-0.4833×10^{-4}	-0.4830×10^{-4}	-0.4827×10^{-4}	-0.4824×10^{-4}	-0.4820×10^{-4}	-0.4818×10^{-4}
M=.5	v'	-0.2967×10^{-4}	-0.2965×10^{-4}	-0.2963×10^{-4}	-0.2961×10^{-4}	-0.2959×10^{-4}	-0.2957×10^{-4}
M=2	v'	-0.1786×10^{-7}	-0.1785×10^{-7}	-0.1783×10^{-7}	-0.1782×10^{-7}	-0.1781×10^{-7}	-0.1780×10^{-7}
M=5	v'	-0.1051×10^{-25}	-0.105×10^{-25}	-0.1049×10^{-25}	-0.1048×10^{-25}	-0.1048×10^{-25}	-0.1047×10^{-25}

Table-(5)

$\boldsymbol{k'}$=0.1, t=0.1

	Z	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
M=0	y'	0.000	-0.1707	-0.3414	-0.5121	-0.6828	-0.8535
M=.5	y'	0.000	-0.1665	-0.3330	-0.4994	-0.6559	-0.8324
M=2	y'	0.000	-0.1144	-0.2280	-0.3433	-0.4577	-0.5721
M=5	y'	0.000	-0.0140	-0.0280	-0.0420	-0.0560	-0.0701

Table-(6)

$\boldsymbol{k'}$=0.2, t=0.1

	Z	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
M=0	y'	0.000	-0.2814	-0.5628	-0.8443	-1.1257	-1.4071
M=.5	y'	0.000	-0.2745	-0.5489	-0.8225	-1.0979	-1.3724
M=2	y'	0.000	-0.1886	-0.3773	-0.5659	-0.7546	-0.9432
M=5	y'	0.000	-0.0231	-0.0462	-0.0693	-0.0924	-0.1155

Table-(7)

$\boldsymbol{k'}$=0.1, t=0.1

	Z	0.0	0.2	0.4	0.6	0.8	1.0
M=0	y'	0.000	-0.3505×10^{-11}	-0.7010×10^{-11}	-1.0516×10^{-11}	-1.4021×10^{-11}	-1.7526×10^{-11}
M=.5	y'	0.000	-0.2126×10^{-11}	-0.4251×10^{-11}	-0.6378×10^{-11}	-0.8504×10^{-11}	-1.0630×10^{-11}
M=2	y'	0.000	-0.1156×10^{-14}	-0.2352×10^{-14}	-0.2352×10^{-14}	-0.4703×10^{-14}	-0.5879×10^{-14}
M=5	y'	0.000	-0.6761×10^{-33}	-1.3521×10^{-33}	-2.0282×10^{-33}	-2.7043×10^{-33}	-3.3804×10^{-33}

Table-(8)

k'=0.2, t=0.1

	Z	0.0	0.2	0.4	0.6	0.8	1.0
M=0	y'	0.000	-0.7721×10^{-7}	-1.5442×10^{-7}	-2.3163×10^{-7}	-3.0884×10^{-7}	-3.8605×10^{-7}
M=.5	y'	0.000	-0.4683×10^{-7}	-0.9366×10^{-7}	-1.4049×10^{-7}	-1.8732×10^{-7}	-2.3415×10^{-7}
M=2	y'	0.000	-0.2590×10^{-10}	-0.5180×10^{-10}	-0.7770×10^{-10}	-1.0360×10^{-10}	-1.295×10^{-10}
M=5	y'	0.000	-0.1489×10^{-28}	-0.2978×10^{-28}	-0.4467×10^{-28}	-0.5957×10^{-28}	-0.7446×10^{-28}

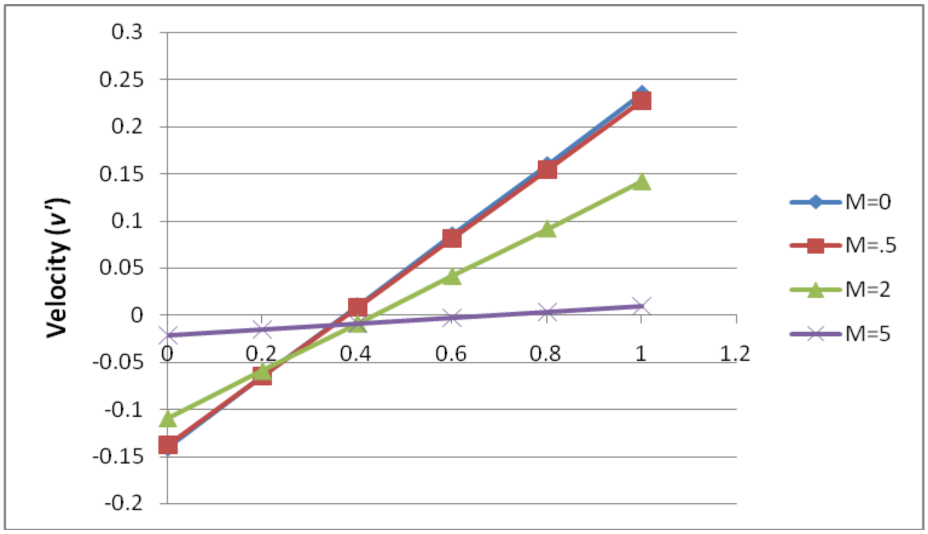

Fig. (1) Vorticity Profile at t=0.1

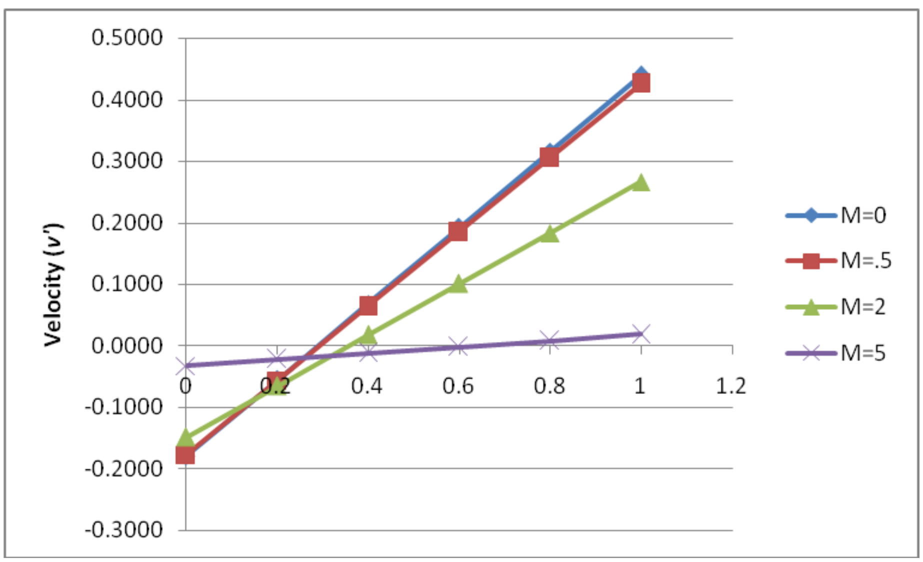

Fig. (2) Vorticity Profile at t=0.1

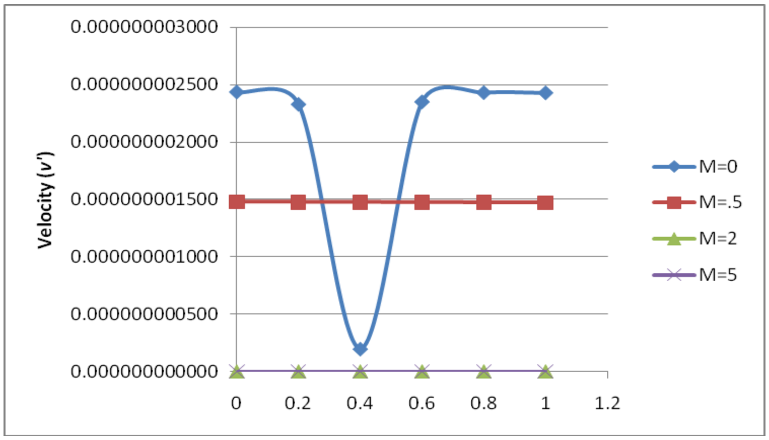

Fig. (3) Vorticity Profile at t=2

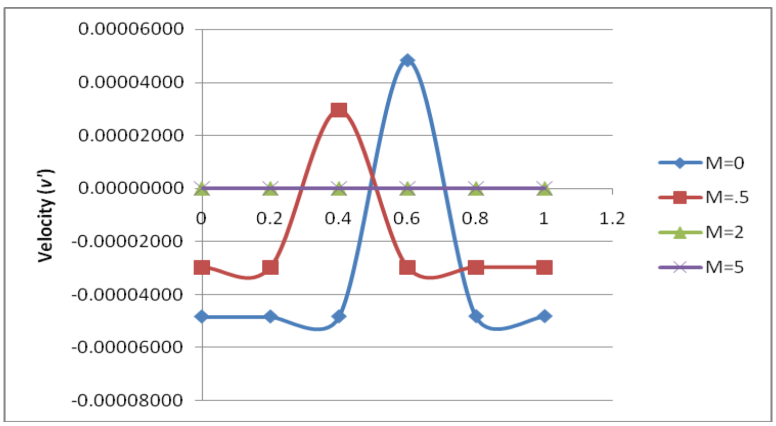

Fig. (4) Vorticity Profile at t=2

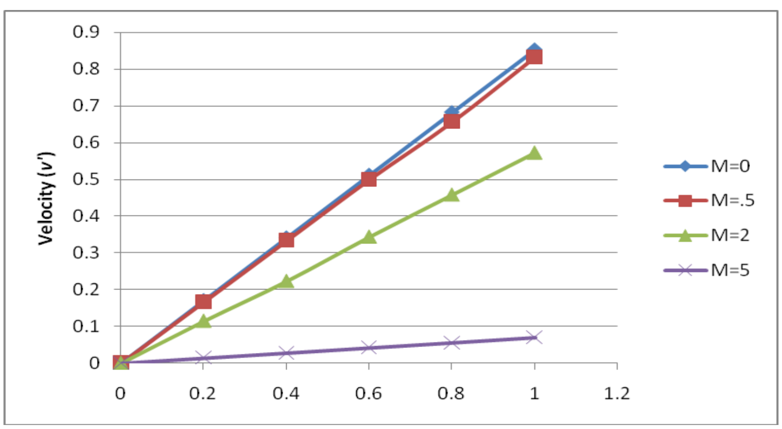

Fig. (5) Vorticity Profile at t=0.1

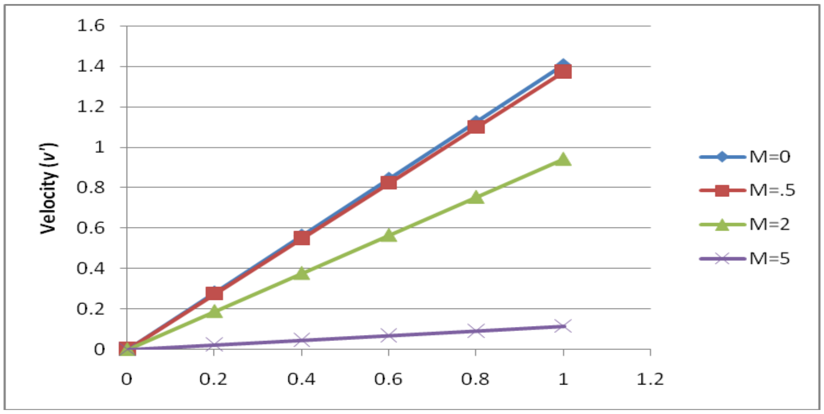

Fig. (6) Vorticity Profile at t=0.1

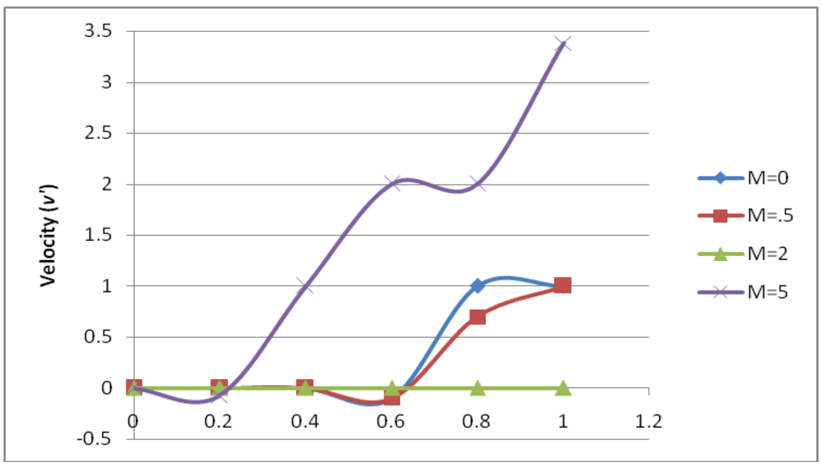

Fig. (7) Vorticity Profile at t=2

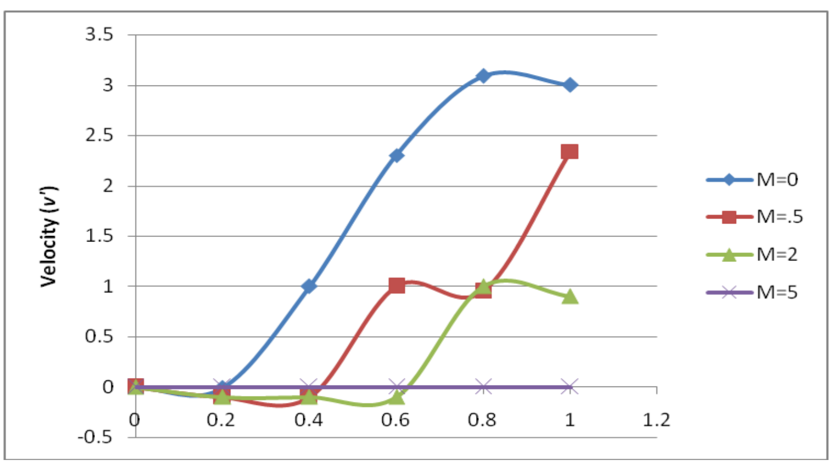

Fig. (8) Vorticity Profile at t=2

RESULTS & DISCUSSION:

From Tables 1 to 4 and figs. (1), (2), (3), (4) we observe that

As we move away from the axis of the tube the velocity decreases. Somewhere near the axis the velocity of flow becomes zero and then it again increases continuously. They region of zero velocity exists slightly away from the axis. As the value of M increases the rate of increase of velocity is almost constant. It is noticeable that at the moment when velocity is zero, vorticity does not vanish.

As the value of magnetic parameter M increases the velocity of flow decreases throughout. For small values of t, with increases in magnetic the parameter M, the velocity decreases slowly. The effect of the permeability parameter k' is to increases the velocity of the fluid. For large value of t, the velocity of fluid decreases sharply.

From Tables 5 to 8 and figs (5), (6), (7), (8) we observe that

Vorticity is zero at the axis of the cylindrical tube (with elliptic section), *i.e.*, the flow is irrotational at the axis of the tube , and as we move away from the axis of the tube vorticity comes into picture and increases with the increases in distance from the axis of the tube. For fixed t and k' with increases in magnetic parameter M the vorticity decreases slowly. For the fixed t and M the vorticity increases with increases in k', the permeability parameter. For fixed k' with increases in time t the vorticity decreases sharply. As the value of t and k' increases, value of vorticity although increase but its rate of increase decreases continuously. But for increased k' and t the role of vorticity does not remain predominant and the flow remains almost irrotational.

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