Acta Ciencia Indica, Vol. XLVI-M, No. 1 to 4, 27 (2020)

AN ESTIMATE OF ORTHOGONAL SERIES BY GENERALIZED (N, p, q) MEANS

NEETESH KUMAR

(T.G.T. Science) J.P.N. Inter College, Nawabganj, Bareilly (Uttar Pradesh)-262406

RECEIVED : 26 November, 2020

DEFINITIONS AND NOTATIONS

Let $\{\varphi_n(x)\}$ be an Orthogonrmal system defined in the interval (*a*, *b*). We suppose that *f*(*x*) belongs to L^2 (*a*, *b*) and

$$f(x) \sim \sum_{n=0}^{\infty} a_n \ \varphi_n \ (x)$$

We also write

$$B_n = (\in * \lambda)_n = \sum_{v=0}^{n} \in_{n-v} \lambda_v = \sum_{v=0}^{n} \in_{n-v} \delta_v$$

$$B_n^j = \sum_{k=j}^{n} \in_{n-k} \lambda_k \text{ and } B_n^{n+1} = 0$$

Then we have $B_n^o = B_n$

INTRODUCTION

OKUYAMA [1] proved the following theorem

Theorem : If the series

$$\sum_{n=1}^{\infty} \left\{ \sum_{j=1}^{n} \left(\frac{R_n^j}{R_n} - \frac{R_{n-1}^j}{R_{n-1}} \right)^2 |a_j|^2 \right\}^{1/2}$$

Converges, then the orthogonal series

$$\sum_{n=0}^{\infty} a_n \varphi_n \ (x) \text{ is summable } |N, p_n, q_n| \text{ almost everywhere.}$$

3. The object of the present paper is to prove the following theorem by using *G* (*N*, *p*, *q*) means of the Fourier series of function *f*. However, our theorem is as follows:

Theorem : If the series

$$\sum_{n=1}^{\infty}\left\{\sum_{j=1}^{n}\left(\frac{B_n^j}{B_n}-\frac{B_{n-1}^j}{B_{n-1}}\right)^2 \left|a_j\right|^2\right\}^{1/2}$$

Converges, then the orthogonal series

$$\sum_{n=0}^{\infty} a_n \varphi_n \ (x) \text{ is summable } |G(N,p,q)| \text{ almost every where.}$$

PROOF OF THE THEOREM:-

Let $g_n^{p,q}\ (x)$ be the $n-$ th G (N,p,q)mean of the series

$$\sum_{n=0}^{\infty} a_n \varphi_n \ (x). \text{Then}$$

we have

$$g_n^{p,q}\ (x) = \frac{1}{B_n}\sum_{k=0}^{\infty} \in_{n-k}\ \mu_k s_k\ (x)$$

$$= \frac{1}{B_n}\sum_{k=0}^{n} \in_{n-k}\ \mu_k \sum_{j=0}^{k} a_j \varphi_j\ \ (x)$$

$$= \frac{1}{B_n}\sum_{j=0}^{n} a_j \psi_j (x) \quad \sum_{k=j}^{n} \in_{n-k}\ \mu_k$$

$$= \frac{1}{B_n}\sum_{j=0}^{n} B_n^j a_j \varphi_j (x)$$

where $$s_k\ (x) = \sum_{k=0}^{n} a_k \varphi_k (x)$$

Thus we obtain

$$g_n^{p,q}(\mathrm{x}) - g_{n-1}^{p,q}(x) = \frac{1}{B_n}\sum_{j=0}^{n} B_n^j a_j \varphi_j (x) - \frac{1}{B_{\mathrm{n}-1}}\sum_{j=0}^{n-1} B_{n-1}^j a_j \varphi_j (x)$$

$$= \frac{1}{B_n}\sum_{j=1}^{n} B_n^j a_j \varphi_j (x) - \frac{1}{B_{\mathrm{n}-1}}\sum_{j=1}^{n-1} B_{n-1}^j a_j \varphi_j (x)$$

$$= \frac{1}{B_n}\sum_{j=1}^{n} B_n^j a_j \varphi_j (x) - \frac{1}{B_{\mathrm{n}-1}}\sum_{j=1}^{n-1} B_{n-1}^j a_j \varphi_j (x)$$

$$= \sum_{j=1}^{n}\left(\frac{B_n^j}{B_n}-\frac{B_{n-1}^j}{B_{n-1}}\right)\ a_j \varphi_j (x)$$

Using the Schwarz's inequality and the orthogonality, we obtain

$$\int_a^b \left|\Delta g_n^{p,q}(x)\right| dx \le (b-a)^{\frac{1}{2}} \left\{\int_a^b \left|\Delta g_n^{\epsilon,\mu}(x)\right|^2 dx\right\}^{\frac{1}{2}}$$

$$= (b-a)^{\frac{1}{2}} \left\{\sum_{j=1}^{n} \left(\frac{B_n^j}{B_n} - \frac{B_{n-1}^j}{B_{n-1}}\right)^2 \left|a_j\right|^2\right\}^{\frac{1}{2}}$$

and then, $$\sum_{n=1}^{\infty}\int_a^b \left|\Delta g_n^{p,q}(x)\right| dx \le (b-a)^{\frac{1}{2}} \sum_{n=1}^{\infty}\left\{\sum_{j=1}^{n} \left(\frac{B_n^j}{B_n} - \frac{B_{n-1}^j}{B_{n-1}}\right)^2 \left|a_j\right|^2\right\}^{\frac{1}{2}}$$

which is convergent by the assumption and from the Beppo-Leni lemma.

REFERENCES

1. OKUYAMA, Y. On absolute generalized Nörlund summability of orthogonal series, *Tamkang Journal of Mathematics*, Vol. 33, Number 2, summer, 161-165 (2002).

❑