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CERTAIN CONNEXIONS ON A DIFFERENTIABLE MANIFOLD EQUIPPED WITH UNIFIED STRUCTURES

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Here we have taken unified structure defined by Singh and Singh [6] and obtained some results keeping in view of certain connections & some structure induced in suc-manifolds.

PRELIMINARIES

Let us consider an odd dimensional real differentiable manifold $V_n (n = 2m+1)$ of class C^∞. Let there exists in V_n a vector valued C^∞ function F, a C^∞ vector T & 1 forms A satisfying Singh & Singh [6].

(a) $\overline{X} = a^2 X + cA(X)T$ …(1.1)

(b) $A(T) = \dfrac{-a^2}{x}$

(c) $A(\overline{X}) = A(FX) = 0$

(d) $\overline{T} = 0$

where a non-zero complex number and c is an integer giving the following classes:

(i) $(a = \pm i,\ c = 1)$ an almost contact structure

(ii) $(a = 1,\ c = -1)$ an almost paracontact structure

(iii) $(a = \pm 1,\ c = 1)$ an almost hyperbolic structure

Let us consider a Riemannian metric G satisfying

(a) $G(\overline{X}, \overline{Y}) = -(a^2 G(X,Y) + cA(X)\,A(Y))$, …(1.2)

(b) $G(X,Y) = A(X)$

if we consider $'F(X,Y) = G(FX,Y) = -G(X,FY)$,

then we have

(a) $'F(\overline{X}, \overline{Y}) = a^2 F(X,Y)$, …(1.3)

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(b) $'F(X,Y) = -'F(Y,X)$

(c) $'F(\bar{X},\bar{Y}) = -'F(X,Y)$

if D is the Riemannian connection on V_n then

$$(D_X\, 'F)(Y,\bar{\bar{Z}}) = (D_X\, 'F)(\bar{\bar{Y}},\bar{\bar{Z}}) \qquad \ldots(1.4)$$

V_n is said to possess a unified structure satisfying (1.1) to (1.2) if it holds,

$$'F(X,Y) = (D_x F)(Y) - (D_Y F)(X)$$

then V_n is denoted by V_n^*.

We see that in

$$'F(X,Y) = (D_x F)(Y) - (D_Y A)(X) = dA(X,Y) \qquad \ldots(1.5)$$

It is found that in V_n^*.

$$\frac{C}{(X,Y,Z)}(D_X\, 'F)(Y,Z) = 0\,, \qquad \ldots(1.6)$$

where $\dfrac{C}{(X,Y,Z)}$ is cyclic sum.

If in V_n^* there is also

$$(D_X A)(Y) + (D_Y A)(X) = 0 \qquad \ldots(1.7)$$

then V_n^* will be denoted by V_n^{**}, where we get

$$'F(X,Y) = 2(D_X A)(Y) = -2(D_Y A)(X) \qquad \ldots(1.8)$$

Mishra considered a Nijenhuis tensor $N(X,Y)$ as follows :

$$N(X,Y) = (D_{\bar{X}}\bar{Y}) - (D_{\bar{Y}}\bar{X}) + \overline{\overline{D_X Y}} - \overline{\overline{D_Y X}} - \overline{D_X Y} - \overline{D_Y X} + \overline{D_Y X} \qquad \ldots(1.9)$$

AFFINE CONNECTION

If in V_n we have

$$(D_X F)(Y) = 0 \qquad \ldots(2.1)$$

then D is called F connection.

From (2.1), using (1.1), we get

$$a^2 A(D_X Y) = -cA(X)A(D_X T) \quad \ldots(2.2)$$

Let

$$s(X,Y) = D_X Y - D_Y X - [X,Y] \quad \ldots(2.3)$$

then from (1.2), (2.2) and (1.9) we have

$$N(X,Y) = 0$$

Let us define

$$\mu(X,Y) = (D_Y A)(\bar{X}) - (D_X A)(\bar{Y}) + (D_{\bar{Y}} A)(X) - (D_{\bar{X}} A)(Y) \quad \ldots(2.4)$$

$$Y(X) = (D_T F)(X) - (D_X F)(T) - (D_{\bar{X}} T). \quad \ldots(2.5)$$

and $$\sigma(X) = (D_X A)(T) - (D_T A)(X) \quad \ldots(2.6)$$

From (2.4), (2.5) & (2.6) and using $(D_X A)(\bar{Y}) = 0$, the above reduces to

$$a^2 \mu(X,Y) = CA(X)A(D_{\bar{Y}} T) - CA(Y)A(D_{\bar{X}} T). \quad \ldots(2.7)$$

If D is an F connection then from (2.4) to (2.7), we get

$$Y(X) = -(D_{\bar{X}} T),$$

and also

$$A(X)Y(\bar{Y}) - A(Y)\sigma(\bar{X}) = Y(\bar{X}) - \sigma(\bar{Y}).$$

NEW TYPE AFFINE CONNECTION

A new type affine connection $'D$ is defined as follows

$$A(Y)D'_X T = -(D'_X A)(T)T, \quad \ldots(3.1)$$

then from (1.1) & (3.1) and on barring, we get

$$a^2 D'_X T = -cA(D'_X T)T, \quad \ldots(3.2)$$

$$Div\, X = (C : \nabla)X, \quad \ldots(3.3)$$

and $$(\nabla_X Y) \underline{\underline{def}}\, D_Y X, \quad \ldots(3.4)$$

then from (3.1) & (3.3) & (3.4), we have

$$a^2 Div\, T = -CA(D_T T). \quad \ldots(3.5)$$

Let us now take a different affine connection D^0 defined in V_n as follows

$$A(Y)D_X{}^0 T = -(D_X^0)(Y)T \quad \ldots(3.6)$$

and $$A(x)(D_X{}^0F)(Y)+(D_Y^0F)(X)=0 \qquad \ldots(3.7)$$

(3.7) reduces to

$$\overline{D_Y^0 Y}=a^2D_T^0Y+cA(D_T^0Y)T\ ,$$

where we have used (1.2) and (3.6).

MORE GENERALIZED AFFINE CONNECTION

If we use a more generalized from of affine connection D^* as follows

$$A(Y)D_X^0T+(D_X^0A)(Y)T=0 \qquad \ldots(4.1)$$

$$(D_{\bar{X}}^0F)(Y)+(D_{\bar{X}}^0F)(\bar{Y})=0 \qquad \ldots(4.2)$$

from this on using (1.1) and replacing *X* by *T*, we get

$$D_T^*\bar{Y}=\overline{D_T^*Y}\ , \qquad \ldots(4.3)$$

& $$\overline{D_T^*Y}=a^2D_T^*Y+cA(D_T^*Y)T\ . \qquad \ldots(4.4)$$

Let us define

$$B_XY\ \underline{\underline{def}}\ D_XY+H(X,Y)+H(X,Y)\ , \qquad \ldots(4.5)$$

We see that

(a) $$H(X,Y)=\frac{1}{2}\{S(X,Y)+P(X,Y)+P(Y,X)\}\ , \qquad \ldots(4.6)$$

where

(b) $g(S(Z,X),Y)=G(P(X,Y),Z)$,

(c) $P(X,Y)=A(X)Y-G(X,Y)T$

Thus we have

(a) $H(X,Y)=A(Y)X-g(X,Y)T$, …(4.7)

(b) $H(X,Y)=P(X,Y)$

also (c) $g(S(X,Y),T)=0,\quad H(X,T)=S(X,T)$.

If we define

(a) $S(X,Y,Z)\ \underline{\underline{def}}\ g(S(X,Y),z)$, …(4.8)

(b) $H(X,Y,Z)=g(H(X,Y),z)$,

Thus we get

$$'S(X,Y,Z) = A(Y)g(X,Y) - A(X)g(Y,Z)\,, \qquad \ldots(4.9)$$

(a) $c'H(\bar{X},T,\bar{Y}) = -a^2 g(\bar{X},\bar{Y}) = c'(\bar{X},T,\bar{Y})$, …(4.10)

(b) $c'H(\bar{X},T,\bar{Y}) = -a^4 g(\bar{X},\bar{Y}) = c'S(\bar{X},T,\bar{Y})$,

(c) $c'H(\bar{X},T,Y) = -a^2 F(X,Y) = c'S(\bar{X},T,Y)$

(d) $c'H(\bar{X},T,\bar{Y}) = -a^2\, 'F(X,Y) = c'S(\bar{X},T,\bar{Y})$

Theorem (4.1) : In a manifold V_n of class C^∞, we have

$$C(B_{\bar{X}} A)(\bar{\bar{Y}}) = C(X_{\bar{X}} A)(\bar{\bar{Y}}) - A^4\, 'F(X,Y)\,, \qquad \ldots(4.11)$$

From (4.5), we get

$$B_{\bar{X}} \bar{Y} = D_{\bar{X}} \bar{Y} - A^2\, 'F(X,Y)T \qquad \ldots(4.12)$$

On barring and using (1.1) and (4.6), we find

$$c(B_{\bar{X}} A)(\bar{\bar{Y}}) = C(D_{\bar{X}} A)(\bar{\bar{Y}}) - A^4 G(\bar{X},\bar{Y})\,,$$

further, we assume

$$B_X Y = D_X Y - A^2[X,Y]\,, \qquad \ldots(4.13)$$

and

$$S^*(X,Y) = B_X Y - B_Y X - [X,Y] \qquad \ldots(4.14)$$

$$= D_X Y - D_Y X - [X,Y][1 + 2A^2]$$

$$= S(X,Y) - 2A^2[X,Y]\,.$$

Theorem (4.2) : In a differential manifold V_n of class C^∞, the connection *B* will the same type as connection *D*.

Proof : Suppose *D* is F^*, M^* & O^* connection as follows

(a) $(D_X F)(Y) = 0$ …(4.15)

(b) $(D_X F)(Y) + (D_Y F)(X) = 0$

(c) $(D_X F)(\bar{Y}) + (D_{\bar{X}} F)(\bar{Y}) = 0$

keeping in view of (4.13) and (4.12) we can easily show that *B* is also F^*, M^* & O^* type connection. If we define

$$c^*(X,Y) = B_X Y - B_Y X = D_X Y - D_Y X - 2a^2[X,Y] \qquad \ldots(4.16)$$

then $$c^{*}(X,Y) = S(X,Y) - [X,Y][2a^2 - 1] \qquad \ldots(4.17)$$

From (4.16), (1.1) and barring X, Y etc., and putting $Y = T$, we get

$$c^{*}(\overline{T, \bar{X}}) = -2a^2[\overline{T, \bar{X}}] = 2a^2[\overline{\bar{X}, T}]$$

NIJENHUIS TENSOR

Let $N(X,Y) = D_{\bar{X}}\bar{Y} - D_{\bar{Y}}\bar{X} - \overline{\overline{D_Y X}} - \overline{D_{\bar{X}} Y} + \overline{D_{\bar{Y}} X} + \overline{\overline{D_X Y}} - \overline{D_X \bar{Y}} + \overline{D_Y \bar{X}}$ ...(5.1)

Similarly Nijenhuis Tensor of connection B^{*} is given by

$$N^{*}(X,Y) = B_{\bar{X}}\bar{Y} - B_{\bar{Y}}\bar{X} - \overline{\overline{B_Y X}} - \overline{B_{\bar{X}} Y} + \overline{B_{\bar{Y}} X} + \overline{\overline{B_X Y}} - \overline{B_X \bar{Y}} + \overline{B_Y \bar{X}}$$

Theorem (5.1) : If Nijenhuis Tensor N^{*} and *N* respectively are of type B^{*} and *D* affine connection then in V_n, we have

$$A(N^{*}(X,Y)) = A(N(X,Y)) - 2a^2(A(\bar{X}, \bar{Y}])$$

From (5.1) and (5.2) using (1.1), we get

$$N^{*}(X,Y) = N(X,Y) - 2a^2[(\bar{X}, \bar{Y}] + [\overline{\bar{X}, \bar{Y}}] - a^2[X,Y] - cA([X,Y])T] \qquad \ldots(5.4)$$

or from (5.4), using (1.1), we can write

$$A(N^{*}(X,Y)) = A(N(X,Y) - 2a^2[A[\bar{X}, \bar{Y}])) - a^2 A[X,Y] + a^2 A[X,Y].$$

Thus finally we get

$$A(N^{*}(X,Y)) = A(N(X,Y)) - 2a^2(A[\bar{X}, \bar{Y}])$$

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