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MULTIPLE SYNCHRONOUS VACATIONS MODEL (M/M/C) : (FCFS, SY, MV)

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In this section we consider a M/M/C queuing system with synchronous multiple working vacation policy. In multiple working vacation (MWV) policy the servers continue to take vacation till they find the system non-empty at a vacation completion.

Keywords: Multiple synchronous vacation, M/M/C queue, infinitesimal generator.

INTRODUCTION

The analysis of multi-server vacation models is far more complex when compared to single server vacation models and therefore a limited information in the literature is available for multi-server vacation models. A multi-server queuing model with MarKovian arrival and synchronous phase type vacations was formulated by Chakravarthy (2007) with the help of probabilistic role and control thresholds. Xu *et al.* (2013) derived with steady State distribution of the queue length of an M/M/C queuing system by using quasi-birth-and-death (QBD) process and a matrix-geometric solution method.

Description of the model: In this model we consider an M/M/C queuing with synchronous vacation policy. Where the customers arrive poission process with rate λ and service rate with μ and order FCFS.

If $\rho = \lambda (c\mu)^{-1} < 1$, the system is positive recurrent, and these exists the stationary distribution of the queue length.

In steady state the number of waiting customers given that all servers are busy, denoted by $Q_0^{(c)}$, follows a geometric distribution with parameter ρ. That is

$$P\left\{Q_0^{(c)} = K\right\} = (1-\rho)e^k, \quad k \geq 0 \qquad \ldots(1)$$

When the customer arrives at a state all the server are busy, this customers, conditional waiting time $W_0^{(c)}$ follows on exponential distribution with parameter $c\mu(1-\rho)$. Therefore its distribution function and LST are,

$$W_0^{(c)}\ (X) = 1 - \rho^{-c\mu\ (1-e)\chi},\ \chi \geq 0 \qquad \text{... (2)}$$

$$W_0^{*(c)}(s) = \frac{c\mu\ (1-\rho)}{S + c\mu\ (1-\rho)} \qquad \text{...(3)}$$

In multiple synchronous vacations in an M/M/C system. We take the multiserver vacation model denoted by M/M/C (SY, MV). In such a system, all *c* servers start taking a vacation together when the system becomes empty. If the system remains empty. These service take another vacation together; if there are $1 \leq i < c$ customers in the system, then *i* servers start serving customers and *c*-*i* servers stay idle. If there are $i \geq c$ customers in the system, all c servers start serving the customers and *i*-*c* customers wait in the line. We assumed that the vacation times, the service times, and the inter arrival times are mutually independent.

Let $Q_v(t)$ be the number of customers in the system at time *t* and define.

$$J\ (t) = \begin{cases} 0 \text{ the servers are not on vacations,} \\ i \ \ \text{the servers are on vacation at phase } i,\ i = 1, 2, 3,,, n \end{cases}$$

Since the vacations are synchronous, at least one server is busy during the nonvacation period, and some servers may be idle. Then the servers being idle is different from their being on vacation with (*SY*, *MV*) policy ($Q_v(t)$, J (*t*), $t > 0$) is a *QBD* process with state space.

$$\upsilon = \{(0,i) : 1 \leq i \leq \text{n}]\, U\ \{(k,i) : k \geq 1,\ 0 \leq i \leq n\} \qquad \text{... (4)}$$

The infinitesimal generator can be written as in the form

$$L = \begin{bmatrix} A_0 & C_0 & & & & \\ B_1 & A & C & & & \\ & B & A & C & & \\ & & B & A & C & \\ & & & . & . & . \\ & & & . & . & . \end{bmatrix} \qquad \text{...(5)}$$

A_0 is a square matrix of order $n^* = (c\text{-}1)\ (n+1) + n$, representing the transitions among the boundary states, where the number of customers in the system is no more than *C*-1. B_1 and C_0 are the $(n+1)\ n^*$ and $n^*\ (n+1)$ matrices.

There matrices can be written as

$$A_0 = \begin{bmatrix} A_0 & C_0 & & & & \\ B_1 & A_1 & C & & & \\ & B_2 & A_2 & C & & \\ & . & . & . & & \\ & . & . & . & & \\ & & B_{C-2} & A_{C-2} & C & \\ & & & B_{C-1} & A_{C-1} & \end{bmatrix} \qquad ...(6)$$

$$B_1 = (0, B_0), \qquad C_0 = \begin{pmatrix} 0 \\ C \end{pmatrix}$$

where $A_0 = -\lambda I + T + T^{o}\alpha$ is the square matrix of order n, $C_0 = (0, \pi I)$ is the n (n+1) matrix and $C = \lambda I$ is the square matrix of order (n+1), we have

$$A_K = \begin{bmatrix} -(\lambda + k\mu\ ) & 0 \\ T^0 & -l\ I + T \end{bmatrix}_{(n+1)\times(n+1)} \qquad 1 \le k \le C-1$$

$$B_1 = \begin{pmatrix} \mu\ \alpha \\ 0 \end{pmatrix}_{(n+1)\times n}, B_K = \begin{bmatrix} k\mu & 0 \\ 0 & 0 \end{bmatrix}, \ {}_{(n+1)\times(n+1)}, \ 2 \le k \le C-1$$

A, B and C in are all the square matrices of order n+1, as follows:

$$A = \begin{bmatrix} -(\lambda + c\mu\ ) & 0 \\ T^0 & -\lambda\ I + T \end{bmatrix}, B = \begin{bmatrix} K\mu & 0 \\ 0 & 0 \end{bmatrix}, \ C = \lambda\ I$$

Theorem : If $\rho = \lambda\,(c\mu\,)^{-1} < 1$, the matrix equation $R^2B + RA + C$=0 has the minimum non-negative solution

$$R = \begin{bmatrix} \rho & 0 \\ \rho^{e} & \lambda\,(\lambda\ I - T)^{-1} \end{bmatrix}$$

Proof: Since A, B and C are all the lower Triangular matrices, the solution to the matrix equation must have the same form. Assume that

$$R = \begin{bmatrix} r & 0 \\ \xi & H \end{bmatrix} \qquad ...(7)$$

where r is a real number, H is a square matrix of order n, and ξ is a $n \times 1$ column vector. Substituting R into the matrix equation, we have

$$C\mu\, r^2 - (\lambda + c\mu)\, r + \lambda = 0 \qquad ...(8)$$

$$H\,(-\lambda\ I + T) + \lambda\ I = 0 \qquad ...(9)$$

$$C\mu\,(r\ I + H)\,\xi - (\lambda + c\mu\,)\,\xi + HT^0 = 0 \qquad ...(10)$$

If $\rho < 1$, the equation (8) has the minimum non-negative solution $r = \rho$: The equation (9) gives $H = \lambda\,(\lambda\,I - T)^{-1}$ which is non negative, put the value of ρ and H in equation (10) and $-T_e = T^0$. Then we have

$$\xi = \frac{\lambda}{c\mu}\left\{I - \lambda\,(\lambda\,I - T)^{-1}\right\}^{-1}\left(\lambda\,I - T\right)^{-1}T^0$$

$$= \rho\left\{(\lambda\,I - \mathrm{T})\,[I - \lambda\,(\lambda\,I - \mathrm{T})^{-1}]\right\}^{-1}T^0$$

$$= \rho\,(-T)^{-1}T^0 = \rho\,e$$

Which has minimum non-negative solution.

CONCLUSION

In this paper, multi-server synchronous vacation model denoted by M/M/C is studied. Where only a certain number of servers (not all) are allowed to take a vacation each time. Based on this, the theorem is proved taking different value for the parameter C to get desirable results.

SCOPE OF FUTURE WORK

The results obtained in this paper may have potential applications in production inventory system and communication networks. This model is also applicable in certain banks and cross- border stations. This field is full of prospects for future researchers.

REFERENCES

1. Zhang, Z.G., Vickson, R.G. and Love, G.E.: The optimal service policies in an M/G/I queuing system with Multiple vocation types : *INFOR*, **39**, 357-366 (2001).
2. Zhang, Z.G. and Tian, N. (2003a): Analysis of queueing system with synchronous vacations of partial servers. *Perform, education*, **52**, 282-296.
3. Zhang, Z.G. and Tian, N. (2003b): Analysis of queuing systems with synchronous single vacations for servers. *Queuing sys.*, **45**, 161-175.
4. Chakravarthy, S.R. : A multi-server synchronous vacation model with threshold and a probabilistic decision rule, *Euro. J. Oper. Res.* Vol. **182**, p. 305-320 (2007).
5. Xu, X., Geng. J., : Stationary analysis for the fluid model driven by M/M/C working vacation queue *J. Math. Anal. Appl.* Vol. **403** P. 423-433 (2013).

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