Acta Ciencia Indica, Vol. XLVI-P, No. 1 to 4, 27 (2020)

SUPERCONDUCTIVITY IN SUPERSYMMETRIC GAUGE THEORIES

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RECEIVED : 1 March, 2021

The studies of superconductivity, dual superconductivity and color superconductivity have been undertaken through the breaking of supersymmetric gauge theories which automatically incorporate the condensation of monopoles and dyons leading to confining and superconducting phases. Constructing the total effective Lagrangian of N=2 SU(2) gauge theory with N_f=2 quark multiplets and quark chemical potential at classical and quantum levels, it has been demonstrated that baryon number symmetry is spontaneously broken as a consequence of the SU(2) strong gauge dynamics and the color superconductivity dynamically takes space at the non-SUSY vacuum.

Key words: Super symmetry; confinement; dyonic condensation;

PACS no.s: 11.10-z; 11.15-q; 11.15Ex; 11.25-w; 12.38Aw; 14.80

INTRODUCTION

All supersymmetric theories with holomorphic super-potentials have moduli space as the result of flat directions in such potentials[1-7]. All points located on these flat directions represent degenerate but physically inequivalent ground states. Such a moduli space incorporates Higgs, Coulomb and confinement phases in the theory where superconductivity, dual superconductivity and color superconductivity occur in confinement phase. Supersymmetric quantum field theories are easier to analyze and are much more tractable than non-supersymmetric theories due to the constraints which follow from supersymmetry. In particular, the homomorphicity of the superpotential when combined with global symmetries enables one to find many exact results[8,9]. In all these exact solutions, the singularities of quantum moduli space of the theory correspond to the appearance of mass less monopoles and dyons. Consequently, the microscopic superpotential, explicitly breaking N=2 to N=1 supersymmetry, has been introduced [10,11] to explore physics near N=2 singularities and it has been found that the generic N=2 vacuum is lifted leaving only a singular loci of moduli space at the N=1 vacua where monopole and dyons can condensate leading to confinement and superconductivity.

In the present paper we have undertaken the studies of superconductivity and dual superconductivity through the breaking of supersymmetric gauge theories which automatically incorporate the condensation of monopoles and dyons leading to confinement phase and superconducting phase. Constructing the effective Lagrangian near singularity u = $\wedge^2$ in moduli space for N=2 supersymmetric theory with SU(2) gauge group, it has been shown that when the mass term is added to this Lagrangian, the N=2 supersymmetry is reduced to N = 1 supersymmetry containing mass less monopoles (0, 1) at u = $\wedge^2$. The addition of a mass less term to the low energy Lagrangian near the singularity u=-$\wedge^2$ has been shown to yield the dyonic condensation which leads to confinement and superconductivity as the consequence of generalized Meissner effect [12-14].

SUPERSYMMETRIC DYONS IN N = 2 THEORY

The simplest four dimensional supersymmetric model in which boundary terms enter as central charge[15] may be constructed in terms of the following $N = 2$ supersymmetric dyonic Lagrangian[16] in SU(2) gauge theory with the generalized gauge field strength $G_{\mu\nu}$;

$$L = -\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + \frac{1}{2}(D_\mu\phi)^a(D_\mu\phi)_a + \frac{1}{2}(D_\mu P)^a(D^\mu P)_a$$

$$+i\bar{\psi}a\gamma^\mu(D_\mu\psi)_a - |q|\in_{abc}\bar{\psi}^a\gamma_5\psi^c P^b - i|q|\in_{abc}\bar{\psi}^a\psi^b P^c - V(\phi,P) \quad ...(2.1)$$

where

$$V(\phi,P) = \frac{1}{4}(|q|\in_{abc}\phi^b\phi^c)^2 + \frac{1}{4}[|q|\in_{abc}P^bP^c]^2$$

$$-\frac{1}{4}(|q|\in_{abc}\phi^bP^c)^2 \quad ...\ (2.2)$$

ψ is Dirac spinor, ϕ is scalar field and P is pseudo scalar field (all these fields are in the adjoint representation of the gauge group). In this model the vacuum energy V (ϕ, P) is independent of values of ϕ and P in certain directions in field space. As long as ϕ and P commute, the vacuum energy is classically zero. ϕ and/or P may have non-zero vacuum expectation values, spontaneously breaking some of gauge symmetries.

The nature of dyons is strongly perturbed by fermionic sector which couples with them since the zero energy solutions for dyons continue to exist for both isospinor and isovector fermions. The simplest four-dimensional model, in which boundary terms enter as external electric and magnetic charges in $N = 2$ supersymmetric theory, may be obtained [17,18] in terms of the Lagrangian density given by equation (2.1) where fields are in adjoint representation of SU(2) and the classical potential $V(\phi, P)$ is given by

$$V(\phi, P) = \frac{1}{|q|^2} t_r (\phi, P)^2 \qquad \text{...(2.3)}$$

In Prasad – Sommerfield limit [19], we have

$$V(\phi, P) = 0 \qquad \text{... (2.4)}$$

but $$< 0 | \phi | 0 >= \nu \neq 0 \qquad \text{... (2.4a)}$$

where $$\bar{\phi}^a = A\phi^a + BP^a \qquad \text{... (2.5)}$$

with A and B as constants satisfying the condition

$$A^2 + B^2 = 1$$

Then the Lagrangian density (2.1) yields the following expressions for electric and magnetic fields as zero order solution of equations (2.4);

$$E_i^a = (D_i \phi)^a \sin\alpha$$

$$B_i^a = (D_i \phi)^a \cos\alpha \qquad \text{... (2.6)}$$

where $\alpha = \tan^{-1} e / g$

In the classical potential (2.3) of $N = 2$ theory without hypermultiplets (*i.e.* without quarks) let us set $P = \phi^\dagger$ such that

$$V(\phi, P) = V\ |\phi| = \frac{1}{(g')^2} tr(\phi, \phi^\dagger)^2 \qquad \text{... (2.7)}$$

where g' is the gauge coupling constant of the underlying microscopic theory. As long as ϕ and $\phi^\dagger$ commute, the scalar potential $V(\phi) = 0$, even for non-vanishing expectation value of ϕ, given by equation (2.4a), which spontaneously breaks SU(2) to U(1) showing that the theory has a continuum of gauge in-equivalent vacua called the classical moduli space parameterized by[7]

$$u = tr\phi^2 = \frac{1}{2}v^2 \qquad \text{... (2.8)}$$

where for SU(2) gauge group we have set

$$\phi = \frac{1}{2}v\,\sigma^3$$

with $$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This parameter u is a good local coordinate on the classical moduli space. Such a classical moduli space is the consequence of existence of flat directions along which the scalar potential (2.7) vanishes. All points located on these flat directions represent degenerate but physically inequivalent ground states. Defining electric and magnetic charge numbers n_e and n_m respectively, as follows in the classical moduli space;

$$n_e = \frac{e}{\sqrt{2}} \quad \text{and} \quad n_m = \left(\frac{g^{i2}}{8\pi}\right)\frac{g}{\sqrt{2}}, \qquad \text{... (2.9)}$$

the dyonic charge may be written as

$$q = e - ig = \sqrt{2}\left(n_e - \frac{8\pi i n_m}{g^{i\,2}}\right) \qquad \text{... (2.10)}$$

and the dyonic mass may be written as

$$M = \sqrt{2}v\left|n_e - \frac{8\pi i n_m}{g'^2}\right| = \sqrt{2}v\,|\,n_e - \tau_{cl}n_m\,|, \qquad \text{... (2.11)}$$

where $$\tau_{cl} = \frac{8\pi i}{g'^2} \qquad \text{...(2.12)}$$

Then equation (2.10) may also be written as

$$q = \sqrt{2}(n_e - \tau_{cl}\,n_m) = (n_e\,, n_m) \qquad \text{... (2.13)}$$

Setting $$v = a$$

and $$\tau_{cl}v = a_D\ , \qquad \text{... (2.14)}$$

the equation (2.11) for mass of BPS state may be written as

$$M = \sqrt{2}\,|\,Z\,| \qquad \text{... (2.15)}$$

where $$Z = an_e - a_D\,n_m \qquad \text{... (2.16)}$$

is the central charge of supersymmetric algebra. The explicit form of $a(u)$ and $a_D(u)$ is given by the following relations [17, 18, 20]

$$a(u) = \sqrt{\frac{1+u}{2}} F\left(-\frac{1}{2}, \frac{1}{2}, 1;\ \frac{2}{1+u}\right) \qquad \text{... (2.17)}$$

and
$$a_D(u) = i\sqrt{\frac{u-1}{2}} F\left(-\frac{1}{2}, \frac{1}{2}, 1;\ \frac{1-u}{2}\right) \qquad \text{... (2.18)}$$

where $F(\alpha, \beta, \gamma; x)$ is usual hyper geometric function defined as

$$F(\alpha,\ \beta, \gamma, x) = \frac{1}{B(\beta, \gamma-\beta)} \int_0^1 (1-t)^{\gamma-\beta-1} t^{\beta-1} (1-xt)^{-\alpha}\, dt$$

with $B(\beta, \gamma-\beta)$ as the usual β-function. From these equations we get

$$\tau = \frac{da_D}{da} = \frac{iF\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{u-1}{u+1}\right)}{F\left(\frac{1}{2}, \frac{1}{2}, 1;\ \frac{2}{u+1}\right)} = \tau_{11} \qquad \text{... (2.19)}$$

which blows up at $u = \pm 1$ and $u = \infty$, showing that the branch points $u = \pm 1$ are the singularities of the moduli space.

For the dynamically generated mass scale $\wedge$ different from 1, we get the following generalization of equations (2.17) and (2.18)

$$a(u) = \sqrt{\frac{\wedge^2 + u}{2}} F\left(-\frac{1}{2}, \frac{1}{2}, 1;\ \frac{2}{1+u/\wedge^2}\right) \qquad \text{... (2.20)}$$

and
$$a_D(u) = i\sqrt{\frac{\wedge^2 + u}{2}} \left[F\left(-\frac{1}{2}, \frac{1}{2}, 1;\ \frac{u-\wedge^2}{\mu+\wedge\pi^2}\right) - F\left(\frac{1}{2}, \frac{1}{2}, 1;\ \frac{u-\wedge^2}{u+\wedge^2}\right)\right] \qquad \text{... (2.21)}$$

$$= i\sqrt{\frac{u-\wedge^2}{2}} F\left(\frac{1}{2}, \frac{1}{2}, 2;\ \frac{1-u/\wedge^2}{2}\right), \qquad \text{... (2.22)}$$

where we have used the linear transformations of hyper-geometric functions. From these relations we get

$$\tau_{11} = \frac{\partial a_D}{\partial a} = \frac{iF\left(\frac{1}{2}, \frac{1}{2}, 1;\ \frac{u-\wedge^2}{u+\wedge^2}\right)}{F\left(\frac{1}{2}, \frac{1}{2}, 1;\ \frac{2\wedge^2}{u+\wedge^2}\right)} \qquad \text{... (2.23)}$$

which is simple generalization of equation (2.19). It blows up at the cuts $u = \pm\wedge^2$. At a point near the infinity, the asymptotic behavior of the functions $a(u)$ and $a_D(u)$, given by equations (2.17) and (2.18), for $\wedge = 1$, may readily be derived using the asymptotic form of hyper- geometric function. We thus get the following asymptotic behavior of these functions;

$$a(u) \to \sqrt{\frac{u}{2}}$$

$$a_D(u) \to \frac{i}{\pi}\sqrt{2u}\ [\ln u + 3\ln 2 - 2] \qquad \text{... (2.24)}$$

Relation (2.14) may also be written as

$$\frac{\partial F}{\partial a} = a_D,$$

where
$$F(v) = \frac{4\pi i v^2}{g'^2} \qquad \text{... (2.25)}$$

is a holomorphic function.

Under the duality transformation

$$\tau_{cl} \to \tau_{cl} + 1 \qquad \text{... (2.28)}$$

we have
$$(n_e, n_m) \to (n_e - n_m,\ n_m) \qquad \text{...(2.29)}$$

which incorporates the transformation of a monopole (0, n_m) to a dyon ($-n_m$, n_m) and the transformation of a dyon (1,1) to a monopole (0,1).

In Coulomb gauge the gauge theory has a massless photon and hence it is subject to the standard electric–magnetic-duality,

$$q = (n_e, n_m) \to (-n_m, n_e) = \left(\frac{-1}{\tau_{cl}}\right) q \qquad \text{... (2.30)}$$

which incorporates the inversion of τ_{cl}. The transformations (2.28) and (2.30) generate an infinite duality group SL(2,Z). Thus there is a natural family of parameters related by SL(2, Z) and in classical moduli space the singular points are associated with extra massless particles.

If a general matrix $\Gamma \in SL(2, Z)$ is taken as

$$\Gamma = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = M, \qquad \text{... (2.31)}$$

then the transformation

$$\begin{pmatrix} a_D \\ a \end{pmatrix} \to \begin{pmatrix} a'_D \\ a' \end{pmatrix} = M \begin{pmatrix} a_D \\ a \end{pmatrix}, \qquad \text{... (2.32)}$$

gives

$$a_D = \alpha a_D + \beta a \qquad \text{...(2.33)}$$

$$a' = \gamma a_D + \delta a$$

which give the following transformation of the periodic matrix τ_{11}

$$\tau'_{11} = \frac{\beta + \alpha\tau_{11}}{\delta + r\tau_{11}} \qquad \text{... (2.34)}$$

It gives the following modular transformation of the holomorphic function *F* (a);

$$F'(a) = F_\tau(a') = \frac{1}{2}\beta\delta a^2 + \frac{1}{2}\alpha\gamma a_D^2 + \beta\gamma a_D a + F(a) \qquad \text{... (2.35)}$$

Thus for

$$M = S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \text{... (2.36)}$$

we get

$$a'_D = a; \;\; a' = -a_D; \qquad \text{...(2.37)}$$

$$\tau'_{11} = \tau^S_{11} = -\frac{1}{\tau_{11}} \qquad \text{...(2.38)}$$

and

$$F'(a) = F_s(a^s) = -aa_D + F(a) \qquad \text{... (2.39)}$$

Similarly, for

$$M = T = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \qquad \text{... (2.40)}$$

we have

$$a'_D = a_D - a = a^T_D;$$

$$a' = a = a^T; \;\; \tau_{11} = 1 + \tau_{11} \qquad \text{... (2.41)}$$

and

$$F'(a) = F_T(a^T) = \frac{1}{2}a^2 + F(a) \qquad \text{... (2.42)}$$

The transformation (2.30) incorporates the transformation of an electric charge (1, 0) to a monopole (0, 1) *i.e.* it leads to the monopole region. On the other hand the transformation (2.28) transforms a monopole (0, 1) to a dyon (–1, 1) *i.e.* it leads to the dyon region. The corresponding equations (2.37) and (2.41) show that in these monopole and dyon regions the natural independent variables to be used are

$$a^{(m)} = -a_D \qquad \text{... (2.43)}$$

and

$$a^{(d)} = a_D - a \qquad \text{... (2.44)}$$

respectively, with the corresponding prepotentials $F^{(m)}_{(a^m)}$ and $F^{(d)}_{(a^d)}$, given by equations (2.39) and (2.42) respectively.

SUPERCONDUCTIVITY THROUGH BREAKING OF N = 2 SUPERSYMMETRY TO N=1 THEORY IN THE ABSENCE OF HYPERMULTIPLETS

In the absence of flavor degree of freedom, the effective Lagrangian near the singularity $u = \wedge^2$ in the mass less theory contains the monopole fields $(M, \tilde{M})$ [21]. For the prepotential $F(a)$, given by equation (2.25) with $v = a$ as the scalar component of vector multiplet A^{α} associated with the generators of SU(2) and superpotential W^{a}, the effective Lagrangian is [8,11]

$$L_{eff} = L_{HM} + L_{VM} \qquad \text{... (3.1)}$$

where $$L_{VM} = \frac{1}{4\pi} I_m \left[\int d^4\theta \frac{\partial F}{\partial A} \bar{A} + \frac{1}{2} \int d^2\theta \frac{\partial^2 F}{\partial A^2} W_{\alpha} W^{\alpha} \right] \qquad \text{... (3.2)}$$

and $$L_{HM} = \int d^4\theta [M^{\uparrow} e^{(2n_m V_D + 2n_e V)} M + \widetilde{M}^{\uparrow} e^{(-2n_m V_D - 2n_e V)} \widetilde{M}]$$

$$+ \int d^2\theta \{(n_m A_D + n_e A) M \widetilde{M} + H.C\} \qquad \text{... (3.3)}$$

where V_D is the dual of vector superfield V and A_D is the dual of chiral superfield A, and the effective coupling τ in the vacuum parameterized by a is given by equation (2.12) which may also be written as $\tau(v) = \frac{\partial^2 F}{\partial v^2}$ with $v = a$. The variable θ in these equations denotes the vacuum deformation due to SU(2) strong gauge dynamics such that the relations (2.12) becomes

$$\tau = \frac{\partial^2 F}{\partial a^2} = \frac{\partial^2 F}{\partial A^2} = \frac{8\pi i}{g'^2} + \frac{\theta}{2\pi} \qquad \text{... (3.4)}$$

A_D, dual of A, in these relations is the chiral multiplet in $N = 2$ vector multiplet of the dual photon with A as its scalar component. n_e and n_m are the electric and magnetic charge numbers defined by equations (2.9). Let us add the mass term

$$m\phi^2 |_F = mU |_F \qquad \text{... (3.5)}$$

to the Lagrangian (3.1). Let us denote this mass as adjoint mass m_{ad}, then the equation (3.1) for Lagrangian takes the following form

$$L_{eff} = L_{HM} + L_{VM} + L_{soft}, \quad \text{...(3.6)}$$

where

$$L_{soft} = m_{ad} \int d^2\theta tr A^2 + H.C. \quad \text{...(3.7)}$$

Then the exact full superpotential (in absence of hyper multiplets) is

$$V = \sqrt{2}(n_m A_D + n_e A) M \widetilde{M} + mU(A, A_D) \quad \text{... (3.8)}$$

and the supersymmetry $N = 2$ is reduced to $N = 1$ supersymmetry.

At the singularities $u = \wedge^2$ of the quantum moduli space, a monodromy arises from the massless monopole (0, 1) (*i.e.* for $n_e = 0$). Then the condition of masslessness, when imposed on equation (2.11) for mass, gives

$$a_D = 0, \quad \text{... (3.9)}$$

which implies that

$$A_D = 0 \quad \text{...(3.10)}$$

The same result may be obtained by extremizing V of equation (3.8) with respect to M for $n_e = 0$, *i.e.*

$$\frac{\partial V}{\partial M} = 0$$

which implies $A_D = 0$.

Extremization of V of equation (3.8) with respect to A_D, yields the monopole condensation

$$< M > = < \bar{M} > = [-mU'(o)/\sqrt{2}]^{1/2}$$

$$= (2im\wedge)^{1/2} \quad \text{... (3.11)}$$

where $U'(0)$ is the derivative with respect to A_D at $A_D = 0$. The value of condensation

$$< u > = < tr\phi^2 >, \quad \text{... (3.12)}$$

is fixed at $\wedge^2$. This monopole condensation at singularity $u = \wedge^2$ of quantum moduli space leads to confinement and superconductivity as the consequence of dual Meissner effect [22,23].

Similarly at the singularity $u = -\wedge^2$, another monodromy arises from the vanishing mass of dyons (1, –1) for which equations (2.41) become

$$a_D \to a'_D = a_D + a$$

indyonic region. Thus near the singularity $u = -\wedge^2$ of the quantum moduli space, the low energy Lagrangian has the same structure as (3.1) with the replacement

$$M, \widehat{M} \to N, \widehat{N}, \quad A_D' \to A_D' A_D + A \qquad ...(3.13)$$

where $N, \widehat{N}$ denotes dyonic field. The addition of mass term (3.5) here yields the dyonic condensation

$$< N > = < \bar{N} > = [-mU'(o)/\sqrt{2}]^{1/2} \qquad ... (3.14)$$

where $U'(o)$ is the derivative with respect to A_D' at $A_D' = 0$. This dyonic condensation in the breaking of $N = 2$ supersymmetry to $N = 1$ theory leads to confinement and superconductivity as the consequence of generalized Meissner effect[12-14].

DISCUSSION

The simplest four-dimensional model, in which boundary terms enter as external electric and magnetic charges in $N = 2$ supersymmetric theory, has been obtained in terms of the Lagrangian density given by equation (2.1) where fields are in adjoint representation of SU(2) and the classical potential $V(\phi, P)$ is given by equation (2.3). Equation (2.15) gives the mass of BPS state in terms of supersymmetric central charge given by equation (2.16). Equation (2.29) incorporates the transformation of a monopole $(0, n_m)$ to a dyon $(-n_m, n_m)$ and the transformation of a dyon (1,1) to a monopole (0,1) under the duality transformation (2.28). In Coulomb gauge the gauge theory has a massless photon and hence it is subject to the standard electric – magnetic-duality given by equation (2.30) which incorporates the inversion of τ_{cl}. The transformations (2.28) and (2.30) generate an infinite duality group SL(2,Z). Thus there is a natural family of parameters related by SL(2, Z) and in classical moduli space the singular points are associated with extra massless particles. The transformation (2.30) incorporates the transformation of an electric charge (1, 0) to a monopole (0, 1) *i.e.* it leads to the monopole region. On the other hand the transformation (2.28) transforms a monopole (0, 1) to a dyon (–1, 1) *i.e.* it leads to the dyon region. The transformation (2.30) incorporates the transformation of an electric charge (1, 0) to a monopole (0, 1) *i.e.* it leads to the monopole region. On the other hand the transformation (2.28) transforms a monopole (0, 1) to a dyon (–1, 1) *i.e.* it leads to the dyon region. The corresponding equations (2.37) and (2.41) show that in these monopole and dyon regions the natural independent variables to be used are those given by equations (2.43) and (2.44) respectively, with the corresponding prepotentials $F^{(m)}_{(a^m)}$ and $F^{(d)}_{(a^d)}$, given by equations (2.39) and (2.42) respectively.

Monopole condensation given by equation (3.11) at singularity $u=\wedge^2$ of quantum moduli space leads to confinement and superconductivity as the consequence of dual Meissner effect. Similarly at the singularity $u = -\wedge^2$, another monodromy arises from the vanishing mass of dyons (1,-1) in the dyonic region.Thus near the singularity $u = -\wedge^2$ of the quantum moduli space, the low energy Lagrangian has the same structure as (3.1) with the replacement given by equation (3.13). The addition of mass term (3.5) here yields the dyonic condensation given by equation (3.14). This dyonic condensation in the breaking of $N = 2$ supersymmetry to $N = 1$ theory leads to confinement and superconductivity as the consequence of generalized Meissner effect.

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