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SUPERCONDUCTIVITY DUE TO CONDENSATION OF MONOPOLES

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The study of the condensation of monopoles and the resulting chromo magnetic superconductivity has been undertaken in restricted chromo dynamics of SU(2) gauge theory. Constructing the RCD Lagrangian and the partition function for monopoles in terms of string action and the action of the current around the strings, the monopole current in RCD chromo magnetic superconductor has been derived and it has shown that in London' limit the penetration length governs the monopole density around RCD string in chromo magnetic superconductors while with finite (non-zero) coherence length the leading behavior of the monopole density at large distances from the string is controlled by the coherence length and not by the penetration length.

Key words : Restricted chromo dynamics; chromo magnetic superconductivity; monopole condensation; partition function; penetration length; coherence length.

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INTRODUCTION

Quantum chromo dynamics (QCD) is the most favored color gauge theory of strong interaction where as superconductivity is a remarkable manifestation of quantum mechanics on a truly macroscopic scale. In the process of current understanding of superconductivity, Rajput *et al* [1-4] have conceived its hopeful analogy with QCD and demonstrated that the essential features of superconductivity i.e., the Meissner effect and flux quantization, provided the vivid models [5-9] for actual confinement mechanism in QCD. Mandelstam [10-12] propounded that the color confinement properties may result from the condensation of magnetic monopoles in QCD vacuum. In a series of papers [13-16] Izawa and Iwazaki made an attempt to analyze a mechanism of quark confinement by demonstrating that the Yang-Mills vacuum is magnetic superconductor and such a superconducting state is considered to be a condensed state of magnetic monopole. The condensation of magnetic monopole incorporates the state of magnetic superconductivity [17] and the notion of chromo magnetic superconductor where the Meissner effect confining magnetic field in ordinary superconductivity would be replaced by the chromo-electric Meissner effect (*i.e.*, the dual Meissner effect), which would confine the color electric flux. As such one conceives the idea of correspondence between quantum chromo dynamic situation and chromo-magnetic superconductor. However, the crucial ingredient for condensation in a chromo magnetic superconductor would be the non-Abelian force in contrast to the Abelian ones in ordinary superconductivity. Topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles [18]. The method of Abelian projection is one of the popular approaches to confinement problem, together with dual superconductivity [19,20] picture, in non-Abelian gauge theories. Monopole condensation mechanism of confinement (together with dual superconductivity) implies that long-range physics is dominated by Abelian degrees of freedom [21] (Abelian dominance).

Evaluating Wilson loops under the influence of the Abelian field due to all monopole currents, monopole dominance has been demonstrated [21, 22]. In the Abelian projection the quarks are the electrically charged particles and, if monopoles are condensed, the dual Abrikasove string carrying electric flux is formed between quark and anti-quark. Due to non-zero tension in this string, the quarks are confined by the linear potential. The conjecture that the dual Meissner effect is the color confinement mechanism is realized if we perform Abelian projection in the maximal gauge where the Abelian component of gluon field and Abelian monopoles are found to be dominant [23, 24]. Then the Abelian electric field is squeezed by solenoidal monopole current [25]. The vacuum of gluodynamics behaves as a dual superconductor and the key role in dual superconductor model of QCD is played by Abelian monopole. Therefore an important problem, before studying the vacuum properties of non-Abelian theories, is to abelianize them so as to make contribution of the topological magnetic degrees of freedom to the partition function explicit. To meet this end, a dual gauge theory called restricted chromo dynamics (RCD) (*i.e.*, an Abelian version of non-Abelian QCD) has been constructed out of QCD in SU(2) theory [26-29] by imposing an additional internal symmetry named magnetic symmetry [30-34] which reduces the dynamical degrees of freedom. Attempts have been made [1-4] to establish an analogy between superconductivity and the dynamical breaking of magnetic symmetry, which incorporates the confinement phase in RCD vacuum.

In the present paper this structure of RCD has been used to undertake the study of condensation of monopoles and the resultant chromo-magnetic superconductivity in SU(2) gauge theory. The RCD Lagrangian density for monopoles has been derived in magnetic gauge and the resulting partition function has been computed in terms of string action and the action of current around the strings. Using this partition function, the quantum average of Wilson loop for monopoles has been computed and the sources of electric flux (*i.e.* quarks) running along the trajectory have been introduced with the help of Wilson loop.

The monopole current in RCD chromo magnetic superconductor has been derived in London limit which corresponds to infinitely deep Higgs potential leading to vanishing coherence length. It has been shown that the squared monopole current in RCD chromo magnetic superconductor in the London limit has a maximum at the distance of the order of penetration length and it (the penetration length) governs the monopole density around the string in RCD chromo-magnetic superconductor. The monopole current has also been derived in RCD chromo magnetic superconductor with non-zero finite coherence length and it has been shown that the monopole density is non-zero even in the absence of string. It has also been shown that the quantum correction to the squared monopole density is much more than its vacuum expectation value measured far outside the string. It has been demonstrated that in the chromo magnetic superconductors with finite (non-zero) coherence length the quantum corrections to squared monopole density control the leading behavior of the total monopole density in the vicinity of the RCD string. It has also been shown that the leading behavior of the monopole density at large distances from the string is controlled by the coherence length and not by the penetration length.

SUPERCONDUCTIVITY DUE TO CONDENSATION OF MONOPOLES IN SU(2) GAUGE THEORY

In SU(2) gauge theory of QCD the Abeliazation may be achieved by the constraint given by[1-4]

$$D_{\mu}\hat{m} = \partial_{\mu}\widehat{m} + ig\vec{V}_{\mu} \times \hat{m} = 0 \qquad \text{... (2.1)}$$

where D_{μ} is covariant derivative for the gauge group, μ = 0, 1, 2, 3, V_{μ} is the generalized gauge potential and g is magnetic charge on monopole. The vector sign and cross product in this equation are taken in internal group space and $\hat{m}$ characterizes the additional Killing symmetry (magnetic symmetry) which commutes with the gauge symmetry itself and is normalized to unity *i.e.*

$$\hat{m}^2 = 1$$

This magnetic symmetry obviously imposes a strong constraint on the connection and hence may be regarded as symmetry of gauge potential. This gauge symmetry restricts not only the metric but also the gauge potential. Such a restricted theory (RCD) may be extracted from full QCD on restricting the dynamical degrees of freedom of theory, keeping full gauge degrees of freedom intact, by imposing magnetic symmetry which ultimately forces the generalized non-Abelian gauge potential $V_{\mu} = (A_{\mu}, B_{\mu})$ of monopole to satisfy a strong constraint given by eqn. (2.1) which gives the following form of the generalized restricted potentials,

with $$\vec{B}_{\mu} = A_{\mu}^{*}\hat{m} - \frac{1}{g}\hat{m} \times \partial_{\mu}\hat{m} \qquad \text{... (2.2)}$$

$$\vec{A}_{\mu} = B_{\mu}^{*}\hat{m}$$

where A_{μ} and B_{μ} are the electric and magnetic constituents of gauge potential. These equations give

$$\hat{m}.\hat{A}_\mu = B^*_\mu$$

and $$\hat{m}.\hat{B}_\mu = A^*_\mu \qquad \text{... (2.3)}$$

as unrestricted Abelian components of the restricted potentials. If $\vec{A}_\mu = 0$ in the original QCD then unrestricted potential is only B^*_μ and the restricted part of the potential is given as

$$\vec{B}_\mu = -\frac{1}{g}\vec{m}\times\partial_\mu\hat{m} \qquad \text{... (2.4)}$$

$$= -\overset{\iota}{W}_\mu$$

where W_μ is the potential of topological monopoles in magnetic symmetry which is entirely fixed by $\hat{m}$ up to Abelian gauge degrees of freedom. The unrestricted part B^*_μ of the gauge potential describes the monopole flux of color isocharges. The unrestricted part B^*_μ is the dual potential associated with charged gluons $W^\pm_\mu$ and leads to condensation of monopoles and the resultant state of chromo magnetic superconductivity as shown in our earlier papers[1-4].

In the presence of a complex scalar field ϕ (Higg's field) and in the absence of quarks or any colored object, the RCD Lagrangian in magnetic gauge may be written as

$$L = \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}|D_\mu\phi|^2 - V(\phi^*\phi) \qquad \text{... (2.5)}$$

where $$V(\varphi^*\varphi) = -\eta(|\varphi|^2 - v^2)^2 , \qquad \text{... (2.6)}$$

$$D_\mu\phi = (\partial_\mu + igW_\mu)\phi , \qquad \text{... (2.7)}$$

and $$H_{\mu\nu} = W_{\nu,\mu} - W_{\mu,\nu} \qquad \text{... (2.8)}$$

with η as coupling constant of Higgs field and v as the vacuum expectation value *i.e.*

$$v = \langle\phi\rangle_0$$

In Prasad – Sommerfeld limit[35]

$$V(\phi) = 0$$

but $$v \neq 0. \qquad \text{... (2.9)}$$

Here W_μ may be identified as the potential of topological monopoles in magnetic symmetry entirely fixed by $\hat{m}$ up to Abelian gauge degrees of freedom. Thus in the magnetic gauge, the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_μ by removing all non-essential gauge degrees of freedom and hence the topological structure of the theory may be brought into dynamics explicitly where monopoles appear as point-like Abelian ones and the gauge fields are expressible in terms of purely time-like non-singular physical potential W_μ. Under the condition (2.9) the monopoles have lowest possible energy for given magnetic charge.

The Langarian (2.5) of RCD in the absence of quark or any colored object looks like Ginsburg – Landau Lagrangian for the theory of superconductivity. The dynamical breaking of the magnetic symmetry, due to the effective potential $V(\phi^*\phi)$, induces magnetic condensation of vacuum leading to the magnetic super current which screens the magnetic flux that confines the electric color iso-charges (due to dual Meissner effect). In other words, the dual Meissner effect expels the electric field between static colored charges into a narrow flux tube, giving rise to a linearly rising potential and to confinement. In this Abelian Higgs model of RCD in magnetic symmetry the $W\mu$, defined by equation (2.8), is dual gauge field with the mass of dual gauge boson given by

$$M_B = gv, \qquad \text{...(2.10)}$$

and ϕ is the monopole field with charge g and mass

$$M_\phi = \sqrt{(8\eta)}v \qquad \text{... (2.11)}$$

With these two mass scales the coherence length ε and the penetration length λ are given by

$$\varepsilon = 1/M_\phi = 1/[\sqrt{(8\eta)}v]$$

and

$$\lambda = 1/M_B = 1/(gv) \qquad \text{...(2.12)}$$

The region in phase diagram space, where $\varepsilon = \lambda$, constitutes the border between type-I and type-II superconductors. The dual superconductivity model proposed recently by Alessandro *et al* [20] places the Yang-Mills vacuum close to the border between type-I and type II superconductors and marginally on the type-II side. Comparing this penetration length λ with that of relativistic superconducting model *i.e.*

$$M_s = \sqrt{2}e\,|\phi| = \sqrt{2}ev = \frac{1}{\lambda_s},$$

we get

$$\frac{\lambda}{\lambda_s} = \sqrt{2}\cot\theta \qquad \text{... (2.13)}$$

where

$$\cot\theta = e/g,$$

e being the electric charge of gluons $W_\mu^\pm$. This relation shows that with a suitable choice of the charge-space parameter θ, the tube of confining flux can be made thin giving rise to a higher degree of confinement of color flux by magnetically condensed vacuum.

BEHAVIOUR OF MONOPOLES AROUND RCD STRINGS IN SU(2) THEORY

Lagrangian, given by eqn. (2.5), yields the following field equations

$$\partial_\nu H^{\mu\nu} = i\,\varphi^\dagger D_\mu \varphi = j_\mu^0 \qquad \text{... (3.1)}$$

and
$$D_\mu^2 \varphi = 4\beta[|\varphi|^2 - 1]\varphi \qquad \text{... (3.2)}$$

where
$$j_\mu^0 = i\,\varphi^\dagger[\partial_\mu + iW_\mu]\varphi\,,$$

$$\beta = \frac{\eta}{g^2}$$

and
$$j_\mu^a = i\,[\varphi^\dagger \tau^a D_\mu \varphi]$$

$$= i\,[\phi^\dagger \tau^a (\partial_\mu + iW_\mu]\varphi]$$

with a = 1, 2, 3 and τ^a, Pauli matrices , constitute the conserved Notherian current. Using relation (2.8), we may write equation (3.1) as

$$W_\mu - \partial^\nu \partial_\mu W_\nu = i\,\varphi^\dagger \varphi \left[\frac{\partial_\mu \varphi}{\varphi} + iW_\mu\right] \qquad \text{... (3.3)}$$

which reduces to the following form in the Lorentz gauge

$$W_\mu = i\,\varphi^\dagger \varphi \left[\frac{\partial_\mu \varphi}{\varphi} + iW_\mu\right]$$

which further reduces in to following simple form for the small variation in ϕ

$$W_\mu + |\varphi|^2 W_\mu = 0 \qquad \text{...(3.4)}$$

which is a massive vector type equation where the equivalent mass of the vector particle state (*i.e.* condensed mode) may be identified as

$$M = |\phi|$$

with its expectation value

$$<M> = \nu$$

which gives
$$M_B = g < M >= \nu g = \frac{1}{\lambda} \qquad \text{... (3.5)}$$

where λ is penetration length. Thus the penetration length directly follows from the field equation (3.1) obtained from the Lagrangian (2.5) of the extended Abelian Higgs model in restricted chromo- dynamics.

Magnetically condensed vacuum of action of Lagrangian of eqn. (2.5) is characterized by the presence of two massive modes. The mass of scalar mode, M_ϕ given by equation (2.11), determines how fast the perturbative vacuum around a colored source reaches the condensation and the mass M_D of the vector mode determines the penetration length of the colored flux. The masses of these magnetic glue balls may be estimated [32, 36, 37] by evaluating string tension of the classical string solutions of quark pairs, since the extended Abelian Higgs model in restricted chromo dynamics, admits string-like solutions [38]. Let us examine the behavior of monopoles around such RCD strings. The classical field equations (3.1) and (3.2) contain a solution corresponding to the RCD string with a quark and an anti-quark at its ends. We consider such strings which are stationary and translationaly invariant along the third direction $Z = x^3$ of the reference frame used in Lagrangian (2.5). Let us consider the following ansatz [39, 40] for the four components of the vector field $\hat{W}_\mu$ and the two complex components ϕ_1 and ϕ_2 of the Higgs field ϕ;

$$W_\mu = \{W_i(\rho), W_\alpha(\rho)\} \quad \text{... (3.6)}$$

and

$$\phi_i = f_i(\rho)[e^{i(w_\alpha x_\alpha)\delta_{i2}}]e^{i\psi(p)} \quad \text{... (3.7)}$$

where $i = 1, 2$, $\alpha = 3, 4$, $f_i(\rho)$ are complex functions of $\rho = (x_1^2 + x_2^2)^{1/2}$ and ω_3 and ω_4 are real parameters. Here ω_4 is the relative rotation and ω_3 is the relative twist along z-axis between the components ϕ_1 and ϕ_2 of the Higg's field ϕ. This ansatz breaks the originally present global SU(2) symmetry to U(1) and the various terms of the Lagrangian (2.5) reduce from four-dimensional configuration to the two-dimensional configuration in the following manner;

$$H_{\mu\nu}H^{\mu\nu} \to H_{ij}^2 - 2[(\partial_i W_\alpha)(\partial_i W^\alpha)],$$

$$(D_\mu\varphi)(D^\mu\varphi) \to \omega^\alpha(\omega_\alpha + 2W_\alpha)|\varphi_2|^2 - |D_i\varphi_a|^2 + W_\alpha W^\alpha |\varphi_a|^2,$$

and

$$[|\phi|^2 - 1]^2 \to [|\phi_a|^2 - 1]^2 \quad \text{... (3.8)}$$

where i, j and $a = 1, 2$, $W^\alpha W_\alpha = W_3^2 - W_4^2$, and $\omega^\alpha\omega_\alpha = \omega_3^2 - \omega_4^2$

Then the action of Lagrangian (2.5) reduces to

$$A \to \frac{v^2}{|q|^2}\int dx^4 dx^3 \int d^2x\, [W_\alpha W^\alpha |\varphi_a|^2 + \frac{1}{2}\omega^\alpha(\omega_\alpha + 2W_\alpha)|\varphi|^2$$

$$-\frac{1}{2}|D_i\varphi_a|^2 + \frac{1}{4}H_{ij}^2 - \frac{1}{2}(\partial_i W_\alpha)(\partial_i W^\alpha) + \beta(|\varphi_a|^2 - 1)^2] \quad \text{... (3.9)}$$

where $H_{12} = -H_{21} = \frac{\partial W_2}{\partial x_1} - \frac{\partial W_1}{\partial x_2}$

With this Ansatz the field equation (3.1) and (3.2) take the following forms in the $x_1 - x_2$ plane

$$\Box W_\alpha = -\omega_\alpha|\varphi_2|^2 - W_\alpha|\varphi_a|^2, \quad \text{... (3.10)}$$

$$\partial_j H_{jk} = i[\phi_a D_k \phi_a] \quad \text{... (3.11)}$$

and $$D_i^2\varphi_1 = -4\beta[|\varphi_a|^2 - 1]\varphi_1 - 2W_\alpha W^\alpha \varphi_1 \qquad \text{... (3.12)}$$

and $$D_2^2\varphi_2 = -4\beta[|\varphi_a|^2 - 1]\varphi_2 - 2W_\alpha W^\alpha \varphi_2 + \omega^\alpha(\omega_\alpha + 2W_\alpha)\varphi_2 \qquad \text{...(3.13)}$$

where equation (3.10) may also be written as

$$\Delta W_\alpha = \omega_\alpha |\varphi_2|^2 + W_\alpha |\varphi_a|^2, \qquad \text{... (3.14)}$$

with $\Delta = -\Box$

Let us consider the solutions of these equations in the following simple case of the ansatz used in equations (3.6) and (3.7)

$$W_1 = \frac{\hat{x}_2 h(\rho)}{|g|\rho^2}; \quad W_2 = -\frac{\hat{x}_1 h(\rho)}{|g|\rho^2}; \; W_3 = 0; \; W_4 = 0 \qquad \text{... (3.15)}$$

where $\hat{x}_1$ and $\hat{x}_2$ are unit vectors along x_1 and x_2 directions. In this case equation (3.14) gives

$$\omega_\alpha = 0 \qquad \text{...(3.16)}$$

and then equations (3.12) and (3.13) reduce to

$$D_i^2\varphi_a = -4\beta[|\varphi_a|^2 - 1]\varphi_a \qquad \text{... (3.17)}$$

and relation (3.7) becomes

$$\varphi_i = f_i(\rho)e^{i\psi(\rho)} \qquad \text{... (3.18)}$$

showing that there is neither relative rotation nor relative twist between the components of ϕ_1 and ϕ_2 of the Higgs field ϕ. The solutions (3.15) and (3.18) are static and untwisted semilocal solutions. Here ρ is the transverse distance to the string

and $$\psi = \arg(x_1 + ix_2), \qquad \text{... (3.19)}$$

$$\lim_{\rho\to 0} f(\rho) = \lim_{\rho\to 0} h(\rho) = 0,$$

$$\lim_{\rho\to \alpha} f(\rho) = \lim_{\rho\to 0} h(\rho) = 1 \qquad \text{... (3.20)}$$

where $$f(\rho) = f_1(\rho) \quad \text{and} \quad h(\rho) = f_2(\rho) \qquad \text{...(3.20a)}$$

From equation (3.1) the monopole current as

$$k_\mu = g\,\mathrm{Im}[\phi^+ D_\mu \phi] = g\,|\phi|^2\,[\partial_\mu \arg\phi + gW_\mu] \qquad \text{... (3.21)}$$

Equation (3.19) gives

$$\frac{\partial\psi}{\partial x_1} = -\frac{x_1}{\rho^2} \quad \text{and} \quad \frac{\partial\psi}{\partial x_2} = \frac{x_2}{\rho^2} \qquad \text{... (3.22)}$$

Substituting relations (3.18), (3.15), (3.19), (3.20a) and (3.22) into equation (3.21), we get

$$k_i = -\frac{(v^2 \in_{ij} x_j)}{\rho^2} g f^2(\rho)[1-h(\rho)]$$

$$k_3 = 0 \quad \text{and} \quad k_4 = 0 \qquad \text{... (3.23)}$$

where $\in_{12} = -\in_{21} = 1$ and $\in_{11} = \in_{22} = 0$, summation over repeated index is conventionally involved. Substituting relations (3.18), (3.15) and (3.22) into field equation (3.17), we have

$$f''(\rho) + f'(\rho)/\rho - f(\rho)/\rho^2[1-h(\rho)]^2 + \left(\frac{M_\phi^2}{2}\right)[1-f^2(\rho)]f(\rho) = 0 \qquad \text{...(3.24)}$$

where dash devotes derivatives with respect to ρ. At large distance, in view of equations (3.20), we may have

$$f(\rho) = 1 - \varepsilon(\rho), \qquad \text{... (3.25)}$$

where ε (ρ) is infinitesimally small at large distance such that

$$\lim_{\rho \to \infty} \varepsilon(\rho) = 0$$

Then equation (3.24) may be written as

$$\varepsilon''(\rho) + \varepsilon'(\rho)/\rho - M_\phi^2 \varepsilon(\rho) = 0$$

Substituting $r = M_\phi \rho$ into this equation, we have

$$\frac{d^2\varepsilon(r)}{dr^2} + \left(\frac{1}{r}\right)\frac{d\varepsilon(r)}{dr} - \varepsilon(r) = 0$$

which is modified Bessel's equation of zero order, with its solution given as

$$\varepsilon(r) = AI_0(r) = AI_0(M_\phi \rho), \qquad \text{... (3.26)}$$

where I_0 is the modified Bessel's function of zero order. In the similar manner, the field equation (3.3) may be written into the following form by using relations (3.15) and (3.23);

$$h''(\rho) - h'(\rho)/\rho + M_B^2[1-h(\rho)]f^2(\rho) = 0 \qquad \text{... (3.27)}$$

At large distance we may have

$$h(\rho) = 1 - \xi(\rho), \qquad \text{...(3.28)}$$

where $$\lim_{\rho \to \infty} \xi(\rho) = 0$$

Then equation (3.27) reduces to

$$\frac{d^2\xi(r)}{dr^2} - \frac{d\xi(r)}{dr} - \xi(r) = 0 .$$

where $r = M_B \rho$. Let us substitute $\xi(r) = r\chi(r)$ in to this equation. Then we have

$$r\frac{d^2\chi(r)}{dr^2}+\frac{d\chi(r)}{dr}-\chi(r)\left[1+\frac{1}{r^2}\right]=0$$

which is modified Bessel's equation [41] of order-one with its solution given by

$$\chi(r)=\frac{\xi(r)}{r}=BI_1(r) \qquad \text{... (3.29)}$$

where $I_1(r)$ is modified Bessel's function of order one. Thus we have

$$\xi(\rho)=B(M_B\rho)I_1(M_B\rho)\,, \qquad \text{... (3.30)}$$

Substituting relations (3.26) and (3.30) into equations (3.25) and (3.28), we have at large value of ρ,

$$f(\rho)=1-AI_0(M_\phi\rho) \qquad \text{... (3.31)}$$

and $$h(\rho)=1-B(M_B\rho)I_1(M_B\rho)$$

Substituting these results in to equation (3.15) and (3.18) with equation (3.19), we get the solution of classical field equations (3.2) and (3.17) corresponding to the RCD string with a quark and an anti-quark at its ends. The infinitely separated quark and anti-quark correspond to an axially symmetric solution of the string. For such a string solution with a lowest non-trivial flux, the coefficient *A* in the solution (3.26) is always equal to one while the coefficient *B* in the solution (3.30) is unity in the Bogomolnyi limit exactly on the border between the type I and type II superconductors where $M_B = M_\phi$ *i.e.* coherence length and the penetration length coincide with each other. Thus in RCD close to border, we set $B = 1$ besides $A = 1$ and then we have

$$f(\rho)=1-I_0(M_\phi\rho)=-\sum_{n=1}^{\infty}\frac{(M_\phi\rho/2)^{2n}}{(n!)^2}$$

and $$h(\rho)=1-(M_B\rho)I_2(M_B\rho)$$

$$=1-\frac{(M_B\rho/2)^2}{2}-M_B\rho\sum_{n=1}^{\infty}\frac{(M_B\rho/2)^{2n+1}}{\Gamma(n)\Gamma(n+1)} \qquad \text{... (3.32)}$$

The RCD string is well defined by these solutions.

Discussion

The Lagrangian, given by equation (2.5) for RCD in magnetic gauge in the absence of quarks or any colored objects, establishes an analogy between superconductivity and the dynamical breaking of magnetic symmetry which incorporates the confinement phase in RCD vacuum where the effective potential V(θ* θ), given by equation (2.6), induces the magnetic condensation of vacuum. This gives rise to magnetic super current which screens the magnetic flux and confines the color iso-charges as the result of dual Meissner effect. The confinement of color is due to the spontaneous breaking of magnetic symmetry which yields a non-vanishing magnetically charged Higg's condensate, where the broken magnetic group is chosen by Abelianization process and hence the magnetic condensation mechanism of confinement in RCD is dominated by Abelian degrees of freedom. Such Abelian dominance in connection with monopole condensation has recently been demonstrated by Boykov *et al* [42]. The similar result has also been obtained more recently in a dual superconductivity model [20].

In the confinement phase of RCD, the monopoles are condensed under the condition (2.9) where the transition from $<\phi>_0 = 0$ to $<\phi>_0 = v \neq 0$ is of first order, which leads to the vacuum becoming a chromo magnetic superconductor in the analogy with Higg's- Ginsburg-Landau theory of superconductivity. Magnetically condensed vacuum is characterized by the presence of two massive modes given by equations (2.10) and (2.11) respectively, where the mass of scalar mode M_ϕ determines how fast the perturbative vacuum around a color source reaches condensation and the mass M_B of vector mode determines the penetration length of the colored flux. With these two scales of dual gauge boson and monopole field, the coherence length ε and the penetration length λ have been constructed by equations (2.12) in RCD theory. These two lengths coincide at the border between type – I and type –II- superconductors.

The ansatz given by relations (3.6) and (3.7) shows that there is a non-trivial coordinate dependent relative phase between the components of SU(2) doublets. This anastz breaks the originally present global SU(2) symmetry to U(1) and reduces the four-dimensional action of Lagrangian (2.5) to the two dimensional one given by equation (3.9) with the field equation given by equations (3.10), (3.11), (3.12) and (3.13). For the special case with the static solution given by equations (3.15), (3.17) and (3.18) there is neither a relative rotation nor a relative twist between the components of Higg's field. Substituting relations (3.32) into equations (3.15) and (3.18), the solutions of classical field equations (3.1) and (3.2), corresponding to the RCD string with a quark and antiquark at its ends, readily follows. The RCD string is well defined by solutions (3.32) where the monopole current given by equation (3.21) near the RCD string, is zero at the center of the string and also zero at points far away from the string.

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