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SUPERCONDUCTIVITY DUE TO CONDENSATION OF EMBEDDED MONOPOLES

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Exploring the role of quark monopoles (*i.e.* embedded monopoles) in restoration of chiral symmetry in SU(2) gauge theories, it has been shown that these monopoles play very important part in confining properties including color superconductivity.

Key-Words: Embedded monopoles; Chiral symmetry; Gauge theories; Confinement; Condensation; Supersymmetry.

INTRODUCTION

Physicists were fascinated by magnetic monopole since its ingenious idea was given by Dirac [1,2] and also by Saha [3,4] by showing that the mere existence of monopole implies the quantization of electric charge in the Abelian theory. In the mean time, it became clear [5-7] that monopole and dyon [8-9] (a particle carrying electric and magnetic charges) can be understood better in non-Abelian gauge theories. Such non-Abelian monopoles are known to arise as classical solutions in field theoretical models like the Georgi-Glashow model and also in pure Yang-Mills theories where the role of fundamental Higgs scalars could eventually be played by some composite fields. In any case, these non-Abelian monopoles can be understood, in the framework of these models, as defects in space-time of U(1) gauge fields which arise once the unitary gauge is chosen [10-11]. Julia and Zee [8] extended the idea of non-Abelian monopole proposed by t' Hooft [5] and Polyakov [6] and constructed classical solutions for non-Abelian dyons. Now it is widely recognized [12] that SU(5) grand unified model is a gauge theory that contains monopole solution and it has been demonstrated by Witten [13] that non-Abelian monopoles are necessarily dyons which arise as quantum mechanical excitation of fundamental monopoles. Thus monopoles and dyons became intrinsic part of all current grand unified theories (GUT's) and super symmetrical models [14-19]. Perhaps the most important aspect of monopoles and dyons in physics is their role in the mechanism of quark confinement [20-25] along the lines of dual Meissner effects [26-30] leading to dual superconductivity as discussed in our recent papers [31-36] by employing dual gauge potential where magnetic degree of freedom manifestly appears in the partition function.

Embedded monopoles are gauge-invariant composite objects made of quark and gluon fields. These monopoles constitute a new class of defects of quantum chromo-dynamics (QCD) and proliferate in the quark-gluon plasma phase. This proliferation is associated with the well defined boundary [37] known as Kretesz-line, which separates the hadronic phase (*i.e.* the confinement phase) and the quark-gluon phase (*i.e.* deconfinement phase) of QCD with realistic quark masses and vanishing chemical potential μ. Hence these embedded monopoles are also called quark-monopoles [37]. At larger chemical potential, the phase transition reemerges at a tricritical point and then continues as the first-order phase transition. At even higher temperature, more exotic phases such as color superconductivity and the color-flavor locking appear [38]. Quark-monopoles (*i.e.*, embedded monopoles) in QCD are analogues of the embedded Nambu-monopoles [39, 40] in standard Electro-Weak model[40]. There should be an indirect relation between quark monopoles and confining properties including superconductivity since the confinement phenomenon and the chiral symmetry are intimately related in QCD and the quark monopoles (embedded monopoles) are considered as agents of chiral symmetry restoration [41].

Extending the restricted chromodynamics (RCD) [31, 32] in SU(2) gauge theories in the present paper by including quarks and gluons, the study of dyonic condensation, quark confinement and superconductivity (dual superconductivity as well as color superconductivity) has been undertaken in extended RCD. In this paper the study of superconductivity due to embedded monopoles [37, 41] in these gauge theories has also been carried out by exploring the role of quark monopoles (*i.e.* embedded monopoles) in restoration of chiral symmetry and the related confining properties. The bilinear functions of the fermion fields have been constructed in SU(2) theory as scalar and axial vector from the point of view of space-time transformations. The corresponding Georgi-Glashow multiplets have been used to construct gauge invariant t' Hooft tensors in color space and the currents of quark-monopoles of three types have been shown to possess δ- singularities at the corresponding world lines. These quark monopoles have been shown to carry the magnetic charges with respect to scalar, axial and chiral invariant components of gauge fields in SU(2) theory. It has been shown that these quark monopoles are tightly related to the chiral symmetry restoration and the resulting color superconductivity in QCD.

SUPERCONDUCTIVITY DUE TO CONDENSATION OF EMBEDDED MONOPOLES IN SU(2) THEORY

Monopole condensation mechanism of confinement, together with dual superconductivity, implies that long range physics is dominated by Abelian degrees of freedom and the method of Abelian projection (*i.e.* Abelianization) is one of the popular approaches to the problem of confinement, and hence superconductivity, in non-Abelian gauge theories. In SU(2) gauge theory of QCD this Abeliazation is achieved by the constraint given by [31-33]

$$D_\mu \hat{m} = \partial_\mu \hat{m} + ig\vec{V}_\mu \times \hat{m} = 0 \qquad ...(2.1)$$

where D_μ is covariant derivative for the gauge group,μ = 0, 1, 2, 3, V_μ is the generalized gauge potential and g is magnetic charge on monopole. The vector sign and cross product in this equation are taken in internal group space and $\hat{m}_c$ characterizes the additional Killing symmetry (magnetic symmetry) which commutes with the gauge symmetry itself and is normalized to unity *i.e.*

$$\hat{m}^2 = 1$$

This magnetic symmetry obviously imposes a strong constraint on the connection and hence may be regarded as symmetry of gauge potential. This gauge symmetry restricts not only the metric but also the gauge potential. Such a restricted theory (RCD) may be extracted from full QCD on restricting the dynamical degrees of freedom of theory, keeping full gauge degrees of freedom intact, by imposing magnetic symmetry which ultimately forces the generalized non-Abelian gauge potential $V_\mu = (A_\mu, B_\mu)$ of monopole to satisfy a strong constraint given by eqn. (2.1) which gives the following form of the generalized restricted potentials,

$$\vec{B}_\mu = A_\mu^* \hat{m} - \frac{1}{g}\hat{m} \times \partial_\mu \hat{m} \qquad ...(2.2)$$

with $$\vec{A}_\mu = B_\mu^* \hat{m}$$

where A_μ and B_μ are the electric and magnetic constituents of gauge potential. These equations give

$$\hat{m}.\hat{A}_\mu = B_\mu^*$$

and $$\hat{m}.\hat{B}_\mu = A_\mu^* \qquad (2.3)$$

as unrestricted Abelian components of the restricted potentials. If $\vec{A}_\mu = 0$ in the original QCD then unrestricted potential is only B_μ^* and the restricted part of the potential is given as

$$\vec{B}_\mu = -\frac{1}{g}\vec{m} \times \partial_\mu \hat{m} \qquad ...(2.4)$$

$$= -\vec{W}_\mu$$

where W_μ is the potential of topological monopoles in magnetic symmetry which is entirely fixed by $\hat{m}$ up to Abelian gauge degrees of freedom. The unrestricted part B_μ^* of the gauge potential describes the monopole flux of color isocharges. The unrestricted part B_μ^* is the dual potential associated with charged gluons $W_\mu^\pm$ and leads to condensation of monopoles and the resultant state of chromomagnetic superconductivity as shown in our earlier papers [31-36].

In the presence of a complex scalar field ϕ (Higg's field) and in the absence of quarks or any colored object, the RCD Lagrangian in magnetic gauge may be written as

$$L = \frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{1}{2} | D_\mu \phi |^2 - V(\phi^* \phi) \qquad \text{... (2.5)}$$

where
$$V(\varphi^* \varphi) = -\eta (|\varphi|^2 - V^2)^2, \qquad \text{... (2.6)}$$

$$D_\mu \phi = (\partial_\mu + igW_\mu)\phi, \qquad \text{...(2.7)}$$

and
$$H_{\mu\nu} = W_{\nu,\mu} - W_{\mu,\nu} \qquad \text{...(2.8)}$$

In the presence of quarks (and gluons), the RCD Lagrangian (2.5) may be generalized to the following form ;

$$L_R = \frac{1}{4} H^a_{\mu\nu} H_a^{\mu\nu} + \bar{\psi}^a (i\gamma^\mu D'_\mu)\psi_a + m\bar{\psi}^a \psi_a + \frac{1}{2} \left| D_\mu \phi \right|^2 - V(\phi^* \phi) \ , \qquad \text{...(2.9)}$$

where a = 1, 2, 3, Ψ represents quark field with mass m, and $\vec{H}_{\mu\nu}$ has been constructed as $\vec{H}_{\mu\nu} = H_{\mu\nu}\vec{\xi}_3$ in the magnetic gauge by aligning $\hat{m}$ along a space- time independent direction (say $\widehat{\varepsilon_3}$ in isospin space) on imposing a gauge transformation U such that

$$\hat{m} \xrightarrow{U} \hat{\varepsilon}_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \text{... (2.10)}$$

with $H_{\mu\nu}$ defined by eqn. (2.8) where W_μ may be identified as the potential of topological dyons in magnetic symmetry which is entirely fixed by $\hat{m}$ up to Abelian gauge degrees of freedom. Thus in the magnetic gauge, the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_μ by removing all non-essential gauge degrees of freedom and hence the topological structure of the theory may be brought into dynamics explicitly. It assures a non-trivial dual structure of the theory of monopoles in magnetic gauge in which these objects appear as point-like Abelian ones and the gauge fields are expressible in terms of purely time-like non-singular physical potential W_μ. Lagrangian (2.9) can be used to represent the interactions between quarks and monopoles in the theory. It can be viewed as the effective Lagrangian used to describe the dual dynamics of RCD at the phenomenological level just as the Ginsburg – Landau Lagrangian is used in the theory of superconductivity. With this Lagrangian in hand, we have two phases in our theory. The first one is the unconfinement phase (quark-gluon plasma phase), where magnetic symmetry is preserved and the second one is the confinement phase (hadronic phase), where magnetic symmetry is broken dynamically. These two phases are separated by the well defined boundary [37], known as Kreteszline, with which there is associated the proliferation of embedded monopoles which are gauge-invariant composite objects made of quark and gluon fields. Hence these embedded monopoles are also called quark-monopoles [37].

In order to explore the role of quark monopoles (*i.e.*, embedded monopoles) in restoration of chiral symmetry and the related confining properties and superconductivity, in SU(2) theory, let us start with the quark field (*i.e.* fermion field) Ψ, introduced through eqn. (2.9), which transforms in the fundamental representation of gauge group SU(2) in Yang-Mills theory. Then the bilinear functions of this fermion fields may be defined as

$$\hat{S}^a = \bar{\psi}(x)\,\hat{\tau}^a\,\psi(x) \qquad ...(2.11)$$

$$\hat{A}^a = \bar{\psi}(x)(iy_5)\,\hat{\tau}^a\,\psi(x) \qquad ...(2.12)$$

where $\hat{\tau}^a$ are the Pauli matrices and $\hat{S}$ and $\hat{A}$ (the real valued composite fields) are scalar and axial (*i.e.*, pseudo scalar) fields from the point of view of space-time transformations. Both these fields transform as adjoint three- component quantities with respect to the action of the gauge group.

Let $\psi(x)$ used in equation (2.11) and (2.12) be c-valued function as an eigen mode of mass less Dirac operator $\hat{D}$,

$$\hat{D}\psi_\lambda(x) = \lambda\psi_\lambda(x) \qquad ...(2.13)$$

where

$$\hat{D} = \gamma_\mu\left(\partial_\mu + \frac{1}{2}i\,\tau^a B^a_\mu\right) \qquad ...(2.14)$$

with B^a_μ (x) as the gauge fields. Let us consider the axial transformations $U_A(1)$ defined by the global Abelian parameter α as

$$\psi \to \psi' = e^{i\alpha r_5}\psi \quad \text{and} \quad \bar{\psi} \to \bar{\psi}' = \psi e^{-i\alpha\gamma_5} \qquad ... (2.15)$$

Under these transformations $U_A(1)$, the color vector $\hat{S}^a$ and $\hat{A}^a$ given by equation (2.11) and (2.12), transform as

$$\hat{S}^a \to \hat{S}'^a = \hat{S}^a \cos 2\alpha + \hat{A}^a \sin 2\alpha$$

$$\hat{A}^a \to \hat{A}'^a = -\hat{S}^a \sin 2\alpha + \hat{A}^a \cos 2\alpha \qquad \text{... (2.16)}$$

Let us construct the following three unit color vectors in terms of adjoint fields $\hat{S}^a$ and $\hat{A}^a$;

$$\hat{n}_s = \frac{\vec{S}}{|\vec{S}|}; \;\; \hat{n}_A = \frac{\vec{A}}{|\vec{A}|}; \;\; \hat{n}_1 = \frac{\vec{S} \times \vec{A}}{|\vec{S} \times \vec{A}|}, \qquad \text{... (2.17)}$$

where the symbol$\to$ denotes vector in color space and $|\vec{S}|$ and $|\vec{A}|$ are norms of color vectors $\vec{S} = \hat{S}$ and $\vec{A} = \hat{A}$

$$|\vec{S}| = \left\langle \hat{S}, \hat{S} \right\rangle^{1/2},$$

$$|\vec{A}| = \left\langle \hat{A}, \hat{A} \right\rangle^{1/2}$$

The last relation in equation (2.17) gives the normalized vector product (in color space) of the scalar and axial color vectors $\hat{S}$ and $\hat{A}$ respectively. Using equations (2.17) and (2.16), it may readily be shown that the unit vector $\hat{n}_1$ is invariant under the axial transformations (2.15) and (2.16). Unit vectors of equation (2.17) may be interpreted as the directions of the composite adjoint Higgs field. Then we get following three Georgi-Glashow multiplets in SU(2) gauge theory with Higgs fields;

$$(\hat{n}_s^a, B_\mu^a); \; (\hat{n}_A^a, B_\mu^a); \; (\hat{n}_I^a, B_\mu^a) \qquad \text{... (2.18)}$$

These multiplets can be used to construct the gauge invariant, *t* Hooft tensors[5] in the following form in color space:

$$\Im_{\mu\nu}^{s} = \hat{n}_{\dot{s}} \left[\vec{H}_{\mu\nu} - \frac{1}{g} (D_\mu \hat{n}_s) \times (D_s \hat{n}_s) \right];$$

$$\Im_{\mu\nu}^{A} = \hat{n}_{\dot{A}} \left[\vec{H}_{\mu\nu} - \frac{1}{g} (D_\mu \hat{n}_A) \times (D_\nu \hat{n}_A) \right];$$

$$\Im_{\mu\nu}^{I} = \hat{n}_{\dot{I}} \left[\vec{H}_{\mu\nu} - \frac{1}{g} (D_\mu \hat{n}_I) \times (D_\nu \hat{n}_I) \right]; \qquad \text{...(2.19)}$$

where

$$\vec{H}_{\mu\nu} = \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu + g(\vec{B}_\mu \times \vec{B}_\nu) \qquad \text{...(2.20)}$$

is the field strength of the gauge field $\vec{B}_\mu$ with magnetic charge g and

$$(D_\mu)_{ab} = \delta_{ab}\partial_\mu + g \in_{abc} B_\mu^c \qquad ...(2.21)$$

is the adjoint covariant derivative. *t*' Hooft tensors of equation (2.19) are the gauge invariant field strength tensors for the diagonal components

$$\left.\begin{aligned} B_\mu^s &= (\vec{B}_\mu, \hat{n}_s); \\ B_\mu^A &= (\vec{B}_\mu, \hat{n}_A); \\ B_\mu^I &= (\vec{B}_\mu, \hat{n}_I); \end{aligned}\right\} \qquad ...(2.22)$$

of the gauge field with respect to the color directions. The current of the quark monopole of S^{th} type is then given by

$$\begin{aligned} k_\mu^s &= \frac{g}{2k} \Im_{\mu\nu,\nu}^{(d)s} \\ &= \int_{C^s} d\tau \frac{\partial X_\nu^{C^s}(\tau)}{\partial \tau} \delta^{(4)}[x - X^{C^s}(\tau)] \end{aligned} \qquad ...(2.23)$$

where $\Im_{\mu\nu}^{(d)s}$, dual of $\Im_{\mu\nu}^s$, is given by

$$\Im_{\mu\nu}^{(d)s} = \frac{1}{2} \in_{\mu\nu\sigma\rho} \Im_{\sigma\rho}^s \qquad ... (2.24)$$

and C^s is the corresponding world-line while monopole world-line is parameterized by the vector $X_\mu = X_\mu^{C^s}(\tau)$ and the parameter τ. This current, given by equation (2.23), has a δ-like singularity at the world-line C^s. Similarly, the currents of quark monopoles of A^{th} and I^{th} type have δ-like singularities at the world lines C^A and C^I respectively.

The quark monopoles defined by equation (2.23) are quantized and the corresponding monopole charge is conserved. In other words the world lines C^S, C^A and C^I are closed. . In the corresponding unitary gauges

$$n_s^a = \delta^{a3}, \quad n_A^a = \delta^{a3} \quad \text{and} \quad n_I^a = \delta^{a3} \qquad ... (2.25)$$

the quark monopoles correspond to monopoles embedded into the diagonal components given by equations (2.22).

DISCUSSION

In equations (2.11) and (2.12), the bilinear functions of the fermion fields have been constructed to explore the role of quark monopoles (*i.e.*, embedded monopoles) in restoration of chiral symmetry and the related confining properties and superconductivity in SU(2) gauge theory. These scalar and axial fields transform according to equations (2.16) under the axial transformations given by equations (2.15). The unit vectors of equations (2.17) may be interpreted as the direction of the composite Higgs field and consequently, equations (2.18) give three Georgi-Glassow multiplets in SU(2) gauge theory with Higgs field. These multiplets have been used to construct gauge invariant t' Hooft tensors in the form given by equation (2.19) in color space. These tensors are gauge invariant field strength tensors for diagonal components of the gauge field with respect to the color direction. The current of the quark monopole of S^{th} type, given by equation (2.23), has a δ-like singularity at the world line C^S . In the similar manner the currents of quark monopoles of A^{th} and I^{th} type may be constructed and these currents also may be shown to have δ-like singularities on the world lines C^A and C^I respectively. The quark monopoles described by equations (2.23) are quantized and the world lines C^S, C^A and C^I are closed. Thus the quark monopoles of S^{th}, A^{th} and I^{th} types carry the magnetic charges with respect to scalar, axial and chiral invariant components of gauge fields given by equations (2.22). In the corresponding unitary gauges, defined by equations (2.24a), the quark monopoles correspond to monopoles embedded into the diagonal field components given by equations (2.22). In the gauges, where these diagonal components are regular, such monopoles are hedgehogs[41] in the composite quark-antiquark fields. The corresponding quark condensates are characterized by the typical hedgehog behavior $n_s^a \sim x^a$ etc. in the local transverse vicinity of monopoles. The existence of these monopoles and their condensate in QCD is a kinematical consequence of the existence of adjoint real valued fields of equations (2.11), (2.12) and (2.17). There is an infinite number of equivalent formulations of the embedded monopoles (*i.e.*, quark monopoles) associated with triplet isovectors given by chiral rotation (2.16) of isovectors $\hat{S}$ and $\hat{A}$ with an arbitrary angle 2α. Because of the hedgehog behavior of embedded *QCD* monopoles in quark – antiquark condensates, these monopoles are rightly called 'quark monopoles'. These quark monopoles are tightly related to the chiral symmetry restoration and the resulting color superconductivity in *RCD*.

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