Acta Ciencia Indica, Vol. XLVI-P, No. 1 to 4, 69 (2020)

PATTERN CLASSIFICATIONS USING GROVER'S AND VENTURA'S ALGORITHMS IN A TWO-QUBITS SYSTEM

BALWANT SINGH RAJPUT

Deptt. of Physics, Kumaon University, Nainital
I-11, Gamma-2, Greater Noida, Disst. Gautambudh Nagar (U.P) India
email: bsrajp@gmail.com

RECEIVED : 19 February, 2021

Carrying out the classification of patterns in a two-qubit system using both Grover's iterative algorithm and Ventura's model taking different superposition of two-pattern start state and one-pattern start respectively, it has been shown that the exclusion superposition is the most suitable two-pattern search state for the simultaneous classifications of patterns demonstrating that the unknown patterns (not present in the concerned data-base) are classified more efficiently than the known patterns (present in the data-base). It has also been shown that the first and second states $|\psi_1 > and\ |\psi_2 >$ of Singh-Rajput MES [1,2] are most suitable for the simultaneous classification of the patterns $|0?>$ while third and fourth states $|\psi_3>$ and $|\psi_4>$ of these MES simultaneously classify the patterns $|1?>$ most effectively. It has also been demonstrated that these MES are separately most suitable search states for the separate classifications of patterns |00>, $|01>$, |10> and |11> respectively on the second iteration of Grover's method or the first operation of Ventura's algorithm.

INTRODUCTION

In recent years the quantum entanglement has played important role in the fields of quantum information theory [3], quantum computers [4], universal quantum computing network [5], teleportation [6], dense coding [7,8], geometric quantum computation [9, 10] and quantum cryptography [11-13]. Singh and Rajput have recently explored [14] the entanglement as one of the key resources required for quantum neural network (QNN), established [15] the functional dependence of the entanglement measures on spin correlation functions, worked out the correspondence between evolution of new maximally entangled states (Singh-Rajput MES) of two-qubit system and representation of SU(2) group, and investigated the evolution of MES under a rotating magnetic field. They have also performed very recently the pattern recall (quantum associative memory) [16], pattern classifications [17, 18] and pattern association [19] by employing the method of Grover's iteration [20] on Bell's MES and Singh-Rajput MES in two-qubit system and demonstrated that for all the related processes (memorization, recalling, and pattern classification) in a two-qubit system Singh-Rajput MES provide the most suitable choice of memory states and the search states. We have very recently undertaken [21] the study of the classification of patterns in the framework of quantum neural network (QNN) in a three-qubit system using the method of repeated iterations in Grover's algorithm and the algorithm of Ventura and demonstrated that the superposition of exclusion is the most suitable choice as the search state for these classifications with two-pattern start-state as well as one-pattern start state. It has also been shown that in a three-qubit system the method of Grover is most effective for classification of unknown patterns (not present in the search states) with the largest data base (size of search state) while in the method of Ventura the classification of patterns is done more effectively with the smallest data-base.

In the present paper, the classification of patterns has been carried out in a two-qubit system using both Grover's iterative algorithm and Ventura's model taking different superposition of two-pattern start state and one-pattern start respectively. Comparative probabilities of simultaneous classification of patterns $|1?>$, and the simultaneous classification of patterns $|0?>$ where ? denotes 0 and 1, on applying Grover's method of repeated iterations on all the three superposition for two- patterns start-state, have been plotted in the graphs showing that the exclusion superposition is the most suitable two-pattern search state for such simultaneous classifications of patterns and hence demonstrating that the unknown patterns (not present in the concerned data-base) are classified more efficiently than the known patterns (present in the data-base). Applying Grover's algorithm and Ventura's method on all the possible superposition as the search states obtained for one-pattern start states, it has been shown that the superposition of phase-invariance are the best choice as the respective search state for simultaneous classifications of the patterns in both Grover's and Ventura's methods of classifications of patterns. These states respectively are identical to the third and fourth states $|\psi_3>$ and $|\psi_4>$ of Singh-Rajput MES [15] for the simultaneous classification of patterns $|1?>$ and the first and second states $|\psi_1> and |\psi_2$ for the simultaneous classification of the patterns $|0?>$. It has also been shown that these MES ($|\psi_1>, |\psi_2>, |\psi_3>$ and $|\psi_4>$) obtained from the corresponding self-single-pattern start states are the most suitable search states for the classification of patterns $|00>, |01>$, |10> and |11>respectively on the second iteration of Grover's method or the first operation of Ventura's algorithm. Each of these MES consists of the entire data base of a two-qubit system and the higher effectiveness of Grover's algorithm for such large search states is obvious as shown in our earlier papers[22] but the suitability of these states in Ventura's algorithm also, in contrast to the earlier results about its higher effectiveness for smaller data base for higher–qubit systems [23], is worth mentioning here. Finally, it has been demonstrated in this paper that the states $|\psi_{exc}>$ separately obtained from the single-pattern start-states consisting of patterns |10>, |11> , |01> and |00> respectively, are the most suitable search states for the classification of patterns |11>, |10> , |00> and |01> respectively on the first iteration in Grover's method or on the second operation of Ventura's algorithm.

METHODS OF GROVER AND VENTURA FOR PATTERN CLASSIFICATIONS IN TWO-QUBIT SYSTEMS

The pattern classification may be performed in straight forward approach employing the method of Grover's iterate which is described as a product of unitary operators $\hat{D}=\hat{G}\hat{R}$ applied to the chosen quantum search state iteratively and probability of desired result maximized by measuring the system after appropriate number of iterations. Here the operator $\hat{R}$ is phase inversion of the state(s) that we wish to observe upon measuring the system. It is represented by identity matrix I with diagonal elements corresponding to desired state(s) equal to -1 and the operator $\hat{G}$ is descried as an inversion about average:

$$G = 2|\Psi><\Psi| - I \qquad \text{... (2.1)}$$

where $|\Psi>$ represents the full data base available in the given quantum system *i.e.* for a two-qubit system it contains all the four possible patterns as

$$|\Psi> = \frac{1}{2}[\ |00> +|01> +|10> +|11>] \qquad \text{... (2.2)}$$

and hence for a two-qubit system we have the following inversion operator $\hat{G}$;

$$\hat{G} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \qquad \text{... (2.3)}$$

Basic idea of Grover' algorithm is to invert the phase of the desired basis state and then to invert all the basis states about the average amplitude of all the states. The number (r) of times the classification will have to be repeated in Grover's method in a 2-qubit system is $r = \frac{\pi}{4}\sqrt{N} = \frac{\pi}{2}$, where $N = 4$. It gives $1 \leq r < 2$. It may also be written approximately as $\frac{1}{P_C}$ where P_C is the probability of correct classification in the first operation.

On the other hand Ventura proposed [24, 25] an algorithm as generalized Grover's one and insisted that it is effective in all cases. This algorithm may be written for the present case in the following simplified manner;

$$|\Psi> = \hat{G}I_\rho\hat{G}I_\tau|\psi>$$

$$\text{Repeat } \frac{\pi}{4}\sqrt{N} - 2 \approx 1 \; time$$

$$\text{and take } |\hat{\Psi}> = \hat{G}I_\tau|\Psi> \qquad \text{... (2.4)}$$

for measuring the probability of desired classifications where $|\psi>$ is the search state (stored data base), I_τ inverts the sign of the pattern to be classified, I_ρ inverts the signs of all patterns in the stored data base and the operator $\hat{G}$ is given by eqn. (2.3).

SIMULTANEOUS CLASSIFICATIONS OF PATTERNS $|1?>$

For the simultaneous classification of patterns |1?>, through Grover's method of repeated iterations, the phase inversion operator $\hat{R}$ and the iteration operator $\hat{D}$ are respectively given as

$$\hat{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \ldots (3.1)$$

and

$$\hat{D} = \frac{1}{2}\begin{bmatrix} -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix} \quad \ldots (3.2)$$

which may be used to operate iteratively on the different search states in the following cases.

(a) Two-patterns start-States : Let us first apply Grover's algorithm by choosing two pattern search states consisting of patterns |10> and |11> with the following possible general superposition (inclusion, exclusion and phase-inversion) respectively;

$$|\psi_{inc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}; \quad |\psi_{exc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}; \quad |\psi_{phi}> = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad \ldots (3.3)$$

The comparative probabilities of simultaneous classifications of the patterns |10> and |11> on repeatedly applying the operator $\hat{D}$, given by eqn. (3.2), on all these three superposition are given by graphs of figures-1,where RES denotes the number of iterations, red curve gives the probabilities of classification on different iterations of $|\psi_{exc}>$, and blue and green curves give the probabilities of classification for desired patterns on different iterations of $|\psi_{inc}>$ and $|\psi_{phi}>$ respectively.

These graphs show that on any number of iterations of Grover's algorithm the phase invariance superposition $|\psi_{phi}>$ the probability of classification of desired patterns |1?> never exceeds 50% and the first iteration of the inclusion superposition $|\psi_{inc}>$ does not classify the desired patterns while the first iteration of exclusion superposition $|\psi_{exc}>$, with the given two-patterns start states, gives the fifty percent probability of simultaneous classifications of each of the patterns and the 100% total probability of the classification of desired patterns |1?>. With inclusion superposition such situation comes on second iteration but the allowed number of iterations r of Grover's algorithm for 2-qubit system is given by $1 \leq r < 2$ and hence the exclusion superposition is the most suitable two-pattern search state for simultaneous classification of patterns |10> and |11> while none of these patterns occurs in this search state (data-base). while none of these patterns occurs in this search state (data-base). It is interesting to note here that in spite of the fact that the phase-inversion superposition $|\psi_{phi}>$ as the search state is the full data base for the 2-qubit system, the Grover's algorithm does give satisfactory classification at all here contradicting the claim [25] that Grover's algorithm is most effective when the number of patterns in the search state (the number of stored data) is large.

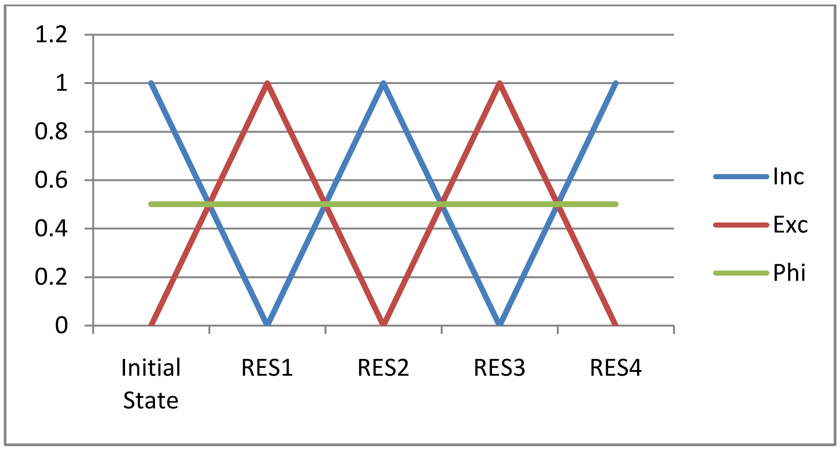

Figure-1: Graph for Total Probabilities of classification of patterns |1?> with 2 pattern start state.

On applying Ventura's method described by eqn. (2.4) for classification of desired patterns |1?>, we have found that inclusion superposition $|\psi_{inc}>$ of eqn. (3.3) gives the zero probability and phase invariance superposition $|\psi_{phi}>$ gives only 25% probability of this classification on any number of repetition of Ventura's algorithm while exclusion supervision $|\psi_{exc}>$ gives the fifty percent probability of the simultaneous classification of the patterns |10> and |11> and hundred percent total probability of the classification of desired patterns '1?' on any number of repetition of the algorithm. It shows the perfect suitability of the exclusion superposition as two-pattern search state for simultaneous classification of patterns |1?> using Ventura's method also demonstrating its suitability for smaller data base.

Other possible two-state start systems may be obtained by choosing any of the following maximally entangled Bell's states as the possible search states (data-base):

$$|\phi_1>= -\frac{i}{\sqrt{2}}(|00>-|11>); \quad |\phi_2>= \frac{1}{\sqrt{2}}(|00>+|11>)$$

$$|\phi_3>= -\frac{i}{\sqrt{2}}(|01>+|10>); |\emptyset_4>= \frac{1}{\sqrt{2}}(|01>-|10>), \qquad ...(3.4)$$

Iterating these states repeatedly by the iteration operator $\widehat{D}$, given by eqn. (3.2), we find that the patterns |10> and |11> are never classified simultaneously and the probability of the irrelevant classification never falls below 50% with any of these MES as the search state. Thus none of the Bell's MES is the suitable choice of the two-pattern search state for the simultaneous classification of patterns |10> and |11> by using Grover's method of repeated iterations. None of these states has been found suitable as the search state for the classification of desired patterns |1?> by using Ventura's algorithm described by eqn. (2.4).

It also follows from all these results that the superposition of exclusion, represented by the state $|\psi_{exc}>$ given by the second of the eqns. (10) in a two-qubit system is the most suitable choice as the search state (data base) with two-patterns start state for both of Grover's and Ventura's algorithms in spite of the fact that this state does not contain any of the classified patterns. This result obtained here for two-qubit system supports the earlier result [21] for higher qubit-systems that the unknown patterns (not present in the concerned data-base) are classified more efficiently than the known patterns (present in the data-base).

(b) One pattern start state: Let us start with one pattern start state consisting of the pattern '11'. Then we have the following usually possible superposition with inclusion, exclusion and phase-inversion respectively as search states (data-base):

$$|\psi_{inc}>=\begin{bmatrix}0\\0\\0\\1\end{bmatrix};\quad |\psi_{exc}>=\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\\0\end{bmatrix};\quad |\psi_{phi}>=\frac{1}{2}\begin{bmatrix}1\\1\\1\\-1\end{bmatrix} \qquad \ldots(3.5)$$

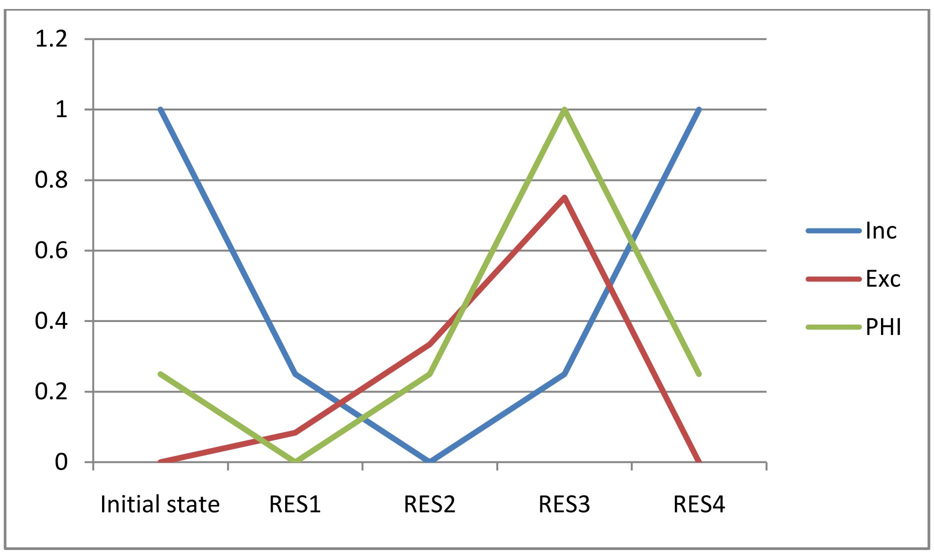

Figure-2: Graph for Probabilities of Pattern Classification of pattern |11> with self- start state

The comparative probabilities of classification of patterns |10> and |11> on different number of iterations of operator $\widehat{D}$, given by eqn. (3.2), on all these superposition as respective search states are shown in graphs of figure-2 and figure-3 respectively where RES denotes the number of iterations, red curve gives the probabilities of classification on different iterations of $|\psi_{exc}>$, and blue and green curves give the probabilities of classification for desired patterns on different iterations of $|\psi_{inc}>$ and $|\psi_{phi}>$ respectively.

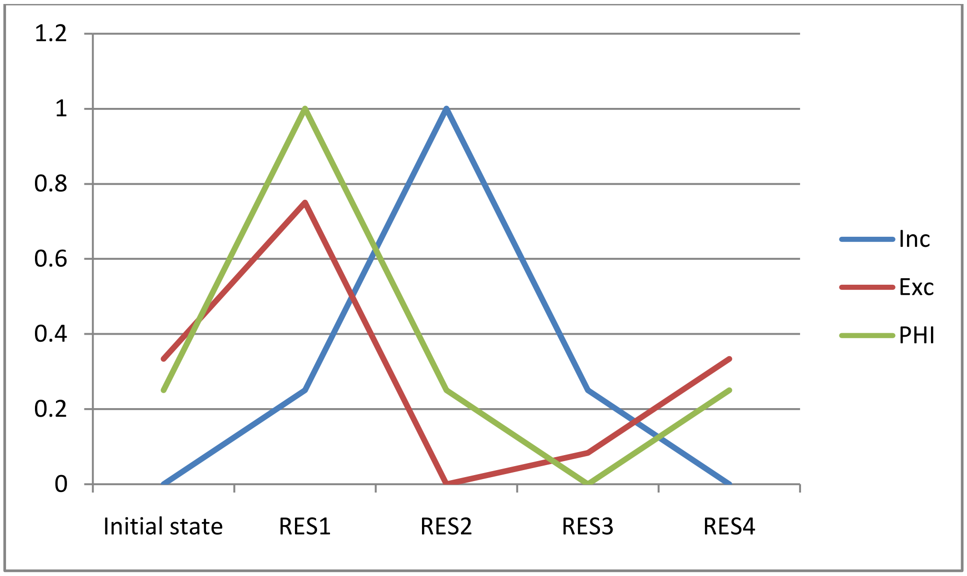

Figure-3: Graph for Probabilities of Classification of Pattern |10> with |11> as start state

On applying Ventura's method described by eqn. (2.4) for classification of desired patterns |1?> using the one pattern start-state |11>, we have found that inclusion superposition gives 25% probability of classification of desired pattern on any number of repetition of Ventura's algorithm; exclusion superposition gives 75% probability of classification of pattern |11> and 8% probability of classification of pattern |10> on first application of algorithm and vice a versa on its second application; and the superposition of phase-invariance, given by third of eqns. (3.5), yields 100% probability of classification of pattern |11> on first application and 100% probability of classification of |10> on second application of the algorithm. Thus in the case of one-pattern start-state |11> the superposition of $\left|\Psi_{phi}\right>$, given by third of eqns. (3.5), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns |1?> in two-qubit systems with the difference that while on the first iteration of Grover's algorithm the pattern |10> (absent from the one-pattern start-state) is classified with 100% probability, the first application of Ventura's method classifies the pattern |11> (already present in the one-pattern start-state) with 100% probability. These results demonstrate that in Grover's method the probabilities of correct classifications are higher for unknown patterns (not present in the one-pattern start-state and this method is more effective when the stored data is large (*i.e* $\left|\Psi_{phi}\right>$) but Ventura's method does not give better results for unknown patterns and also for smaller data base (*i.e* $|\psi_{inc}>$) in two-qubit system in contrast to the case of higher-qubits systems. These results also demonstrate that in a 2-qubit system the maximum probability in

Ventura's algorithm is not obtained in the case when the number of stored data $= m = \frac{N}{4} + 2 = 3$ (*i.e*, the search state $|\psi_{exc}>$, given by second of eqns. (3.5) as claimed for higher-qubits systems in an earlier paper [21].

Let us now consider one-pattern start-state consisting of the pattern |10> and construct the following superposition as the possible search states:

$$|\psi_{inc}> = \begin{bmatrix}0\\0\\1\\0\end{bmatrix};\ \ |\psi_{exc}> = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\0\\1\end{bmatrix};\ \ |\psi_{phi}> = \frac{1}{2}\begin{bmatrix}1\\1\\-1\\1\end{bmatrix} \qquad \text{... (3.6)}$$

The comparative probabilities of classification of patterns |10> and |11> on different number of iterations of all these superposition as respective search states are shown in graphs of figures-2 and fig-3 respectively (with roles of these patterns interchanged). For all these superposition, given by eqns. (3.6) corresponding to one-pattern start-state |10> and also those given by eqns. (3.5) corresponding to one pattern start-state |11>, the comparative total probabilities of classifications of desired patterns |1?> and the comparative probabilities of irrelevant classifications (of patterns different from desired ones) are shown by graphs given in figure-4 and figure-5 respectively.

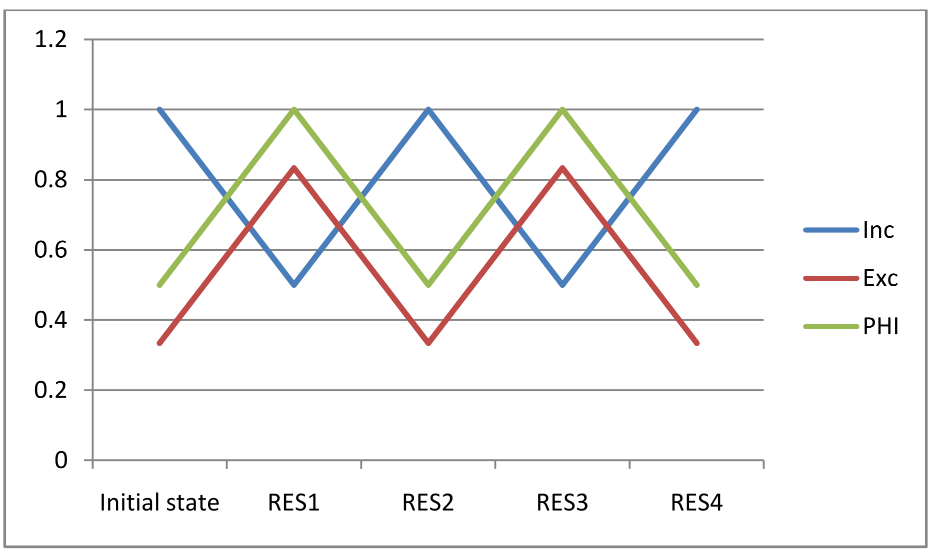

Figure-4: Graph for Total Probabilities of Classifications of desired Patterns |1?> with one-pattern start-state

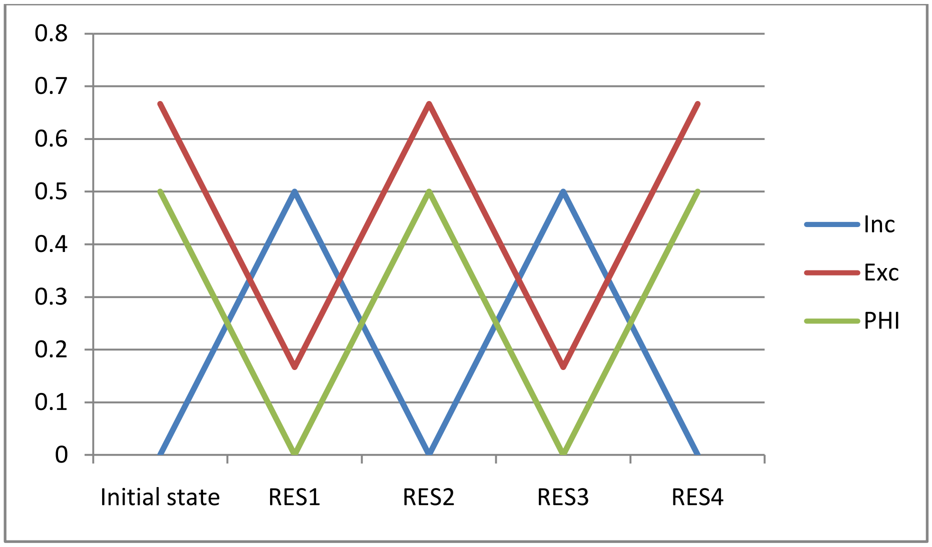

Figure-5: Graph for Probabilities of Irrelevant Classification with one-pattern start-state

Using Ventura' method the superposition of phase-invariance, given by third of eqns. (3.6), yields 100% probability of classification of pattern |10> on first application and 100% probability of classification of pattern |11> on second application of the algorithm. These results also show that Ventura's method gives better results neither for unknown patterns (not present in the search states) nor for smaller data base (*i.e.*, $|\psi_{inc}>$, given by first of eqns. (3.6)) in two-qubit system in contrast to the case of higher-qubits systems [23] . These results demonstrate further that in this case also the maximum probability in Ventura's algorithm is not obtained when the number of stored data $= m = \frac{N}{4} + 2 = 3$ (*i.e.*, the search state $|\psi_{exc}>$, given by second of eqns. (3.6) as claimed for higher-qubits systems.

These results show that in the case of one-pattern star- state |10> also the superposition of $|\Psi_{phi}>$, given by third of eqns. (3.6), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns in two-qubit systems. These results also demonstrate that in Grover's method the probabilities of correct classifications are higher for unknown patterns (not present in the one-pattern start-state) and this method is more effective when the stored data is large (*i.e.*, $|\Psi_{phi}>$). Thus in both the one-pattern start-states consisting of patterns |10> and |11> respectively the superposition of phase-invariance, given by third of eqns. (3.6) and third of eqns. (3.5) respectively, are the best choice as the respective search state in both Grover's and Ventura's methods of classifications of patterns. These states respectively are identical to the third and fourth states $|\psi_3>$ and $|\psi_4>$ of Singh- Rajput MES (maximally entangled states) [1,2] given as,

$$|\psi_1> = \frac{1}{2}[-|00> +|01> +|10> +|11>],$$

$$|\psi_2> = \frac{1}{2}[\ |00> -|01> +|10> +|11>],$$

$$|\psi_3> = \frac{1}{2}[\,|00> +|01> -|10> +|11>],$$

$$|\psi_4> = \frac{1}{2}[\,|00> +|01> +|10> -|11>] \qquad \ldots(3.7)$$

The concurrence for each of these states is unity (each of these states is maximally entangled) and these states constitute the orthonormal complete set (forming Singh-Rajput Eigen basis) since

$$<\psi_\mu|\psi_\nu> = \delta_{\mu\nu}$$

and $$\sum_{\mu=1}^{4} |\psi_\nu><\psi_\mu| = I \qquad \ldots(3.8)$$

Thus any of the maximally entangled states $|\psi_3>$ and $|\psi_4>$ of Singh-Rajput basis is the most suitable choice as search state for the desired pattern classification '1?' based on both of Grover's iterative search algorithm and Ventura's repeated search algorithm in two-qubit system. Each of these states consists of the entire data base of a two-qubit system and the higher effectiveness of Grover's algorithm for such large search states is obvious but the suitability of these states in Ventura's algorithm also, in contrast to the earlier results about its higher effectiveness for smaller data base for higher – qubit systems, is worth mentioning.

Let us now choose the full data base (search state) given by eqn. (2.2). On different number of iterations of this state by the operator, given by eqn. (3.2), the probabilities of classification of patterns $|10>$ and $|11>$ have been calculated and it has been found that the probability of simultaneous classification of these patterns remains only 25% and the probability of irrelevant classifications does not fall below 50% on any number of iteration. Exactly similar results have been obtained by applying Ventura's algorithm on this state. Such a low efficiency of classification is expected on applying Ventura's algorithm on the largest data base (full data base) considered here but the low efficiency on applying Grover's algorithm on this state contradicts the general result of high efficiency of Grover's algorithm for large data base. Exactly similar results were found on applying Grover's and Ventura's algorithm on the full data base represented by state $|\psi_{phi}>$ given by eqns. (3.3). Thus even the Grover's method is not always effective for larger data base for a two-qubit system. On the other hand full data-base states representing phase-invariance superposition in eqns. (3.5) and (3.6), which are respectively identical to $|\psi_4>$ and $|\psi_3>$ of Singh-Rajput basis given by eqns. (3.7), have been shown as most suitable choice for search state for the desired pattern classification '1?' based on both of Grover's iterative search algorithm and Ventura's repeated search algorithm. Thus for the full data base represented by maximally entangled states in a two-qubit system , like third and fourth states of Singh-Rajput MES, Grover's and Ventura's algorithms both are fully efficient for classification of patterns '1?', where ? denotes 0 or 1. This result supports our earlier results [17, 18].

SIMULTANEOUS CLASSIFICATIONS OF PATTERNS |0? >

Let us find the probability of observing the correct classification of the point '0?', where ? denotes 0 or 1 by using Grover's and Ventura's algorithms respectively. For the given search point the involved qubits are $|00>$ and $|01>$ and therefore the phase inversion operator $\hat{R}$ and the iteration operator $\hat{D}$ are respectively given by

$$\hat{R} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\hat{D} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & -1 \end{bmatrix} \quad \text{... (4.1)}$$

With two pattern search states consisting of patterns |00> and |01> we get the following possible general superposition (inclusion, exclusion and phase-inversion) respectively;

$$|\psi_{inc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}; \ |\psi_{exc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}; \ |\psi_{phi}> = \frac{1}{2}\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \quad \text{... (4.2)}$$

On different iterations of these states by the operator $\hat{D}$ of eqn. (4.1), the graphs of comparative probabilities of simultaneous classification of patterns |00> and |01> have been obtained in the form exactly identical as shown in figure-1 with patterns $|1?>$ replaced by $|0?>$ showing that in this case also the exclusion superposition $|\psi_{exc}>$, given by eqn. (4.2), is the most suitable two-pattern search state for simultaneous classification of patterns |00> and |01> while none of these patterns occurs in this search state (data-base). Here also the Grover's algorithm does not give satisfactory classification with the search state as the phase-inversion superposition $|\psi_{phi}>$ consisting of full data base for the 2-qubit system, contradicting the claim that Grover's algorithm is most effective when the number of patterns in the search state (the number of stored data) is large. Applying Ventura's method, described by eqn. (2.4), on all these superposition, given by eqns. (4.2), we have found the perfect suitability of the exclusion superposition $|\psi_{exc}>$, as two-pattern search state for simultaneous classification of patterns |00> and |01> by using Ventura's method also. Thus superposition of exclusion, represented by the state $|\psi_{exc}>$ given by the second of the eqns. (4.2) in a two-qubit system is the most suitable choice as the search state (data base) with two-patterns start state for the simultaneous classification of patterns |00> and |01> using Grover's and Ventura's algorithms respectively, in spite of the fact that this state does not contain any of the classified patterns. This result also supports the earlier result [21] for higher – qubit systems that the unknown patterns (not present in the concerned data-base) are classified more efficiently than the known patterns (present in the data-base).

Choosing one pattern start state consisting of the pattern '00', we get the following three usually possible superposition as search states (data-base):

$$|\psi_{inc}> = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}; \quad |\psi_{exc}> = \frac{1}{\sqrt{3}}\begin{bmatrix}0\\1\\1\\1\end{bmatrix}; \quad |\psi_{phi}> = \frac{1}{2}\begin{bmatrix}-1\\1\\1\\1\end{bmatrix} \qquad \text{... (4.3)}$$

The graphs of comparative probabilities of classification of patterns |00> and |01> on different number of iterations of operator $\widehat{D}$, given by eqn. (4.1), on all these superposition as respective search states have been obtained exactly similar to those as shown in graphs of figure-2 and figure-3 respectively with patterns |11> and |10> replaced by patterns |00> and |01> respectively. On applying Ventura's method described by eqn. (2.4) for classification of desired patterns |0?> using the one pattern start-state |00>, we have found that the superposition of phase-invariance, given by third of eqns. (4.3), yields 100% probability of classification of pattern |00> on first application and 100% probability of classification of |01> on second application of the algorithm.Thus in the case of one-pattern start-state |00> the superposition of $|\Psi_{phi}>$, given by third of eqns. (4.3), is the best choice as search state in both the algorithms (Grover's and Ventura's) for the classification of patterns |0?> in two-qubit systems with the difference that while on the first iteration of Grover's algorithm the pattern |01> (absent from the one-pattern start-state) is classified with 100% probability, the first application of Ventura's method classifies the pattern |00> (already present in the one-pattern start-state) with 100% probability. These results also demonstrate that in Grover's method the probabilities of correct classifications are higher for unknown patterns and this method is more effective when the stored data is large (*i.e.*, $|\Psi_{phi}>$) but Ventura's method does not give better results for unknown patterns and also for smaller data base (*i.e.*, $|\psi_{inc}>$) in two-qubit system in contrast to the case of higher-qubits systems.

Applying repeatedly the operator $\widehat{D}$, given by eqn. (4.1), on the following possible superposition as the search states with one-pattern start-state consisting of the pattern |01>

$$|\psi_{inc}> = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}; |\psi_{exc}> = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\0\\1\\1\end{bmatrix}; \quad |\psi_{phi}> = \frac{1}{2}\begin{bmatrix}1\\-1\\1\\1\end{bmatrix}, \qquad \text{... (4.4)}$$

We found here also that the superposition of $|\Psi_{phi}>$, given by third of eqns. (4.4), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns |0?> in two-qubit systems. Thus in both the one-pattern start-states consisting of patterns |00> and |01> respectively the superposition of phase-invariance, given by third of eqns. (4.3) and third of eqns. (4.4) respectively, are the best choice as the respective search state in both Grover's and Ventura's methods of classifications of patterns. These states respectively are identical to the first and second states $|\psi_1>$ and $|\psi_2>$ of Singh-Rajput MES (maximally entangled states) given by eqns. (3.7).

CLASSIFICATIONS OF SEPARATE PATTERNS OF A TWO-QUBITS SYSTEM

For the classification of pattern |11> the phase inversion operator $\widehat{R}$ and the iteration operator $\widehat{D}$ are respectively given by

$$\hat{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

and

$$\widehat{\mathrm{D}} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \ldots (5.1)$$

Applying this iteration operator on the different superposition, given by eqns. (3.3) for two pattern search states consisting of patterns |10> and |11>, we find that the probability of correct pattern classification (*i.e.*, the pattern |11>) does not exceed 50% with the search states $|\psi_{inc}>$ and $|\psi_{exc}>$ of eqns. (3.3) while with the search state $|\psi_{phi}>$ the probability of correct pattern classification is zero and that of incorrect classification (*i.e.*, the pattern |10>) is 100% on first iteration using Grover's method. Using Ventura's algorithm described by eqn. (2.4), it has been found that the probability of classification of correct pattern classification with any of the search state of eqns. (3.3) does not exceed 25% on first application of Ventura's method while the probability of incorrect pattern classification is 100% on its second application on the search state $|\psi_{phi}>$of eqn. (3.3). Thus none of the superposition (not even $|\psi_{exc}>$) of eqns. (3.3) for two-patterns start state is suitable as the search state for the correct pattern classification using any of the algorithm, Grover's or Ventura's.

The comparative probabilities of correct pattern classification (the pattern |11>) and irrelevant classifications (patterns other than |1?>) on applying the iteration operator (5.1) of the Grover's algorithm on various superposition given by eqns. (3.5) for single self-start state (consisting of the pattern |11>) have been computed and plotted in the graphs of figure-6 and figure-7, respectively, showing that with $|\psi_{phi}>$ of eqns. (3.5) as the search state the correct pattern is classified with 100% probability (with zero per cent probability of irrelevant classifications) on second iteration while it does not exceed 25% and 75% in first two iterations of $|\psi_{inc}>$ and $|\psi_{exc}>$ respectively.

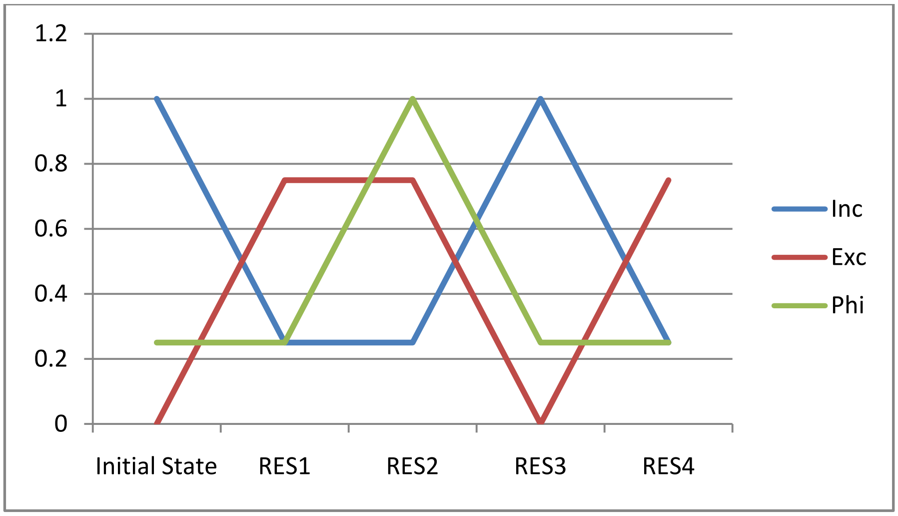

Figure-6: Comparative Probabilities of correct Pattern Classification (pattern |11>)with single self-start state (consisting of pattern |11>)

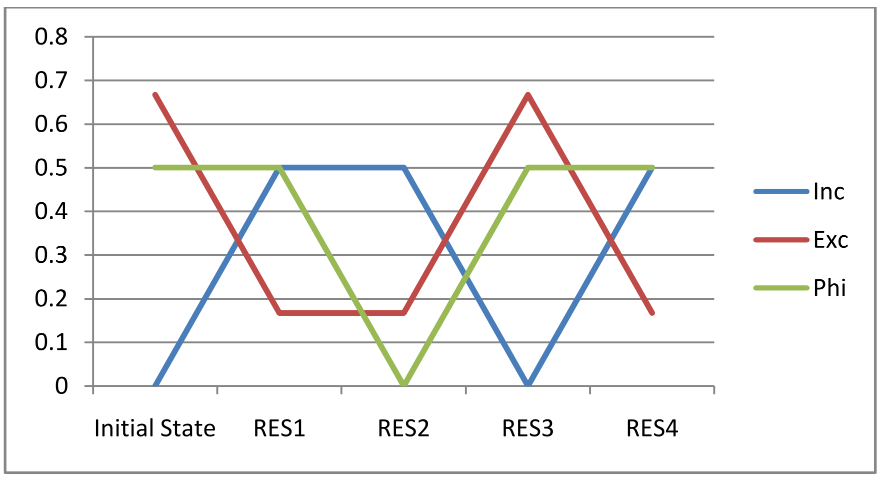

Figure-7: Comparative Probabilities of Irrelevant Classification with self-start state (|11>)

Applying Ventura's algorithm described by eqn. (2.4) on all these superposition, we get the same results as obtained by Grover's method of iterations except that 100% probability of correct pattern classification is obtained on the first application of Ventura's algorithm on the search state $|\psi_{phi}>$ of eqns. (3.5). Thus the superposition of $|\psi_{phi}>$ is the most suitable search state for the classification of pattern |11> by using any of Grover's and Ventura's algorithms. It is noteworthy that this search state is identical to fourth state $|\psi_4>$ of Singh-Rajput MES given by eqns. (3.7).

Probabilities of correct pattern classification (of pattern |11>) have been calculated by using the Grover's method with iteration operator of eqn. (5.1) applied on various superposition given by eqns. (3.6) for the start state of single pattern |10> and the comparative probabilities of correct pattern classification and irrelevant pattern classification have been respectively plotted in the graphs of figure-8 and figure-9 showing that the probability of correct pattern classification never exceeds beyond 25% and that of irrelevant classification does not falls below 50% on first iteration with $|\psi_{inc}>$ or $|\psi_{phi}>$ of eqns. (3.6) as search state while with $|\psi_{exc}>$ as search state the probability of correct classification on the first iteration in Grover's method is 75% and that of irrelevant classification is below 20%.

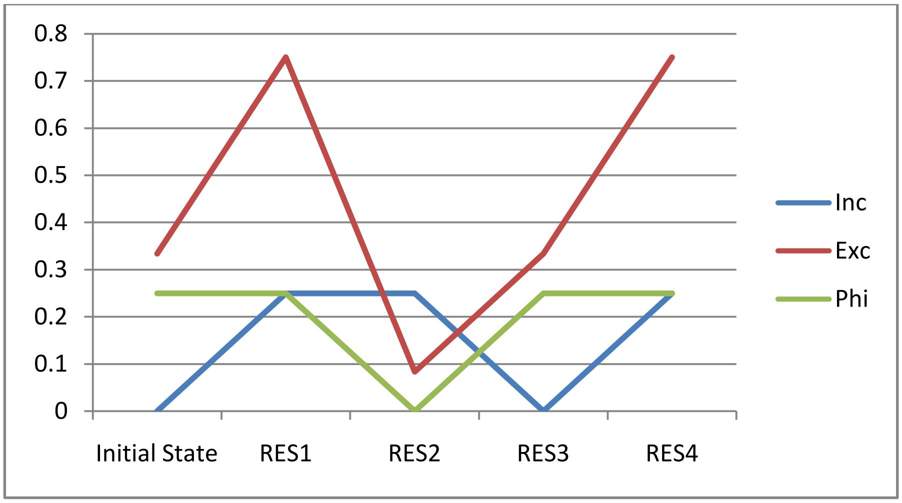

Figure-8: Comparative Probabilities of Classification of Pattern |11> with Start-State of Single Pattern |10>

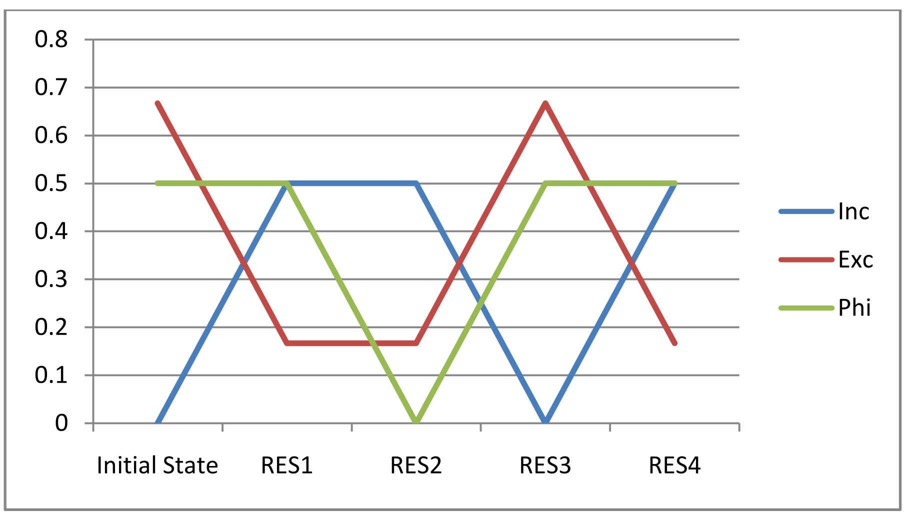

Figure-9: Comparative Probabilities of Irrelevant Classifications with Start-State of Single Pattern |10>

Applying Ventura's algorithm described by eqn. (2.4) on all the superposition given by eqns. (3.6) with the single-pattern start state |10> the probability of classification of pattern |11> has been found below 25% on any number of application of the algorithm on the search state $|\psi_{inc}>$ or $|\psi_{phi}>$ while it is as high as 75% on the second application of the algorithm on the search state $|\psi_{exc}>$ given by second of eqns. (3.6). Thus this superposition

$|\psi_{exc}>$ of equation (3.6) with the start state of single pattern |10> is the most suitable choice as the search state for the correct classification of the pattern |11> with the start state of single pattern |10> on the first iteration of Grover's algorithm and also on the second application of the Ventura algorithm.

For the classification of pattern |10> the phase inversion operator $\widehat{R}$ and the iteration operator $\widehat{D}$ are respectively given by

$$\hat{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\widehat{D} = \frac{1}{2}\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \qquad \ldots (5.2)$$

Applying separately this iteration operator and the Ventura's algorithm described by eqn. (2.4) on the different superposition, given by eqns. (3.3) for two pattern search states consisting of patterns |10> and |11>, we find that none of the superposition (not even $|\psi_{exc}>$) of eqns. (3.3) for two-patterns start state is suitable as the search state for the correct classification of the pattern |10> also. Applying the iteration operator given by eqn. (5.2) for this pattern classification on the superposition of eqns. (3.6) with the self- single-pattern start-state the identical graphs of comparative probabilities of correct classification and the irrelevant classification have been obtained as shown respectively in figure-7 and figure-8 with pattern |11> replaced by pattern |10>. It shows the suitability of the superposition $|\psi_{phi}>$, given as third of the eqns. (3.6) with self-single-pattern start-state, as the search state in Grover's algorithm for the classification of the pattern |10>. Applying Ventura's algorithm described by eqn. (2.4) on all these superposition (3.6) for the classification of pattern |10>, we get 100% probability of correct pattern classification on the first application of the algorithm on the search state $|\psi_{phi}>$ of eqns. (3.6). Thus the superposition of $|\psi_{phi}>$ given by third of eqns. (3.6) is the most suitable search state for the classification of pattern |10> by using any of Grover's and Ventura's algorithms. This search state is identical to third state $|\psi_3>$ of Singh-Rajput MES given by eqns. (3.7). Using Grover's method of pattern classification by applying the iteration operator of eqn. (5.2) separately on the states of eqns. (3.5) obtained from the single-pattern start-state |11>, we calculated the probabilities of correct classification of pattern |10> and those of irrelevant classification and found the graph of these comparative probabilities identical to those shown in figure-8 and figure-9 respectively with patterns |10> and |11> and search states of eqns. (3.6) replaced by those given by eqns. (3.5). These graphs show that on using the Grover's method the probability of correct classification of pattern |10> never exceeds beyond 25% and that of irrelevant classification does not falls below 50% on first iteration with $|\psi_{inc}>$ or $|\psi_{phi}>$ of eqns. (3.6) as search state while with $|\psi_{exc}>$ as search state the probability of correct classification on the first iteration is as high as 75% and that of irrelevant classification is below 20%. Applying Ventura's algorithm on all the search states given by eqns. (3.5) with the single-pattern start state |11>, the probability of classification of pattern |10> has been found below 25% on any number of application of the algorithm on the search state $|\psi_{inc} >$ or $|\psi_{phi} >$ while it is as high as 75% on the second application of the algorithm on the search state $|\psi_{exc} >$ given by second of eqns. (3.5). Thus this superposition $|\psi_{exc} >$ of equation (3.5) with the start state of single pattern |11> is the most suitable choice as the search state for the correct classification of the pattern. Applying Ventura's algorithm described by eqn. (2.4) on all the superposition given by eqns. (3.6) with the single-pattern start state |10> the probability of classification of pattern |11> has been found below 25% on any number of application of the algorithm on the search state $|\psi_{inc} >$ or $|\psi_{phi} >$ while it is as high as 75% on the second application of the algorithm on the search state $|\psi_{exc} >$ given by second of eqns. (3.6). Thus this superposition $|\psi_{exc} >$ of equation (3.6) with the start state of single pattern |10> is the most suitable choice as the search state for the correct classification of the pattern |11> with the start state of single pattern |10> on the first iteration of Grover's algorithm and also on the second application of the Ventura's algorithm.

Let us now carry out the classification of pattern |00> by using Grover's and Ventura's algorithms separately. For this classification the inversion operator and the iteration operator are respectively obtained as

$$\hat{R} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\hat{D} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix} \qquad \text{... (5.3)}$$

Applying this iteration operator on the different superposition, given by eqns. (4.2) for two pattern search states consisting of patterns |00> and |01> , we find that the probability of correct classification of the pattern |00> does not exceed 50% with the search states $|\psi_{inc} >$ and $|\psi_{exc} >$ of eqns. (4.2) while with the search state $|\psi_{phi} >$ the probability of correct pattern classification is zero and that of incorrect classification (*i.e.*, the pattern |01>) is 100% on first iteration using Grover's method. Using Ventura's algorithm, it has been found that the probability of correct pattern classification with any of the search state of eqns. (4.2) does not exceed 25% on first application of Ventura's method and hence none of the superposition (not even $|\psi_{exc} >$) of eqns. (4.2) for two-patterns start state is suitable as the search state for the correct classification of pattern using any of the algorithm, Grover's or Ventura's. Applying the iteration operator of eqn. (5.3) on the search states of eqns. (4.3) obtained for the self-single-particle start-state, the comparative probabilities of correct classification of the pattern |00> and irrelevant classifications (patterns other than |0?>) have been computed and their graphs have been found identical to those given by figure-6 and figure-7, respectively, with pattern |11> replaced by pattern |00> and the search states of eqns. (3.5) replaced by those given by eqns. (4.3) showing that with $|\psi_{phi}>$ of eqns. (4.3) the pattern |00> is classified with 100% probability (with zero per cent probability of irrelevant classifications) on second iteration. We have also found that 100% probability of correct classification of this pattern is obtained on the first application of Ventura's algorithm on the search state $|\psi_{phi}>$ of eqns. (4.3). Thus the superposition of $|\psi_{phi}>$, given by third of the eqns. (4.3) is the most suitable search state for the classification of pattern |00> by using any of Grover's and Ventura's algorithms. It is noteworthy that this search state is identical to first state $|\psi_1>$ of Singh-Rajput MES given by eqns. (3.7). The graphs of comparative probabilities of correct classification of pattern |00> and those of irrelevant classification (patterns other than |0?>) on applying the iteration operator of eqn. (5.3) on the search states given by eqns. (4.4), with one-pattern start-state consisting of the pattern |01>, have been found identical to those given by figure-8 and figure-9 respectively, with patterns |11> and |10> replaced by patterns |00> and |01> respectively and the search states of eqns. (3.6) respectively replaced by those given by eqns. (4.4), showing that with $|\psi_{exc}>$ as search state the probability of correct classification of pattern |00> on the first iteration in Grover's method is 75% and that of irrelevant classification is below 20%. It has also been found that the second operation of Ventura's algorithm on the search state $|\psi_{exc}>$ given by second of eqns. (4.4) gives 75% probability of the classification of the pattern |00>.

For the classification of pattern |01> the inversion operator and the iteration operator are respectively given as

$$\hat{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\hat{D} = \frac{1}{2}\begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \qquad \ldots (5.4)$$

Applying this iteration operator and the algorithm described by eqn. (2.4) on each superposition of eqns. (4.2) for the two-patterns start-state it has been found that none of these superposition (not even $|\psi_{exc}>$) is suitable as the search state for the correct classification of pattern |01> using any of the algorithm, Grover's or Ventura's. On applying the iteration operator of eqn. (5.3) on search states given by eqns. (4.4) for the self-single –pattern start-state it has been found that with $|\psi_{phi}>$ as the search state the pattern |01> is classified with 100% probability (with zero per cent probability of irrelevant classifications) on second iteration of Grover's algorithm. It has also been shown that this pattern is classified with 100% probability on the first application of the Ventura's algorithm on the search state $|\psi_{phi}>$ given by third of the eqns. (4.4). Thus the superposition of $|\psi_{phi}>$, given by third of the eqns. (4.4) is the most suitable search state for the classification of pattern |01> by using any of Grover's and Ventura's algorithms. This search state is identical to second state $|\psi_2>$ of Singh-Rajput MES given by eqns. (3.7). Applying the iteration operator of eqn. (5.3) on the search states of eqns. (4.3) obtained from the single-pattern start-state |00> , it has been found that with $|\psi_{exc}>$ as search state the probability of correct classification of pattern |01> on the first iteration in Grover's method or on the second operation of Ventura's algorithm is 75% and that of irrelevant classification is below 20%.

DISCUSSION

Eqn. (3.2) gives the iteration operator for the simultaneous classification of the patterns |1?> using Grover's algorithm in a two-qubit system, where symbol ? is 0 or 1. Comparative probabilities of simultaneous classification of these patterns on applying Grover's method of repeated iterations on all the three superposition, given by eqns. (3.3) for two-patterns start-state, have been plotted in the graphs of figure-1 showing that the exclusion superposition, given by second of eqns. (3.3), is the most suitable two-pattern search state for simultaneous classification of patterns |11> and |10> while none of these patterns occurs in search state (data-base). It supports the earlier result [21,22] that Grover's method gives better results with the unknown patterns (not present in the search state or data-base). It is also clear from these graphs that though phase inversion superposition, given by third of eqns. (3.3), contains the full database, it is not a better choice for search state for the simultaneous classification of patterns |1?> by using Grover's method of iteration on two-pattern start-states. It is in contrast to the earlier claim [23] about effectiveness of Grover's iterative algorithm in case of higher number of patterns in a search state. Similar results have been observed on applying Ventura's model on these three superposition of two-pattern start state and it has been shown that exclusion superposition is the most suitable choice here also supporting the claim [23] that Ventura's method is more effective in case of smaller database. Similar results have been obtained about the suitability of the exclusion superposition $|\psi_{exc}>$, given by second of eqns. (4.2), for the simultaneous classification of the patterns |00> and |01> on applying the iteration operator of eqn. (4.1) and the corresponding Ventura's algorithm on each superposition of eqns. (4.2). Thus among all the two-patterns start states, the state $|\psi_{exc}>$, given by second of eqns. (3.3) or second of eqns. (4.2) has been shown to be the most suitable choice as search state for the simultaneous classifications of patterns |00> and |01> or the patterns |11> and |10> respectively using Grover's method or Ventura's algorithm in spite of the fact that this state does not contain any of the classified patterns. This result obtained here for two-qubit system supports the earlier result for higher qubit-systems that the unknown patterns (not present in the concerned data-base) are classified more efficiently than the known patterns (present in the data-base).

The graphs of figure-2 and figure-3 of comparative probabilities of classification of patterns |11> and |10> respectively on different number of iterations of operator of eqn. (3.2) and the results of Ventura's algorithm applied separately on the search states of eqns. (3.5), obtained from the single-pattern start-state consisting of pattern |11>, and the search states of eqns. (3.6), obtained from the single-pattern start state consisting of pattern |10>, show that the superposition of $|\Psi_{phi}>$, given by third of eqns. (3.5) and the third of eqns. (3.6), are respectively the best choice as search states in both the algorithms, Grover's and Ventura's, for the classification of patterns |1?> in two-qubit systems with the difference that while on the first iteration of Grover's algorithm the pattern absent from the one-pattern start-state is classified with 100% probability, the first application of Ventura's method classifies the pattern already present in the one-pattern start-state with 100% probability. These results demonstrate that in Grover's method the probabilities of correct classifications are higher for unknown patterns (not present in the one-pattern start- state) and this method is more effective when the stored data is large (*i.e.*, $|\Psi_{phi}>$) but Ventura's method does not give better results for unknown patterns and also for smaller data base (*i.e.*, $|\psi_{inc}>$) in two-qubit system in contrast to the case of higher-qubits systems. These results also demonstrate that in a 2-qubit system the maximum probability in Ventura's algorithm is not obtained in the case when the number of stored data $= m = \frac{N}{4} + 2 = 3$ (*i.e.*, the search state $|\psi_{exc}>$, given by second of eqns. (3.5) or second of eqns. (3.6)) as claimed for higher-qubits systems in an earlier paper [21]. Graphs of figure-4 and figure-5 of the comparative total probabilities of classifications of desired patterns |1?> and the comparative probabilities of irrelevant classifications (of patterns different from desired ones) and the corresponding results obtained by using Ventura's algorithm show that in both the one-pattern start-states consisting of patterns |10> and |11> respectively the superposition of phase-invariance, given by third of eqns. (3.6) and third of eqns. (3.5) respectively, are the best choice as the respective search state in both Grover's and Ventura's methods of classifications of patterns. These states respectively are identical to the third and fourth states $|\psi_3>$ and $|\psi_4>$ of Singh-Rajput MES (maximally entangled states) [ ] given by eqns. (3.7) and hence these states of Singh-Rajput are the most suitable choice as search states forthe simultaneous classification of the patterns |1?> based on both of Grover's iterative search algorithm and Ventura's repeated search algorithm in two-qubit system. The similar suitability of the first and second states $|\psi_1>$ and $|\psi_2>$ of Singh-Rajput MES has been demonstrated for the simultaneous classification of patterns |0?> by using Grover's method or the Ventura's algorithm. Each of these MES, $|\psi_1>, |\psi_2>, |\psi_3>$ and $|\psi_4>$, consists of the entire data base of a two-qubit system and the higher effectiveness of Grover's algorithm for such large search states is obvious as shown in our earlier papers [,] but the suitability of these states in Ventura's algorithm also, in contrast to the earlier results about its higher effectiveness for smaller data base for higher–qubit systems [23], is worth mentioning here.

Graphs of figure-6 and figure-7 of comparative probabilities of correct pattern classification (the pattern |11>) and irrelevant classifications (patterns other than |1?>) on applying the iteration operator (5.1) of the Grover's algorithm on various superposition given by eqns. (3.5) for single self-start state (consisting of the pattern |11>)show that with $|\psi_{phi} >$ of eqns. (3.5) as the search state the pattern |11> is classified with 100% probability (with zero per cent probability of irrelevant classifications) on second iteration. Applying Ventura's algorithm described by eqn. (2.4) on all these superposition, we get the same results as obtained by Grover's method of iterations except that 100% probability of correct pattern classification is obtained on the first application of Ventura's algorithm on the search state $|\psi_{phi} >$ of eqns. (3.5). Thus the superposition of $|\psi_{phi} >$ of eqns.(3.5) is the most suitable search state for the classification of pattern |11> by using any of Grover's and Ventura's algorithms. It is noteworthy that this search state is identical to fourth state $|\psi_4 >$ of Singh-Rajput MES given by eqns. (3.7). Similarly, applying separately the iteration operators of eqn. (5.2), (5.3) and (5.4) and the corresponding Ventura's algorithm separately on the search states of eqns. (3.6), (4.3) and (4.4) obtained from the corresponding self- single-pattern start-states, it has been demonstrated that third state $|\psi_3 >$, the first state $|\psi_1 >$ and the second state $|\psi_2 >$ of Singh- Rajput MES given by eqns. (3.7) are the most suitable search states for the classification of patterns |11>, $|00 >$ and |01> respectively on the second iteration of Grover's method or the first operation of Ventura's algorithm.

Graphs of figure-8 and figure-9 and the corresponding results obtained by applying Ventura's algorithm described by eqn. (2.4) on all the superposition given by eqns. (3.6) demonstrate that the superposition $|\psi_{exc} >$ of equation (3.6), obtained from the start state of single pattern |10>, is the most suitable choice as the search state for the correct classification of the pattern |11> on the first iteration of Grover's algorithm and also on the second application of the Ventura algorithm. Similarly, it has been demonstrated that the states $|\psi_{exc} >$ respectively given by eqns. (3.5), (4.4) and (4.3), obtained from the single-pattern start-states consisting of patterns |11> , |01> and |00> respectively, are the most suitable search states for the classification of patterns |10> , |00> and |01> respectively on the first iteration in Grover's method or on the second operation of Ventura's algorithm.

REFERENCES

1. Manu P. Singh and B.S. Rajput, Maximally Entangled States of Two-Qubit Systems *Int. Journ. Theor. Phys.* **52,** 4237-4255 (2013).
2. Manu P. Singh and B.S. Rajput, New maximally entangled states and Pattern classification, *Int. Journ. Theor. Phys.* **53(9)** 3226-3238 (2014).
3. L.K. Grover, A Fast quantum Mechanical Algorithm for Data Base Search, Proc. 28th Ann. ACM. Symp. *On Theory of Computing*, Philadelphia, Pennsylvania, *ACM Press* 212-221 (1996).
4. D. Simon, On the power of Quantum Computation, SIAM Journ. Comp. 26(5) 1474-1483 (1997).

5. A.A. Ezhov, A.V. Nifanova and Dan Ventura, Quantum Associative Memories with Distributed Queries, information Sciences 128 271-293 (2000).
6. S.S. Li, Y.Y. Nie, Z.H Hong, X.J. Yi and Y.B. Huang, Controlled Teleportation Using Four-Particle Cluster State, *Comm. Theor. Phys.* 50, 633-640 (2008).
7. Y.B. Huang, S.S. Li and Y.Y. Nie, Controlled Dense Coding between Multi Particles, *Int. Journ. Theor. Phys.* 48, 95-100 (2009).
8. S.S. Li, Dense Coding with Cluster State via Local measurement, *Int. Journ. Theor. Phys.* 51, 724-730 (2012).
9. Z.S. Wang, C. Wu, X.L. Feng, L.C. Kwek, C.H. Lai, C.H. Oh, and V. Vedral, Non-adiabatic Geometric Quantum Computation, *Phys. Rev.* A76, 044303-307 (2007).
10. Z.S, Wang, Geometric Quantum Computation and Dynamical Variable Operators, *Phys. Rev.* A79, 024304-308 (2009).
11. T. Jennewein, C. Simon, G. Weihs, H. Weinfurter and A. Zeilinger, Quantum Cryptography with Entangled Photons, *Phys. Rev. Lett.* 84, 4729-4732 (2000).
12. D.S. Naik, C.G. Peterson, A.G. White, A. J. Burglund and P.G. Kwiat, Entangled State Quantum Cryptography, *Phys. Rev. Lett.* 84, 4733-4736 (2000).
13. W. Tittel, J. Bendel, H. Zbinden and N. Gisin, Quantum Cryptography using Entangled Photons in Energy-time Bell States, *Phys. Rev. Lett.* 84, 4737- 4740 (2000).
14. Manu P. Singh and B.S. Rajput, Role of Entanglement in Quantum Neural Networks (QNN), Journ. *Mod. Phys*. **6,** 1908-1920 (2015).
15. Manu P. Singh and B.S. Rajput, Pattern Classification in two-qubit and three-qubit systems, *Euro. Phys. J. Plus* **129**(57) 1-13 (2014).
16. Manu P. Singh and B.S. Rajput, Applications of Singh-Rajput MES in Recall operations of Quantum Associative Memory for a Two-Qubit System, *Int. Journ. Theor. Phys.* **54(10),** 3443-34460 (2015).
17. Manu P. Singh and B.S. Rajput, Processes of Quantum Associative Memory (QuAM) through New Maximally Entangled States (Singh-Rajput MES), *Int. Journ. Theor. Phys.* **55(2),** 124-140 (2016).
18. Manu P. Singh and B.S. Rajput, Pattern Classifications Using Grover's and Venchura's Algorithm in a Two-Qubit System, *Int. Journ. Theor. Phys.* 57, 692-705 (2018).
19. Manu P. Singh and B.S. Rajput, New Maximally Entangled States for Pattern Association Through Evolutionary Process in a Two-qubit System, *Int. Journ. Theor. Phys.* **56**(4), 1274-1285 (2017).
20. L.K. Grover, Quantum Mechanics helps in searching for a needle in haystack, *Phys. Rev. Lett.* **79,** 325-328 (1997).
21. W.K.Wootters, Entanglement of Formation and Concurrence, *Quantum Information and Computation* **1(1)**, 27-44 (2001).
22. Manu P. Singh, K. Radhey, V.K. Saraswat and Sandeep Kumar, Classification of Patterns Representing Apples and Oranges in Three – Qubit System, *Quant. Inf. Proc.* **16(1),** 1-24 (2017).
23. L.K. Grover, Super Classical Search Algorithm to find an Element in Unsorted Array, *Phys. Rev. Lett.* **78,** 325-328 (1997).
24. D. Ventura, On the Utility of Entanglement in Quantum Neural computing, *Proceedings of International Joint Conference on Neural Networks*, 1565-1570 (2001).
25. D. Ventura and T. Martinez, Quantum Associative Memory, *Information Sciences* **124,** 273-296 (2000).

❑