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CLASSIFICATIONS OF PATTERNS BASED ON HAMMING SEPARATIONS IN A TWO-QUBITS SYSTEM

BALWANT SINGH RAJPUT

Deptt. of Physics, Kumaon University, Nainital
I-11, Gamma-2, Greater Noida, Disst. Gautambudh Nagar (U.P) India
email: bsrajp@gmail.com

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Carrying out the classification of pairs of patterns in a two-qubit system by separately using Grover's and Ventura's algorithms on different possible superposition, it has been shown that the unknown patterns (not present in the concerned start state or the date base) are classified more efficiently than the known ones (present in the data-base) in Grover's algorithm.It has also been shown that all possible pairs of Singh-Rajput MES are the most suitable choice as the search states obtained from the corresponding single-pattern start-states for the classification of respective pairs of patterns of a two-qubit system. It has been further demonstrated that the pairs of consecutive states of Singh-Rajput MES are most suitable search states for the classification of pairs of class C_1 with minimum Hamming separations while the pairs of alternative states of these MES are suitable for the simultaneous classification of patterns of pairs of class C_3 with intermediate Hamming separation and the pair of the first and last states of these MES classify most efficiently the patterns of class C_2 with maximum Hamming separation.

Key Words : Pattern Classification; Entanglement; Qubits; Iterations; Search States; Grover's Algorithm; Ventura's Algorithm; Maximally Entangled States (MES); Singh-Rajput MES**,** Hamming Separations

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INTRODUCTION

In recent years the quantum entanglement [1] has played important role in the fields of quantum information theory[2,3], quantum computers[4], universal quantum computing network[5], teleportation[6], dense coding[7,8], geometric quantum computation[9,10] and quantum cryptography[11-13]. From physical point of view, entanglement is still little understood. What makes it too powerful is the fact that since quantum states exist as superposition, these correlations exist in superposition as well and when superposition is destroyed, the proper correlation is somehow communicated between the qubits [14]. It is this communication that is the crux of entanglement. We have recently explored [15] the entanglement as one of the key resources required for quantum neural network (QNN), constructed the complete set of new maximally entangled states (Singh-Rajput MES) different from Bell's MES in a two-qubit system and established [16] the functional dependence of the entanglement measures on spin correlation functions.We have also performed the pattern association (quantum associative memory) [17,18,19] and pattern classifications [20] by employing the method of Grover's iteration [21] on Bell's MES [22] and Singh-Rajput MES [15,16] in two-qubit system and demonstrated that, for all the related processes in a two-qubit system, Singh-Rajput MES provide the most suitable choice of memory states and the search states. Applying the method of Grover's iterate on three different superposition in three-qubit system, it has been shown [23] that the state corresponding to exclusive superposition is the most suitable choice as the search state for the desired pattern classifications.

Using Grover's method of repeated iterations [21, 25] and the algorithm of Ventura [26, 27], Singh and Radhey have recently undertaken [24] the study of the classification of patterns in a three-qubit system and demonstrated that the superposition of exclusion is the most suitable choice as the search state for these classifications. It has also been shown that in a three-qubit system the method of Grover is most effective for classification of unknown patterns (not present in the search states) with the largest data base (size of search state) while in the method of Ventura the classification of patterns is done more effectively with the smallest data-base. They have also carried out [28], very recently, the simultaneous classification of Oranges and Apples using both Grover's iterative algorithm and Ventura's model in a two-qubit system and demonstrated that the exclusion superposition is the most suitable two-pattern search state for simultaneous classification of patterns associated with Apples and Oranges and the superposition of phase-invariance are the best choice as the respective search states based on one –pattern start-states in both Grover's and Ventura's methods of classifications of patterns.

Dividing all the pairs of the patterns of a two-qubit system among three classes corresponding to minimum, maximum and intermediate Hamming separations respectively, in this paper we have used Grover's and Ventura's algorithms separately for simultaneous classification of the patterns of these pairs in terms of the search states obtained from two-patterns start-states and one pattern start states respectively. It has been shown that each superposition of exclusion $|\psi_{exc}>$obtained from the two-pattern start-state, consisting of patterns of the corresponding pair, is the most suitable choice of the search state for the simultaneous classification of these patterns supporting our earlier result [19, 20] that Grover's method is more efficient for the classification of the unknown patterns (not present in the data base) but contradicting the earlier claim [19,21] about effectiveness of Grover's iterative algorithm in case of higher number of patterns in a search state. Similar results have been observed on applying Ventura's model on these superposition of two-pattern start-state and it has been shown that exclusion superposition is the most suitable choice in this case also supporting the claim [24, 29] that Ventura's method is more effective in case of smaller database. It has also been shown that all possible pairs of Singh-Rajput MES [15,16] are the most suitable choice as the search states obtained from the corresponding single-pattern start-states for the classification of respective pairs of patterns of a two-qubit system demonstrating that in Grover's method the probabilities of correct classifications are higher for unknown patterns (not present in the one-pattern start-state) and this method is more effective when the stored data is large (*i,e* $|\Psi_{phi}>$) but Ventura's method does not give better results for unknown patterns and also for smaller data base (*i.e* $|\psi_{inc}>$) in two-qubit system in contrast to the case of higher-qubits systems [24, 29]. It has also been shown that the pairs of consecutive states of Singh-Rajput MES are most suitable search states for the classification of pairs of class C_1 with minimum Hamming separations while the pairs of alternative states of these MES are suitable for the simultaneous classification of patterns of pairs of class C_3 with intermediate Hamming separation and the pair of the first and last states of these MES classify most efficiently the patterns of class C_2 with maximum Hamming separation.

METHODS OF GROVER AND VENTURA FOR PATTERN CLASSIFICATIONS IN TWO-QUBIT SYSTEMS

The pattern classification may be performed in straight forward approach employing the method of Grover's repeated iterations [21, 25], which is described as a product of unitary operators $\hat{D}=\hat{G}\hat{R}$ applied to the chosen quantum search state iteratively and probability of desired result maximized by measuring the system after appropriate number of iterations. Here the operator $\hat{R}$ is phase inversion of the state(s) that we wish to observe upon measuring the system. It is represented by identity matrix I with diagonal elements corresponding to desired state(s) equal to -1 and the operator $\hat{G}$ is described as an inversion about average:

$$G=2|\Psi><\Psi|-I \quad ...(2.1)$$

where $|\Psi>$ represents the full data base available in the given quantum system *i.e.* for a two-qubit system it contains all the four possible patterns as

$$|\Psi>=\frac{1}{2}[\ |00>+|01>+|10>+|11>] \quad ...(2.2)$$

and hence for a two-qubit system we have the following inversion operator $\hat{G}$;

$$\hat{G} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \quad \ldots (2.3)$$

The number (r) of times the classification will have to be repeated in Grover's method in a 2-qubit system is $r = \frac{\pi}{4}\sqrt{N} = \frac{\pi}{2}$ where $N = 4$. It gives $1 \leq r < 2$.

On the other hand Ventura proposed [26, 27] an algorithm which may be written for the present case in the following simplified manner;

$$|\Psi > = \hat{G}I_\rho \hat{G}I_\tau |\psi >$$

Repeat at $\frac{\pi}{4}\sqrt{N} - 2 \approx 1$ time and take $|\hat{\Psi} > = \hat{G}I_\tau |\Psi >$ ...(2.4)

for measuring the probability of desired classifications where $|\psi >$ is the search state (stored data base), I_τ inverts the sign of the pattern to be classified, I_ρ inverts the signs of all patterns in the stored data base and the operator $\hat{G}$ is given by eqn. (2.3).

Different patterns of full data base given by eqn. (2.2) for a two-qubit system may be categorized into the following three classes of pairs of patterns with minimum, maximum and intermediate Hamming separations respectively:

$$C_1 = \{ P_1;\ P_2;\ P_3\}; C_2 = \{ P_4\} \text{ and } C_3 == \{ P_5; P_6\} \quad \ldots(2.5)$$

where P_1; $P_2 and$ P_3 denote the following pairs of patterns with minimum Hamming separation

$$P_1 = (|00 >;\ |01 >) = |0? >;$$

$$P_2 = (|11 >;\ |10 >) = |1? >; P_3 = (|01 >;\ |10 >) \quad \ldots (2.6)$$

and $$P_4 = (|00 >;\ |11 >; P_5 = (|00 >;\ |10 >) = |?\,0 >;$$

$$P_6 = (|01 >;\ |11 >) = |?\,1 > \quad \ldots(2.7)$$

where symbol ? denotes 0 or 1. Here P_1 and P_2 are symmetric pairs $|0?>$ and $|1?>$ of patterns with minimum Hamming separation; P_5 and P_6 are the symmetric pairs $|?\,0 >$ and $|?\,1 >$ of patterns with intermediate Hamming separation and P_3 and P_4 are the single pairs with minimum and maximum Hamming separations respectively.

CLASSIFICATION OF PAIRS OF PATTERNS OF CLASS C_1 (MINIMUM HAMMING SEPARATION)

Let us carry out the simultaneous classification of patterns of pairs P_1, P_2 and P_3 with minimum Hamming separation in the following subsections.

(a) Simultaneous Classification of Patterns of Pair P_1

Let us find the probability of observing the correct classification of the patterns '0?' by using Grover's and Ventura's algorithms respectively. For the given search point the involved qubits are $|00>$ and $|01>$ and therefore the phase inversion operator $\hat{R}$ and the iteration operator $\hat{D}$ are respectively given by

$$\hat{R} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\hat{D} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & -1 \end{bmatrix} \qquad \text{... (3.1)}$$

With two-patterns search states consisting of patterns | 00> and | 01>, we get the following possible general superposition (inclusion, exclusion and phase-inversion) respectively;

$$|\psi_{inc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}; \ |\psi_{exc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}; \quad |\psi_{phi}> = \frac{1}{2}\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \qquad \text{... (3.2)}$$

On different iterations of these states by the operator $\hat{D}$ of eqn. (3.1), the comparative probabilities of simultaneous classification of patterns |00> and |01>have been calculated and it is found that on any number of iterations of Grover's algorithm on the phase invariance superposition $|\psi_{phi}>$ the probability of classification of desired patterns | 0? > never exceeds 50% and the first iteration of the inclusion superposition $|\psi_{inc}>$ does not classify the desired patterns while the first iteration of exclusion superposition $|\psi_{exc}>$ gives the fifty percent probability of simultaneous classifications of each of the patterns and the 100% total probability of the classification of desired patterns | 0? > . With inclusion superposition such situation comes on second iteration but the allowed number of iterations r of Grover's algorithm for 2-qubit system is given by $1 \leq r < 2$ and hence the exclusion superposition is the most suitable two-pattern search state for simultaneous classification of patterns | 00 > and |01> while none of these patterns occurs in this search state (data-base). On applying Ventura's method described by eqn. (2.4) for classification of desired patterns | 0? >, we have found that inclusion superposition $|\psi_{inc}>$ of eqn. (3.2) gives the zero probability and phase invariance superposition $|\psi_{phi}>$ gives only 25% probability of this classification on any number of repetition of Ventura's algorithm while exclusion supervision $|\psi_{exc}>$ gives the fifty percent probability of the simultaneous classification of the patterns | 00 > and | 01 > and hundred percent total probability of the classification of desired patterns '0?' on any number of repetition of the algorithm. It shows the perfect suitability of the exclusion superposition as two-pattern search state for simultaneous classification of patterns | 0? > using Ventura's method also. It demonstrates the suitability of this method for smaller data base. It follows from all these results that the superposition of exclusion, represented by the state $|\psi_{exc}>$, given by the second of the eqns. (3.2) in a two-qubit system, is the most suitable choice as the search state (data base) with two-patterns start state for both of Grover's and Ventura's algorithms in spite of the fact that this state does not contain any of the classified patterns. This result obtained here for two-qubit system supports our earlier result [24] for higher qubit-systems that the unknown patterns (not present in the concerned data-base) are classified more efficiently than the known ones (present in the data-base).

Choosing one pattern start state consisting of the pattern '00', we get the following superposition as search states (data-base):

$$|\psi_{inc}> = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}; \quad |\psi_{exc}> = \frac{1}{\sqrt{3}}\begin{bmatrix}0\\1\\1\\1\end{bmatrix}; \quad |\psi_{phi}> = \frac{1}{2}\begin{bmatrix}-1\\1\\1\\1\end{bmatrix} \qquad \text{... (3.4)}$$

The graphs of comparative probabilities of classification of patterns of pair P_1 on different number of iterations of operator $\hat{D}$, given by eqn. (3.1), on all these superposition have been shown in figure-1 and figure- 2, respectively, where RES denotes the number of iterations, red curve gives the probabilities of classification on different iterations of $|\psi_{exc}>$, and blue and green curves give the probabilities of classification of desired patterns on different iterations of $|\psi_{inc}>$ and $|\psi_{phi}>$ respectively.

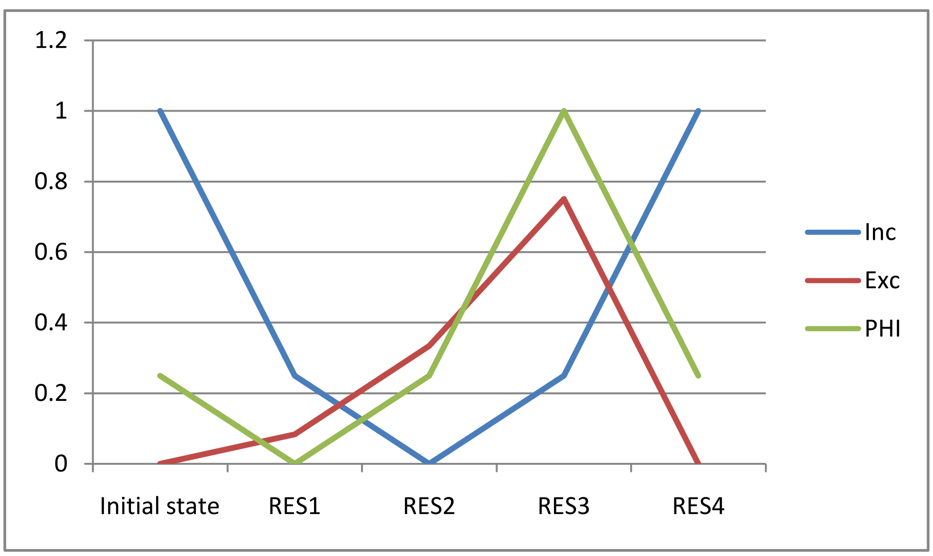

Figure-1: Graph for Probabilities of Classification of pattern | 00> with self- start state

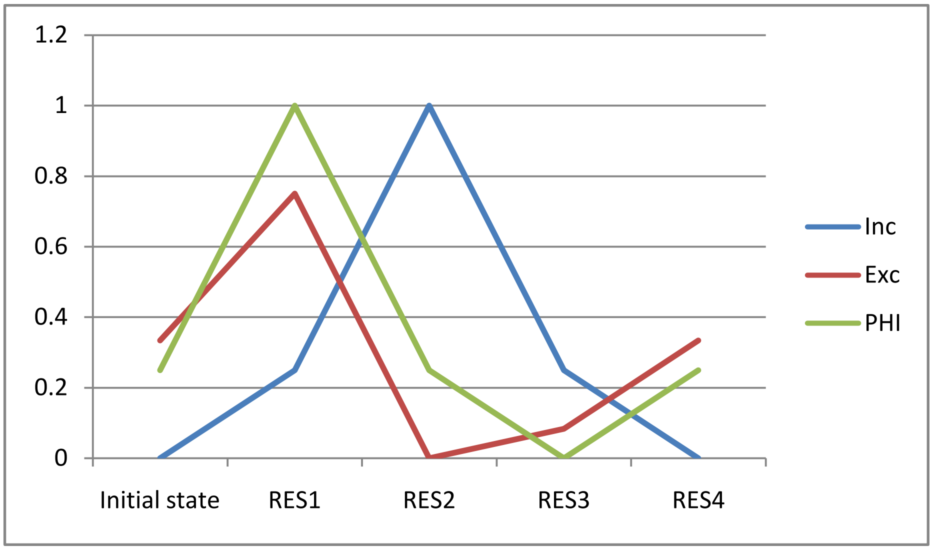

Figure-2: Graph for Probabilities of Classification of Pattern | 01> with | 00> as start state

These graphs demonstrate the superiority and suitability of the superposition of $|\Psi_{phi}>$, given by third of eqns. (3.4), as the search state.

On applying Ventura's method for classification of desired patterns | 0? > by using one pattern start- state | 00 > , we have found that the superposition of phase-invariance, given by third of eqns. (3.4), yields 100% probability of classification of pattern | 00 > on first application and 100% probability of classification of | 01 > on second application of the algorithm. Thus in the case of one- pattern start-state |00> this superposition of $|\Psi_{phi}>$, is the best choice as search state in both the algorithms (Grover's and Ventura's) for the classification of patterns | 0? > in two-qubit systems with the difference that while on the first iteration of Grover's algorithm the pattern | 01 > (absent from the one-pattern start-state) is classified with 100% probability, the first application of Ventura's method classifies the pattern |00> (already present in the one-pattern start- state) with 100% probability.

We have the following possible superposition as the search states with one-pattern start-state consisting of the pattern | 01 >

$$|\psi_{inc} >= \begin{bmatrix}0\\1\\0\\0\end{bmatrix}; \quad |\psi_{exc} >= \frac{1}{\sqrt{3}}\begin{bmatrix}1\\0\\1\\1\end{bmatrix}; \quad \psi_{phi} > = \frac{1}{2}\begin{bmatrix}1\\-1\\1\\1\end{bmatrix}, \qquad \text{... (3.5)}$$

Iterating these superposition repeatedly by the operator given by eqn. (3.1), we found here also that the superposition of $|\Psi_{phi}>$, given by third of eqns. (3.5), is the best choice as search state in Grover's method. Using Ventura's algorithm, we found that this superposition of phase-invariance yields 100% probability of classification of pattern | 01 > on first application and 100% probability of classification of pattern | 00 > on second application of the algorithm. These results show that in the case of one- pattern start- state | 01 > also the superposition of $|\Psi_{phi}>$, given by third of eqns. (3.5), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns of pair P_1 (with minimum Hamming separation) in a two-qubit system. Thus in both the one-pattern start-states consisting of patterns | 00 > and |01> respectively the superposition of phase-invariance, given by third of eqns. (3.4) and third of eqns. (3.5) respectively, are the best choice as the respective search state in both Grover's and Ventura's methods of classifications of patterns | 0? >. These states respectively are identical to the first and second states $|\psi_1>$and $|\psi_2>$ of Singh- Rajput MES (maximally entangled states) [15,16], given as

$$|\psi_1>=\frac{1}{2}[-|00>+|01>+|10>+|11>],$$

$$|\psi_2>=\frac{1}{2}[|00>-|01>+|10>+|11>],$$

$$|\psi_3>=\frac{1}{2}[|00>+|01>-|10>+|11>],$$

$$|\psi_4>=\frac{1}{2}[|00>+|01>+|10>-|11>] \quad \ldots(3.6)$$

which constitutes the orthonormal complete set of maximally entangles states (Singh-Rajput Eigen Basis).

(b) Simultaneous Classification of Patterns of Pair P_2 : For the simultaneous classification of patterns |1?>, through Grover's method of repeated iterations, the phase inversion operator $\hat{R}$ and the iteration operator $\hat{D}$ are respectively given as

$$\hat{R}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix} \quad \ldots(3.7)$$

and

$$\hat{D}=\frac{1}{2}\begin{bmatrix}-1&1&-1&-1\\1&-1&-1&-1\\1&1&1&-1\\1&1&-1&1\end{bmatrix} \quad \ldots(3.8)$$

Let us first apply Grover's algorithm by choosing two pattern search states consisting of patterns |10> and |11> with the following possible general superposition (inclusion, exclusion and phase-inversion) respectively;

$$|\psi_{inc}>=\frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\1\end{bmatrix};\quad |\psi_{exc}>=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\\0\end{bmatrix};\quad |\psi_{phi}>=\frac{1}{2}\begin{bmatrix}1\\1\\-1\\-1\end{bmatrix} \quad \ldots(3.9)$$

On different iterations of these states by the operator $\hat{D}$ of eqn. (3.8), the comparative probabilities of simultaneous classification of patterns | 00 > and | 01 > have been obtained in the form exactly identical to those obtained in previous sub-section with the patterns $|0?>$ replaced by $|1?>$, showing that in this case also the exclusion superposition $|\psi_{exc}>$, given by second of eqns. (3.9), is the most suitable two-pattern search state for simultaneous classification of patterns |10> and |11> while none of these patterns occurs in this search state (data-base. Applying Ventura's method, described by eqn. (2.4), on all these superposition given by eqns. (3.9), we have found the perfect suitability of the exclusion superposition $|\psi_{exc}>$, as two-pattern search state for simultaneous classification of patterns | 10 > and | 11 > by using Ventura's method also. It demonstrates that the superposition of exclusion, represented by the state $|\psi_{exc}>$ given by the second of the eqns. (3.9) in a two-qubit system, is the most suitable choice as the search state (data base) with two-patterns start-state for the simultaneous classification of patterns | 10 > and | 11 > using Grover's and Ventura's algorithms respectively.

Let us start with one-pattern start-state consisting of the pattern | 11 >.Then we have the following usually possible superposition with inclusion, exclusion and phase-inversion respectively as search states (data-base):

$$|\psi_{inc}>=\begin{bmatrix}0\\0\\0\\1\end{bmatrix};\quad |\psi_{exc}>=\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\\0\end{bmatrix};\quad |\psi_{phi}>=\frac{1}{2}\begin{bmatrix}1\\1\\1\\-1\end{bmatrix} \qquad \text{... (3.10)}$$

The comparative probabilities of classification of patterns | 10 > and | 11 > on different number of iterations of operator $\widehat{D}$, given by eqn. (3.8), on all these superposition as respective search states are identical to those shown in graphs of figure-2 and figure- 3 respectively with patterns | 0? > replaced by patterns 1? > . On applying Ventura's method described by eqn. (2.4) on all these superposition, for classification of desired patterns 1?>, we have found that the superposition of phase-invariance, given by third of eqns. (3.10) and obtained from the single-pattern start-state consisting of pattern |11>, yields 100% probability of classification of pattern |11> on first application and 100% probability of classification of |10> on second application of the algorithm. Thus in the case of one-pattern start-state |11> the superposition of $|\Psi_{phi}>$, given by third of eqns. (3.10), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns |1? > with the difference that while on the first iteration of Grover's algorithm the pattern | 10 > (absent from the one-pattern start- state) is classified with 100% probability, the first application of Ventura's method classifies the pattern |11> (already present in the one-pattern start- state) with 100% probability.

Applying repeatedly the operator $\widehat{D}$, given by eqn. (3.8), on the following possible superposition as the search states with one-pattern start-state consisting of the pattern |01>

$$|\psi_{inc} >= \begin{bmatrix}0\\0\\1\\0\end{bmatrix}; |\psi_{exc} >= \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\0\\1\end{bmatrix}; \quad |\psi_{phi} > = \frac{1}{2}\begin{bmatrix}1\\1\\-1\\1\end{bmatrix}, \qquad ...(3.11)$$

we found here also that the superposition of $|\Psi_{phi} >$, given by third of eqns. (3.11), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns | 1? > in two-qubit systems. Thus in both the one- pattern start- states consisting of patterns | 11 > and | 10 > respectively the superposition of phase-invariance, given by third of eqns. (3.10) and third of eqns. (3.11) respectively, are the best choice as the respective search state in both Grover's and Ventura's methods of classifications of patterns. These states respectively are identical to the fourth and third states $|\psi_4 >$and $|\psi_3 >$ of Singh-Rajput MES (maximally entangled states) [15,16], given by eqns. (3.6).

(c) Simultaneous Classification of Patterns of Pair P_3 : For the simultaneous classification of patterns |01> and |10>, through Grover's method of repeated iterations, the phase inversion operator $\hat{R}$ and the iteration operator $\widehat{D}$ are respectively given as

$$\hat{R} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & -1 & 0 & 0\\0 & 0 & -1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \qquad ...(3.12)$$

and

$$\widehat{D} = \frac{1}{2}\begin{bmatrix}-1 & -1 & -1 & 1\\1 & 1 & -1 & 1\\1 & -1 & 1 & 1\\1 & -1 & -1 & -1\end{bmatrix} \qquad ...(3.13)$$

Choosing two-patterns search states consisting of patterns of pair P_3, we get the following possible general superposition (inclusion, exclusion and phase-inversion) respectively;

$$|\psi_{inc} >= \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}; \ \psi_{exc} >= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}; \ |\psi_{phi} >= \frac{1}{2}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} \qquad ...(3.14)$$

The results of computation of comparative probabilities of simultaneous classification of patterns | 01 > and | 10 > on different iterations of the states of eqns. (3.14) by the operator $\widehat{D}$ of eqn. (3.13) and also the applications of Ventura's algorithm on these states demonstrate that in this case also the exclusion superposition $|\psi_{exc} >$, is the most suitable two-pattern search state while none of these patterns occurs in this search state (data-base).

The comparative probabilities of classification of patterns | 01 > and |10> on repeated applications of the operator $\widehat{D}$ of eqn. (3.13), on all the search states, given by eqns. (3.5) obtained from one-pattern start-state consisting of the pattern | 01 >, are exactly the same as shown in graphs figure-2 and figure-3 respectively with patterns | 00 > replaced by patterns | 01 > and the pattern | 01 > replaced by pattern | 10 > . It shows that the patterns | 10 > and

| 01 > are respectively classified with 100% probability on first and third iterations of the search state $|\psi_{phi}>$.On applying Ventura's method described by eqn. (2.4) on these search states given by eqn. (3.5), we have found that the superposition of phase-invariance yields 100% probability of classification of pattern |01> on first application and 100% probability of classification of | 10 > on second application of the algorithm. Thus in the case of one-pattern start- state | 01 > the superposition of $|\psi_{phi}>$, given by third of eqns. (3.5), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns | 01 > and | 10 > with the difference that while on the first iteration of Grover's algorithm the pattern | 10 > (absent from the one-pattern start- state) is classified with 100% probability, the first application of Ventura's method classifies the pattern | 01 > (already present in the one-pattern start- state) with 100% probability.

Repeated applications of the iteration operator $\widehat{D}$ of eqn. (3.13) on the superposition, given by eqns. (3.11) and obtained from the single-pattern start-state |10>, and applying Ventura's algorithm, given by eqn. (2.4), we found that here also the superposition of $|\Psi_{phi},$ given by third of eqns. (3.11), is the best choice as search state in both the algorithms, Grover's and Ventura's, for the classification of patterns | 01 > and | 10 >. Thus in both the one- pattern start- states consisting of patterns | 01 > and | 10 > respectively, the superposition of phase-invariance, given by third of eqns. (3.5) and third of eqns. (3.11) respectively, are the best choice as the respective search state in both Grover's and Ventura's methods of classifications of patterns| 01 > and | 10 > simultaneously. These states respectively are identical to the fourth and third states $|\psi_4>$and $|\psi_3>$ of Singh- Rajput MES (maximally entangled states) [15,16], given by eqns. (3.6).

CLASSIFICATION OF PAIR OF PATTERNS OF CLASS C_2 (MAXIMUM HAMMING SEPARATION) :

For the simultaneous classification of the patterns | 00 > and | 11 > the phase inversion operator $\hat{R}$ and the iteration operator $\widehat{D}$ are respectively given as

$$\hat{R} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

and

$$\widehat{D} = \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix} \qquad \dots (4.1)$$

The results of computation of comparative probabilities of simultaneous classification of patterns |00> and |11> on different iterations of the following states, obtained from the two-patterns start-state consisting of patterns of the pair P_4 by the operator $\widehat{D}$ of eqn. (4.1),

$$|\psi_{inc}> = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix};\ |\psi_{exc}> = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix};\quad |\psi_{phi}> = \frac{1}{2}\begin{bmatrix}-1\\1\\1\\-1\end{bmatrix} \qquad \dots (4.2)$$

and also the applications of Ventura's algorithm on these states demonstrate that in this case also the exclusion superposition $|\psi_{exc}>$, is the most suitable two-pattern search state while none of these patterns occurs in this search state (data-base). The comparative probabilities of classifications of the patterns | 00 > and | 11 > on repeated iterations of the operator of eqn. (4.1) on the states, given by eqns. (3.4) and obtained from the single-pattern start-state consisting of pattern | 00 >, have been computed and plotted in the graphs exactly identical to those shown in figure-1 and figure-2 respectively, with pattern | 01 > replaced by pattern | 11 >. It shows suitability of the superposition $|\Psi_{phi}>$, given by third of eqns. (3.4), for the classification of the patterns of pair P_4 by using Grover's algorithm with the single-pattern start state consisting of the pattern |00>. Similar result has been obtained by using Ventura's algorithm given by eqn. (2.4). Repeated applications of the operator of eqn. (3.15) on the superposition given by eqns. (3.10), obtained from the single-pattern start-state consisting of the pattern |11>, and the applications of Ventura's algorithm demonstrate the superiority and suitability of the superposition $|\Psi_{phi}>$, given by third of the eqns. (3.10) for the simultaneous classification of the patterns of the pair P_4. Thus the superposition $|\Psi_{phi}>$, given by third of eqns. (3.4) and third of eqns. (3.10) respectively are the most suitable choice of the search state, obtained from single-pattern start-states, for the classification of the patterns of the pair P_4 on applying Grover's and Ventura's algorithms respectively with the difference that while the first operation of Grover's algorithm classify the pattern | 11 > in the first case and the pattern | 00 > in the second case with hundred percent probability, the first application of Ventura's algorithm classifies with certainty the pattern | 00 > in the first case and the pattern | 11 > in the second case. In other words, in this case also Grover's algorithm classifies most efficiently the missing patterns (*i.e* absent from the start-state) while Ventura's algorithm is most efficient for the classification of the pattern present in single-pattern start state. It is worth mentioning here also that these states are the states $|\psi_1>$ and $|\psi_4>$ of Singh-Rajput MES (maximally entangled states) [15,16], given by eqns. (3.6).

CLASSIFICATION OF PAIRS OF PATTERNS OF CLASS C_3 (INTERMEDIATE HAMMING SEPARATION)

For the simultaneous classification of patterns of the pair P_5 the phase inversion operator $\hat{R}$ and the iteration operator $\widehat{D}$ may respectively be written as

$$\hat{R} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\widehat{\mathrm{D}} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 \end{bmatrix} \qquad \ldots (5.1)$$

The following possible superposition may be obtained from the two-pattern start-state consisting of the patterns of pair P_5;

$$|\psi_{inc} >= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}; |\psi_{exc} >= \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}; \quad |\psi_{phi} >= \frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \qquad \ldots (5.2)$$

Results of operations of these states by the iterative operator given by eqn. (5.1) and also the applications of Ventura's algorithm, given by eqn. (2.4), on these states demonstrate that the superposition $|\psi_{exc} >$ is the most suitable search state, obtained from the two-pattern start-state, for the simultaneous classification of the patterns of the pair P_5. The graphs of comparative probabilities of classification of patterns of pair P_5 on repeated applications of the operator of eqn. (5.1) on the superposition given by eqns. (3.4), obtained from single-pattern start-states consisting of pattern |00>, have been obtained as shown in figure-1 and figure-2 with pattern | 01 > replaced by pattern | 10 >. These graphs and the results of operation of Ventura's algorithm on these superposition demonstrate that $|\psi_{phi} >$, given by eqns. (3.4) and obtained from the single-pattern start-state consisting of the pattern | 00 >, is the most suitable choice as the search state for the classification of patterns of pair P_5 by Grover's and Ventura's algorithms. The similar operations of Grover's and Ventura's algorithms respectively on the superposition, given by eqns. (3.11), obtained from the single-pattern start-state consisting of the pattern | 10 >, demonstrate that the state $|\psi_{phi} >$of eqns. (3.11) is the most suitable choice as the search state for the simultaneous classification of the patterns of the pair P_5. Thus the superposition of $|\psi_{phi} >$ given by third of eqns. (3.4) and third of eqns. (3.11), obtained from single-pattern start states consisting of the patterns | 00 > and | 10 > respectively, are the most suitable search states for the classifications of the patterns of pair P_5 by the Grover's and Ventura's algorithms. These superposition are the states $|\psi_1 >$and $|\psi_3 >$ of Singh-Rajput MES given by eqns. (3.6).

For the simultaneous classification of the patterns of pair P_6 the operators $\hat{R}$ and $\widehat{D}$ may be written as

$$\hat{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

and $$\widehat{D} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 \end{bmatrix} \quad \ldots (5.3)$$

Operating this operator on the following superposition, obtained from the two-pattern start-state;

$$|\psi_{inc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix};\ |\psi_{exc}> = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix};\ |\psi_{phi}> = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \ldots (5.4)$$

and also applying Ventura's algorithm on these states we found the perfect suitability and superiority of the state $|\psi_{exc}>$ given here as the search state for the simultaneous classification of patterns of the pair P_6. The graphs of the computed comparative probabilities of the classifications of the patterns of this pair on applying operator of eqn. (5.3) on the superposition given by eqns. (3.5) and eqns. (3.10) respectively, obtained from the single-pattern start states consisting of the patterns | 01 > and | 11 > respectively, are just identical to those given by figure-1 and figure-2 with pattern | 00 > and | 01 > replaced by pattern | 11 > and |10 > respectively. These graphs demonstrate the superiority and suitability of the superposition $|\psi_{phi}>$ given by third of eqns. (3.5) and third of eqns. (3.10) respectively, for the classification of patterns of pair P_6 by Grover's algorithm and also by Ventura's algorithm. These states are second and fourth states $|\psi_2>$ and $|\psi_4>$ of Singh-Rajput MES given by eqns. (3.6).

DISCUSSION

Eqns. (3.1), (3.8) and (3.13) give the iteration operators for the simultaneous classification of the patterns of pairs P_1, P_2 and P_3, respectively, of class C_1 with minimum Hamming separation by using Grover's algorithm in a two-qubit system. Comparative probabilities of these respective classifications obtained by applying these respective iteration operators on the respective superposition given by eqns. (3.2), (3.9) and (3.14), obtained from two-pattern start-states consisting of the patterns of pairs P_1 , P_2 and P_3, respectively, demonstrate that each of the exclusion superposition, given by second of eqns. (3.2), second of eqns. (3.8) and second of eqns. (3.14) respectively is the most suitable two-pattern search state for simultaneous classification of patterns of these pairs respectively, while none of these patterns occurs in the corresponding search state (data-base). It supports our earlier result [19, 20] that Grover's method is more efficient for the classification of the unknown patterns (not present in the search state or data- base). It is also clear from these results that though each of phase inversion superposition, given by third of eqns. (3.2), third of eqns. (3.8) and third of eqns. (3.14), respectively, contains the full database, none of these is a better choice for search state for the simultaneous classification of patterns of the corresponding pair by using Grover's method of iteration with two-pattern start-states. Similar results have been observed on applying Ventura's model on these superposition of two-pattern start-state and it has been shown that exclusion superposition is the most suitable choice here also supporting the claim [21] that Ventura's method is more effective in case of smaller database. Similar results have been obtained about the suitability and superiority of each of the exclusion superposition $|\psi_{exc}>$, given by second of eqns. (4.2), second of eqns. (5.2) and second of eqns. (5.4) for the simultaneous classification of the patterns of pair P_4 of class C_2 with maximum Hamming separation and the patterns of pairs P_5 and P_6 respectively of class C_3 with intermediate Hamming separation, by using Grover's method or Ventura's method. Thus among all the two-patterns start states, each of the states $|\psi_{exc}>$, has been shown to be the most suitable choice as search state for the simultaneous classifications of patterns of corresponding pairs of respective classes, using Grover's method or Ventura's algorithm in spite of the fact that this state does not contain any of the classified patterns.

Figures-1 and -2 and the corresponding results obtained by Ventura's algorithm show that the superposition $|\Psi_{phi}>$ of eqns. (3.4) and (3.5); (3.10) and (3.11); and (3.5) and (3.11) respectively, obtained from the single-pattern start-states separately consisting of the patterns of pairs P_1, P_2 *and* P_3 of the class C_1 with minimum Hamming separation, are respectively the best choice as search states in both the algorithms (Grover's and Ventura's algorithms) for the classification of patterns of these pairs with the difference that while on the first iteration of Grover's algorithm the pattern absent from the one-pattern start-state is classified with 100% probability, the first application of Ventura's method classifies the pattern already present in the one-pattern start-state with 100% probability. These superposition are states $|\psi_1>$and $|\psi_2>$; $|\psi_3>$ and $|\psi_4>$; and $|\psi_2>$and $|\psi_3>$ of Singh-Rajput MES for the classification of patterns of pairs for the classification of pairs P_1, P_2 *and* P_3 respectively. In the similar manner superposition $|\Psi_{phi}>$ of eqns. (3.4) and (3.10); (3.4) and (3.11); and (3.5) and (3.10) are the most suitable search states for the classification of patterns of respective pairs P_4; $P_5$and P_6 of classes C_2 and C_3 with maximum and intermediate Hamming separations respectively. These superposition are the states $|\psi_1>$ and $|\psi_4>$ for the classification of pair P_4; states $|\psi_1>$ *and* $|\psi_3>$ for the classification of pair P_5 and the states $|\psi_2>$ *and* $|\psi_4>$ for the classification of pair P_6. These results demonstrate that in Grover's method the probabilities of correct classifications are higher for unknown patterns (not present in the one-pattern start-state) and this method is more effective when the stored data is large (*i.e.* $|\Psi_{phi}>$) but Ventura's method does not give better results for unknown patterns and also for smaller data base (*i.e* $|\psi_{inc}>$) in two-qubit system in contrast to the case of higher-qubits systems [19, 21]. These results also demonstrate that all possible pairs of

Singh-Rajput MES are the most suitable choice as the search states obtained from the corresponding single-pattern start-states for the classification of respective pairs of patterns of a two-qubit system. It is interesting to note that the pairs of consecutive states of these MES are most suitable search states for the classification of pairs of class C_1 with minimum Hamming separations while the pairs of alternative states of these MES are suitable for the simultaneous classification of patterns of pairs of class C_3 with intermediate Hamming separation and the pair of the first and last states of these MES classify most efficiently the patterns of class C_2 with maximum Hamming separation.

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