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PHENOMENOLOGICAL THEORY OF SUPERCONDUCTIVITY AT HIGH TC IN THE FRAMEWORKS OF QCD AND RCD

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Constructing the effective action for dyonic field in Abelian projection of QCD, it has been demonstrated that any charge (electrical or magnetic) of dyon screens its own direct potential to which it minimally couples and anti-screens the dual potential leading to dual superconductivity in accordance with generalized Meissner effect. In this Abelian projection of QCD an Abelian Higgs model, incorporating dual superconductivity and confinement, has been constructed and its string representation has been obtained in terms of average of Wilson loop. The study of the condensation of monopoles and the resulting chromo magnetic superconductivity has been undertaken in restricted chromo dynamics (RCD) of SU(2) and SU(3) gauge theories. Constructing the RCD Lagrangian and the partition function for monopoles in terms of string action and the action of the current around the strings, the monopole current in RCD chromo magnetic superconductor has been derived and it has shown that in London' limit the penetration length governs the monopole density around RCD string in chromo magnetic superconductors while with finite (non-zero) coherence length the leading behavior of the monopole density at large distances from the string is controlled by the coherence length and not by the penetration length.

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INTRODUCTION

In the process of current understanding of superconductivity at high T_c one conceives the notion of its hopeful analogy with quantum chromo-dynamics (QCD) which is the most favored color gauge theory of strong interaction whereas superconductivity is a remarkable manifestation of quantum mechanics on a truly macroscopic scale. In the process of current understanding of superconductivity, Rajput *et al* [1, 2, 3] and S. Kumar [4] have conceived its hopeful analogy with QCD and demonstrated that the essential features of superconductivity *i.e.*, the Meissner effect and flux quantization, provided the vivid models [5-9] for actual confinement mechanism in QCD. Mandelstam [10-12] propounded that the color confinement properties may result from the condensation of magnetic monopoles in QCD vacuum. In a series of papers [13-16] Izawa and Iwazaki made an attempt to analyze a mechanism of quark confinement by demonstrating that the Yang-Mills vacuum is magnetic superconductor and such a superconducting state is considered to be a condensed state of magnetic monopole. The condensation of magnetic monopole incorporates the state of magnetic superconductivity [17] and the notion of chromo magnetic superconductor where the Meissner effect confining magnetic field in ordinary superconductivity would be replaced by the chromo-electric Meissner effect (*i.e.*, the dual Meissner effect), which would confine the color electric flux. As such one conceives the idea of correspondence between quantum chromo-dynamic situation and chromo-magnetic superconductor. However, the crucial ingredient for condensation in a chromo-magnetic superconductor would be the non-Abelian force in contrast to the Abelian ones in ordinary superconductivity. Topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles [18]. The method of Abelian projection is one of the popular approaches to confinement problem, together with dual superconductivity [19-20] picture, in non-Abelian gauge theories. Monopole condensation mechanism of confinement (together with dual superconductivity) implies that long-range physics is dominated by Abelian degrees of freedom [21,22] (Abelian dominance).The conjecture that the dual Meissner effect is the color confinement mechanism is realized if we perform Abelian projection in the maximal gauge where the Abelian component of gluon field and Abelian monopoles are found to be dominant [23-24]. Then the Abelian electric field is squeezed by solenoidal monopole current [25]. The vacuum of gluon-dynamics behaves as a dual superconductor and the key role in dual superconductor model of QCD is played by Abelian monopole. For the self-dual fields, the Abelian monopoles become Abelian dyons [26]. The infra-red properties of QCD in the Abelian projection can be described by the Abelian Higgs Model (AHM) in which dyons are condensed. There exists the model [27-30] of QCD vacuum in which the non-Abelian dyons are responsible for the confinement. The non-Abelian dyons give rise to Abelian dyons in the Abelian projection. Therefore an important problem, before studying the vacuum properties of non-Abelian theories, is to Abelianize them so as to make contribution of the topological magnetic degrees of freedom to the partition function explicit. Such a construction for non-Abelian gauge theories and its relevance to topological magnetic charge and hence to confinement are still lacking in spite of large amount of recent literature [31-36] on the subject.

ELECTROMAGNETIC DUALITY AND DYONIC INTERACTIONS

A gauge invariant and Lorentz covariant quantum field theory of fields associated with dyons has been developed[37-40] in purely group theoretical manner by using two four-potentials and assuming the generalized charge, generalized current and generalized four-potential as complex quantities with their real and imaginary parts as electric and magnetic constituents *i.e.*

generalized charge $q = e - ig$ ... (2.1a)

generalized four-current $J_\mu = j_\mu - ik_\mu$ ... (2.1b)

and generalized four-potential $V_\mu = A_\mu - iB_\mu$ ... (2.1c)

where e and g are electric and magnetic charges on dyon; j_μ and k_μ are electric and magnetic four-current densities and A_μ and B_μ are the electric and magnetic four-potentials associated with dyons. Taking the wave function associated with generalized field as

$$\vec{\psi} = \vec{E} - i\vec{H} \qquad \text{...(2.1d)}$$

The generalized field equations of these fields may be written as

$$\vec{\nabla}.\vec{\Psi} = J_0$$

and
$$\vec{\nabla} \times \vec{\Psi} = -i\vec{J} - i\frac{\partial\vec{\Psi}}{\partial t} \qquad \text{...(2.2)}$$

where J_0 and $\vec{J}$ are the temporal and spatial parts of J_μ defined by eqn. (2.1b).

In the compact form these equations may be written as

$$G_{\mu\nu,\nu} = J_\mu$$

and
$$G^d_{\mu\nu,\nu} = 0 \qquad \text{... (2.3)}$$

where $G_{\mu\nu,}$ the generalized field tensor, is given as

$$G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \qquad \text{... (2.4)}$$

and $G^d_{\mu\nu}$ is its dual given as

$$G^d_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G_{\alpha\beta} \qquad \text{... (2.5)}$$

Equation (2.4) may also be written as

$$G_{\mu\nu} = F_{\mu\nu} - iH_{\mu\nu} \quad \text{... (2.6)}$$

where $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{... (2.6a)}$$

and $$H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{... (2.6b)}$$

Then eqns. (2.3) reduce to the following form

$$F_{\mu\nu,\nu} = j_\mu \quad \text{... (2.7)}$$

and $$H_{\mu\nu,\nu} = k_\mu \quad \text{... (2.7a)}$$

These equations are symmetrical under the duality transformations

$$F_{\mu\nu} \to H_{\mu\nu}; H_{\mu\nu} \to -F_{\mu\nu};\ j_\mu \to k_\mu; k_\mu \to -j_\mu \quad \text{... (2.8)}$$

The Lagrangian density for spin-1 generalized charge (*i.e.* bosonic dyon) of rest mass m_0 may be written as follows in the Abelian theory;

$$L = m_0 - \frac{1}{4}[\alpha\{(A_{\nu,\mu} - A_{\mu,\nu})^2 - (B_{\nu,\mu} - B_{\mu,\nu})^2\} - 2\beta\{(A_{\nu,\mu} - A_{\mu,\nu})(B_{\nu,\mu} - B_{\mu,\nu})\}$$

$$+\{(\alpha A_\mu - \beta B_\mu) j_\mu - (\alpha B_\mu + \beta A_\mu) k_\mu\}]$$

$$= L_P + L_F + L_I \quad \text{... (2.9)}$$

where α and β are real positive unimodular parameters *i.e.*

$$|\alpha|^2 + |\beta|^2 = 1, \quad \text{... (2.10)}$$

L_P, L_F and L_I are free-particle, field and interaction Lagrangians respectively. The action integral may be written as

$$S = \int_{t_1}^{t_2} L dt = S_P + S_F + S_I \quad \text{... (2.11)}$$

Varying the trajectory of particle without changing the field, we get the following equation of motion

$$m\ddot{x}_\mu = \text{Re}(q^* G_{\mu\nu}) u^\nu \quad \text{... (2.12)}$$

where Re denotes the real part and u^ν is the ν^{th} component of four- velocity of dyon.

An Abelian dyon moving in the generalized field of another dyon carries a residual angular momentum [41] (field contribution) besides its orbital and spin-angular momenta. If we consider ith Abelian dyon moving in the field of jth dyon (assumed at rest), its gauge invariant rotationally symmetric orbital angular momentum may be written as [41]

$$\vec{J} = \vec{r} \times (\vec{p} - \mu_{ij}\vec{V}^T) + \mu_{ij}\frac{\vec{r}}{r} \quad \text{... (2.13)}$$

where $\vec{r}$ is the position vector and $\vec{p}$ is the linear momentum of i^{th} dyon, V^T is the transverse generalized vector potential of the field associated with j^{th} dyon and μ_{ij} is the magnetic coupling parameter defined as

$$\mu_{ij} = e_i g_j - e_j g_i \quad \text{...(2.14)}$$

The last term in eqn. (2.13) is the residual angular momentum carried by i^{th} dyon besides its usual orbital angular momentum and spin-angular momentum;

$$\vec{J}_{res} = \mu_{ij}\frac{\vec{r}}{r} \quad \text{... (2.15)}$$

For each pair of dyons, this residual angular momentum generates a one dimensional representation of the pair of four-momentum associated with these particles. This is the subgroup of the Lorentz group which leaves both four-momenta invariant. This residual angular momentum leads to chirality dependent multiplicity in the eigen values of angular momentum of an Abelian dyon.

With the development of non-Abelian gauge theories, Dirac monopole has mutated in another way as we have to take into account not only electromagnetic U(1) gauge group but also the color gauge group SU(3)c describing strong interaction. In QCD, because SU(3) is compact, the color electric charges defined with respect to any maximal Abelian subgroup are quantized. It implies that we can write down gauge field configurations that asymptotically look like magnetic monopole of any chosen Abelian direction. The confinement of color electric charge corresponds to the screening of color magnetic charge. There are monopole field configurations in any non-Abelian gauge theory. The phase structure of any such theory can be probed by adding a scalar field (*i.e.* Higgs field) in the adjoint representation so long as it does not change the nature of flow of the coupling constant with energy. For asymptotically free theories, the low energy behavior is dominated by the Abelian monopoles of almost zero mass which are almost point-like. The interaction of these point-like monopoles with gluons and charged particles can be studied as a dual analogue of point-like charged particle interactions. It leads to condensation of monopole. Thus topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles which undergo condensation leading to confinement. Thus the non-Abelian confinement of dyonic charge is related to linear Abelian theory in a dyonic superconductor.

Let us first consider the effective action for dyonic field in this Abelian projection of QCD in the following manner [2]

$$S = -\frac{1}{4}\int G_{\mu\nu}(x) \notin (x-y) G^{\mu\nu} d^4x d^4y + J_\mu V^\mu \qquad \text{... (2.16)}$$

where $\notin (x-y)$ is the generalized dielectric constant defined as

$$\notin (x-y) = \in (x-y) - i\,\mu(x-y) \qquad \text{... (2.17)}$$

with $\in (x-y)$ as ordinary dielectric constant and $\mu(x-y)$ as magnetic permeability such that

$$\int \in (x-y)\mu(x-z) d^4y = \delta(x-z) \qquad \text{... (2.18)}$$

where $\delta(x)$ is Dirac-Delta function. The generalized field tensor $G_{\mu\nu}(x)$ of eqn. (2.16) satisfies field equations (2.3) or equivalently the field eqns. (2.7). The generalized four-current of field equation (2.3) couples to V_μ, with the current-correlators given by

$$\langle J_\mu \rangle = \frac{\delta S}{\delta V_\mu}$$

and

$$\langle j_\mu(x) J(y) \rangle = \frac{\delta^2 S}{\delta V_\nu(x)\, \delta V_\mu(y)} \qquad \text{... (2.19)}$$

Using eqns. (2.17) and (2.19), we have

$$\langle J_\mu(x) J_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} [k^2 \delta_{\mu\nu} - k_\mu k_\nu] \notin (k^2) \qquad \text{... (2.20)}$$

where $\notin (k^2)$ is Fourier transform of $\notin (x-y)$. For free fields in vacuum, $\notin (k^2) = 1.$ In the perturbation theory the deviation of $\notin (k^2)$ from 1 can be interpreted as the vacuum polarization due to dyon loops. For perturbatively small $\chi(k^2)$, we have

$$\notin (k^2) = 1 + \chi(k^2)\,, \qquad \text{...(2.21)}$$

where

$$\chi(k^2) = \chi_e(k^2) - i\,\chi_g(k^2) \qquad \text{... (2.22)}$$

with $\chi_e(k^2)$ as perturbation related with electric charge loop and $\chi_g(k^2)$ as the perturbation related with magnetic charge loop.

Let us apply eqn. (2.20) to the case of dual superconductivity where $\notin$ includes fully non-perturbative effects. This rigidly excludes generalized electromagnetic field in side dual superconductor in conformity with the generalized Meissner effect with its real and imaginary constituents as the strict Meissner effect and dual Meissner effect respectively. Then the generalized field V_μ can penetrate in to a generalized superconductor up to the generalized London penetration depth

$$\lambda_L = \lambda_e - i\lambda_g \qquad \text{... (2.23)}$$

where λ_e is strict penetration depth due to Meissner effect and λ_g is the dual penetration depth due to dual Meissner effect. For small values of k^2, we have

$$\notin(k^2) = \frac{m_L^2}{k^2} - \frac{ik^2}{m_L^2} \qquad \text{... (2.24)}$$

where $$m_L = \frac{1}{\lambda_L} = m_{l_e} - im_{L_g} \qquad \text{... (2.25)}$$

or $$m_L = \frac{1}{\lambda_e - i\lambda_g} = \frac{\lambda_e + i\lambda_g}{|\lambda_L|^2} \qquad \text{... (2.25a)}$$

It gives $$m_{L_e} = \frac{\lambda_e}{|\lambda_L|^2}$$

and $$m_{L_g} = -\frac{\lambda_g}{|\lambda_L|^2} \qquad \text{... (2.26)}$$

Equations (2.20) and (2.1b) then give

$$\langle[j_\mu(x)j_\nu(y) + k_\mu(x)k_\nu(y)]\rangle = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)}[k^2\delta_{\mu\nu} - k_\mu k_\nu] \in (k^2)$$

and $$\langle[j_\mu(x)k_\mu(y) - j_\nu(x)k_\mu(y)]\rangle = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)}[k^4\delta_{\mu\nu} - k^2 k_\mu k_\nu]\mu(k^2)/m_L{}^2$$

... (2.27)

These equations give the generalized propagator associated with generalized field V_μ .

Let us consider electric and magnetic charges on different particles (*i.e.* not dyons). Then field equations (2.3) reduce to the following form

$$F_{\mu\nu,\nu} = j_\mu;$$

$$F^d_{\mu\nu,\nu} = 0;$$

$$H_{\mu\nu,\nu} = k_\mu;$$

$$H^d_{\mu\nu,\nu} = 0 \quad \text{... (2.28)}$$

or equivalently $\Box A_\mu = j_\mu$

and $\Box B_\mu = k_\mu$ ...(2.29)

and equation of motion (2.12) becomes

$$m\ddot{x}_\mu = (eF_{\mu\nu} + gH_{\mu\nu})u^\nu \quad \text{... (2.30)}$$

All these equations are dual invariant under the transformations (2.8). The effective action in this Abelian projection of QCD may be written as follows from eqn. (2.16)

$$S = -\frac{1}{4}\int F_{\mu\nu}(x) \in (x-y) F^{\mu\nu}(y) d^4x d^4y$$

$$-\frac{1}{4}\int H_{\mu\nu}(x)\mu(x-y)H^{\mu\nu}(y)d^4xd^4y + j_\mu A^\mu + k_\mu B^\mu \quad \text{... (2.31)}$$

The current-correlations (2.19) may then be written as follows

$$\langle j_\mu \rangle = \frac{\delta S}{\delta A_\mu}; \langle k_\mu \rangle = \frac{\delta S}{\delta B_\mu};$$

$$\langle j_\mu(x) j_\nu(y) \rangle = \frac{\delta^2 S}{\delta A_\nu(y) \delta A_\mu(x)}; \quad \text{... (2.32)}$$

$$\langle k_\mu(x) k_\nu(y) \rangle = \frac{\delta^2 S}{\delta B_\nu(y) \delta B_\mu(x)}$$

For the given action in the present case, these relations lead to

$$\langle j_\mu(x) j_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4}[k^2\delta_{\mu\nu} - k_\mu k_\nu] \in (k^2)$$

and
$$\langle k_\mu(x) k_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4}[k^2\delta_{\mu\nu} - k_\mu k_\nu]\mu(k^2) \quad \text{...(2.33)}$$

For small perturbations we have

$$\in (k^2) = 1 \pm \chi_e (k^2)$$

and $$\mu(k^2) = 1 \mp \chi_g (k^2) \qquad ...(2.34)$$

where the upper signs in the right hand sides correspond to vacuum polarization due to charged particle-loops and the lower signs correspond to that due to monopole-loops. Relations (2.33) may also be written as

$$\langle j_\mu (x) j_\nu (y) \rangle = -\int \frac{d^4k}{(2\pi)^4} \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] m_{L_e}{}^2$$

and $$\langle k_\mu (x) k_\nu (y) \rangle = -\int \frac{d^4k}{(2\pi)^4} \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] m_{L_g}{}^2 \qquad ...(2.35)$$

These relations show that charged particles $[\chi_e (k^2) \geq 1]$ produce screening effect for the $A_\mu - propagator$, with the corresponding photon acquiring the mass m_{L_e} and anti-screening effect for the $B_\mu - propagator$. On the other hand, the monopole loops produce screening effect for $B_\mu - propagator$, with corresponding photon acquiring the mass $m_{L_{g,}}$, and anti-screening effect for $A_\mu - propagator$. Thus any particle (electrically charged or a monopole) screens its own direct potential to which it minimally couples, and anti-screens the dual potential (B_μ for electric charge and A_μ for monopole). This dual anti-screening effect leads to dual superconductivity in accordance with generalized Miessner effect.

SUPERCONDUCTIVITY DUE TO CONDENSATION AND CONFINEMENT OF DYONS

The non-Abelian nature of gauge group [SU(3) or SU(2)] is quite crucial to dyon condensation as mechanism of confinement. The method of Abelian projection is one of the popular approaches to the confinement problem in the non-Abelian gauge theories. A general non-Abelian theory of dyons consists of usual four-space (external) and *n* - dimensional internal group space, where the field associated with dyons has *n* - fold internal multiplicity and the multiplets of gauge field transform as the basis of adjoint representation of *n* - dimensional non-Abelian gauge symmetry group. Choosing the internal gauge group as SU(2), the generalized dyonic field tensor may be constructed as

$$\vec{G}_{\mu\nu} = G^a_{\mu\nu} T_a \qquad ... (3.1)$$

with the generalized four-potential defined as

$$\vec{V}_\mu = V^a_{\mu\nu} T_a \qquad ...(3.2)$$

where repeated indices are summed over 1, 2 and 3 (internal degrees of freedom), vector sign is denoted in the internal group space and the matrices T_a are infinitesimal generators of group SU(2), satisfying the commutation relation

$$[T_a, T_b] = i\,\varepsilon_{abc} T_c$$

with ε_{abc} as structure constant of internal group. We may connect $\vec{G}_{\mu\nu}$ and $\vec{V}_\mu$ through the following non-Abelian version of eqn. (2.4);

$$G^a_{\mu\nu} = \partial_\nu V^a_\mu - \partial_\mu V^a_\nu + |q|\,\varepsilon^{abc} V_{\mu b} V_{\nu c} \qquad ... (3.3)$$

where the dyonic generalized charge q is given by eqn. (2.1a).

A suitable Lagrangian density of a spontaneously broken non-Abelian gauge theory SU(2), yielding the classical dyonic solutions, may be constructed as

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2}(D_\mu \phi)^a (D^\mu \phi)_a - V(\phi) = L_{dyon}(A_\mu, B_\mu, \phi)$$

where $$D_\mu \phi = \partial_\mu \phi - i\,\mathrm{Re}(q * V_\mu)\phi = (\partial_\mu - ieA_\mu - igB_\mu)\phi \qquad ...(3.4)$$

with Re denoting the real part and

$$V(\phi) = \frac{1}{4}(\phi^a \phi_a)^2 - \frac{1}{2} v^2 (\phi^a \phi_a)$$

with $$v = \langle \phi \rangle = \langle 0 | \phi | 0 \rangle \qquad ... (3.5)$$

which determines the vacuum expectation value of Higgs field. In simplest manner this equation may be written as

$$V(\phi) = -\eta(|\phi|^2 - v^2)^2 \qquad ... (3.6)$$

with η as a constant.

The gauge dependent part of Lagrangian *i.e.*, first term of rhs in eqn. (3.4) is invariant under the following transformations of the fields A_μ and B_μ;

$$V_\mu = \begin{bmatrix} A_\mu \\ B_\mu \end{bmatrix} \to \begin{bmatrix} A_\mu{}' \\ B_\mu{}' \end{bmatrix} = V_\mu{}' = R(\delta) \begin{bmatrix} A_\mu \\ B_\mu \end{bmatrix} = R(\delta) V_\mu$$

where $$R(\delta) = \begin{bmatrix} \cos\delta \sin\delta \\ -\sin\delta \cos\delta \end{bmatrix} \quad \text{... (3.7)}$$

with $$\delta = \tan^{-1}\left(\frac{g}{e}\right)$$

Using the Lagrangian density given by eqn. (3.4) the electric and magnetic fields of dyons may be calculated by imposing the following ansatz[42]

$$V_{ia} = \varepsilon_{aij} (\vec{r})^j \frac{K(r)-1}{|q| r^2}$$

$$V_{0a} = (\vec{r})_a \frac{J(r)}{|q| r^2} \quad \text{... (3.8)}$$

$$\phi_a = (\vec{r})_a \frac{H(r)}{|q| r^2}$$

where the functions $K(r)$, $J(r)$ and $H(r)$ satisfy the following equations

$$r^2 H''(r) = 2HK^2$$

$$r^2 J''(r) = 2JK^2 \quad \text{... (3.9)}$$

$$r^2 K''(r) = K(K^2 - 1) + K(H^2 - J^2)$$

A solution of these equations may be written as follows

$$J(r) = \alpha\phi(r); H(r) = \beta\phi(r); K(r) = \frac{Cr}{\sinh Cr}$$

where $\beta^2 - \alpha^2 = 1$

and $$\varphi(r) = C(r)\coth Cr - 1 \quad \text{... (3.10)}$$

In the Prasad-Sommerfield limit[43]

$$V(\phi) = 0;$$

but $$v = \langle\phi\rangle \neq 0 \quad \text{... (3.11)}$$

In this limit the dyons have lowest possible energy for given electric and magnetic charges *e* and *g* respectively. Thus we get the following expression for dyonic mass

$$M = v(e^2 + g^2)^{\frac{1}{2}} = v|q| \quad \text{... (3.12)}$$

where the electric and magnetic fields associated with dyons obey the first order equations

$$E^a{}_i = G^a{}_{0i} = \partial^i V^a{}_0 + |q|\,\varepsilon^{abc} V_{ib} V_{0c} = (D_i\,\phi)^a \sin\alpha,$$

$$B^a{}_i = \varepsilon_{ijk} G^{jka} = (D_i\,\phi)^a \cos\alpha$$

and $$D_0(\phi)^a = 0$$

where $$\alpha = \tan^{-1}\frac{e}{g} \qquad \text{... (3.13)}$$

In these equations i and 0 indicate space and time directions and a is an SU(2) vector index. These electric and magnetic fields associated with dyons are non-Abelian in nature having external as well as internal components. In the Abelian projection, obtained by setting

$$K(r) \to 0; J(r) \to b + cr \qquad \text{... (3.14)}$$

where b and c are positive constants having the dimensions of charge and mass respectively, these fields reduce to the following form in the asymptotic limit;

$$E_j{}^a = -\frac{3b}{|q|\,r^4}(\vec{r})^a(\vec{r})_j - \frac{2c}{|q|\,r^3}(\vec{r})^a(\vec{r})_j;$$

$$B_j{}^a = -\frac{(\vec{r})_j(\vec{r})^a}{|q|\,r^4} \qquad \text{... (3.15)}$$

For vanishing c (*i.e.* vanishing mass) these fields corresponds to point-like mass-less dyons with electric charge $\frac{3b}{|q|}$ and magnetic charge $\frac{1}{|q|}$. Thus non-Abelian dyons give rise to the Abelian dyons in the Abelian projection. The infra-red properties of QCD in the Abelian projection can be described in the Abelian Higgs Model (AHM)[2] in which dyons are condensed. In this model the relevant degrees of freedom are two massive gluons $W_\mu{}^{\pm}$, *a* U(1) gluon (associated with generalized field V_μ) and a dyon which we take to be scalar represented by complex field ϕ. Then the Lagrangian (3.4) reduces to

$$L_{dyon}(A_\mu, B_\mu, \phi) = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\left|(\partial_\mu - ieA_\mu - igB_\mu)\phi\right|^2 + \eta(|\,\phi\,|^2 - v^2)^2 \qquad \text{... (3.16)}$$

In terms of this Lagrangian, the partition function in the Euclidean space-time may be written as

$$Z_{dyon} = \int DA_\mu DB_\mu D\phi \exp\{-\int d^4x\; L_{dyon}(A_\mu, B_\mu, \phi)\} \qquad \text{... (3.17)}$$

Applying the transformation (3.7) and integrating over the field A'_μ, this partition function reduces to the following form in AHM;

$$Z_{dyon} = \int DB'_\mu D\phi \exp\{-\int d^4x L_{AHM}(B'_\mu, \phi)\}$$

with $$L_{AHM}(B'_\mu, \phi) = -\frac{1}{4} H'_{\mu\nu} H'^{\mu\nu} + \frac{1}{2}\left|(\partial_\mu - i\bar{g}B'_\mu)\phi\right|^2 + \eta(|\phi|^2 - v^2)^2 \quad \text{... (3.18)}$$

where the Higgs field ϕ has the magnetic charge

$$\bar{g} = |q|$$

and $$H'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu \quad \text{... (3.19)}$$

This model (AHM) incorporates dual superconductivity and hence confinement as the consequence of dyonic condensation since the Higgs type mechanism arises here.

In the dyon theory, specified by partition function (3.17), the quantum average of the Wilson loop is[44]

$$\langle W^c{}_l \rangle_{dyon} = \frac{1}{Z_{dyon}} \int DA_\mu DB_\mu D\phi \exp\{-\int d^4x L_{dyon}(A_\mu, B_\mu, \phi)\} W_l{}^c(A_\mu) \quad \text{...(3.20)}$$

where $$W_l{}^c(A_\mu) = \exp\{ie_0 \int d^4x\, \eta_\mu A^\mu\} \quad \text{...(3.21)}$$

with $$\eta_\mu(x) = \oint_C d\breve{x}_\mu \delta^{(4)}(x - \breve{x}(\tau)) \quad \text{...(3.22)}$$

which creates the particle with electric charge e_0 on the world trajectory C.

Let us apply the transformation (3.7) to the quantum average (3.20) and then integrate over the field A'_μ. Thus we get

$$\langle W_l{}^c \rangle_{dyon} = \langle K^c{}_{(q_l, q_m)}(B'_\mu) \rangle_{AHM} \quad \text{... (3.23)}$$

with the operator $K^c{}_{(q_e, q_m)}$ as the product of *t*' Hooft loop and the Wilson loop W^c;

$$K^c{}_{(q_e q_m)}(B'_\mu) = H^c{}_{q_e}(B'_\mu) W^c{}_{q_m}(B'_\mu)$$

where $$q_e = \frac{e_0 g}{|q|}; q_m = \frac{e_0 e}{|q|} \quad \text{... (3.24)}$$

Then the effective electric and magnetic four-current density may be written as follows :

$$j_\mu = q_e \eta_\mu; k_\mu = q_m \eta_\mu \quad \text{... (3.25)}$$

In eqn. (3.24) the operator $H^c{}_{q_e}(B'_\mu)$ is

$$H^c{}_{q_e}(B'_\mu) = \exp\left\{-\frac{1}{4}\int d^4x\,[(H'_{\mu\nu} - \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta})^2 - H'_{\mu\nu}H'^{\mu\nu}]\right\}$$

where $$H'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu \qquad \text{... (3.26)}$$

and $F_{\alpha\beta}$ is the dual field tensor satisfying

$$F_{\mu\nu,\nu} = j_\mu \qquad \text{... (3.27)}$$

which is identical to eqn. (2.7) for the usual electrodynamic field tensor of the field associated with Abelian dyons. It is what we expect in the Abelian projection of QCD in the present Abelian Higgs Model of Abelian dyons in the Abelian version of QCD.

SUPERCONDUCTIVITY DUE TO DYONIC CONDENSATION IN RESTRICTED CHROMODYNAMICS (RCD)

Using the idea of confinement of electric flux due to condensation of magnetic monopoles, a dual gauge theory called restricted chromo dynamics (RCD) has been constructed out of QCD in SU(2) theory [45-48]. This dual gauge theory incorporates a dynamical dyonic condensation [49], [50] and exhibits the desired dual dynamics that guarantees the confinement of dyonic quark through generalized Meissner effect. This RCD has been extracted from QCD by imposing an additional internal symmetry named magnetic symmetry [45, 51] which reduces the dynamical degrees of freedom . Attempts have been made [52] to establish an analogy between superconductivity and the dynamical breaking of magnetic symmetry, which incorporates the confinement phase in RCD vacuum. Mathematical foundation of RCD [45, 48] is based on the fact that a non-Abelian gauge theory permits some additional internal symmetry *i.e.* the magnetic symmetry. Let us briefly review the RCD in the (4 + *n*) dimensional metric manifold *P* (four – dimensional space-time manifold *M* and *n* - dimensional internal group *G*) with metric g_{AB} (*A*, *B* = 1, 2, ………. 4 + *n*), where the gauge symmetry can be viewed as *n*-dimensional isometry [53, 54] which allows us to view *P* as a principal fibre bundle *P* (*M*, *G*) with *M* = *P*/*G* as the base manifold and *G* as the structure group. Keeping in view the fact that the restricted theory RCD may be extracted from full QCD by imposing an extra internal symmetry, let us now restrict the dynamical degrees of freedom of the theory (keeping full gauge degrees of freedom intact) by imposing an extra magnetic symmetry which ultimately forces the generalized non-Abelian gauge potential V_μ to satisfy a strong constraint given by

$$D_\mu \hat{m} = \partial_\mu \hat{m} + \vec{i} \, |q| \vec{V}_\mu \times \hat{m} = 0 \qquad \text{... (4.1)}$$

where D_μ is covariant derivative for the gauge group, $\mu = 0, 1, 2, 3$, $q = (e - i\,g)$ is the generalized dyonic charge with e and g as electric and magnetic constituents, and the generalized four – potential $\vec{V}_\mu$ is given as

$$\vec{V}_\mu = \vec{A}_\mu - i\,\vec{B}_\mu \qquad \text{...(4.2)}$$

where A_μ and B_μ are electric and magnetic four-potentials respectively. The cross product in eqn. (4.1) is taken in internal group space and $\hat{m}$ characterizes the additional Killing symmetry- (magnetic symmetry) which commutes with the gauge symmetry itself and is normalized to unity *i.e.*

$$\hat{m}^2 = 1 \qquad \text{...(4.3)}$$

It constitutes an adjoint representation of *G*, whose Little group is assumed to be Cartan sub-group [45] at each space-time point. Mathematically, this means that a connection on *P* (*M*, *G*) admits a left isometry of *H*, which formally forms a subgroup of *G* but commutes with *G* (the right isometry). This magnetic symmetry restricts the connection (*i.e.* the space for potential) to those whose holonomy bundle becomes a reduced bundle *P* (*M*, *H*).

Choosing $G = SU(2)$ and $H = U(1)$, the gauge covariant condition (4.1) gives the following form of the generalized restricted potential,

$$\vec{V}_\mu = -iV_\mu^* \hat{m} + \frac{i}{|q|} \hat{m} \times \partial_\mu \hat{m} \qquad \text{... (4.4)}$$

such that $m.V_\mu = -\,iV^*_\mu$ is the unrestricted Abelian component of the restricted potential V_μ while the remaining part is completely determined by magnetic symmetry.

The unrestricted part of the gauge potential describes the dyonic flux of color isocharges and the restricted part describes the flux of topological charges of the symmetry group *G*. The imposed magnetic symmetry, revealing the global topological structure of gauge symmetry, enables us to conceive the gauge theory of non-trivial fibre bundle *P* (*M*, *H*) with only those fields which are defined on global sections where color direction would be chosen by selecting color electric potential of Cartan's sub-group which helps to circumvent the disturbing Schlieder's theorem[55] in defining a meaningful color charge in non-Abelian gauge theory.

The generalized field strength of the gauge field of RCD that describes non-Abelian dyons may be obtained as follows :

$$\vec{G}_{\mu\upsilon} = \vec{G}_{\mu\upsilon} + \left(\frac{i}{|q|}\right)[\vec{V}_{\mu} X \vec{V}_{\upsilon}],$$

$$= (-iF_{\mu\upsilon} + H_{\mu\upsilon})\hat{m} \qquad \text{...(4.5)}$$

where $\vec{G}_{\mu\upsilon} = \vec{V}_{\upsilon,\mu} - \vec{V}_{\mu,\upsilon}$

$$F_{\mu\upsilon} = \overset{*}{V}_{\upsilon,\mu} - \overset{*}{V}_{\mu,\upsilon}$$

and $$H_{\mu\upsilon} = \left(\frac{i}{|q|}\right)\hat{m}.[\partial_{\mu}\hat{m} \times \partial_{\upsilon}\hat{m}] \qquad \text{...(4.6)}$$

Identifying $F_{\mu\upsilon}$ and $H_{\mu\upsilon}$ as the generalized electric and magnetic field strengths respectively, the striking duality between the generalized electric and magnetic fields is obviously manifested in the theory. These field strengths satisfy the following dual symmetric field equations in magnetic gauge

$$F_{\mu\upsilon,\upsilon} = j_{\mu} \quad \text{and} \quad H_{\mu\upsilon,\upsilon} = -k_{\mu} \qquad \text{... (4.7)}$$

where j_μ and k_μ are respectively the electric and magnetic four-current densities constituting the generalized dyonic four-current density

$$J_{\mu} = j_{\mu} - ik_{\mu} \qquad \text{...(4.8)}$$

In order to demonstrate the topological structure, let us introduce magnetic gauge by aligning $\hat{m}$ along a space-time independent direction (say $\hat{\varepsilon}_3$ in isospin space) by imposing a gauge transformation U such that

$$\hat{m} \xrightarrow{\upsilon} \hat{\varepsilon}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \text{...(4.9)}$$

and the potential and field strength transform as

$$\vec{V}_{\mu} \rightarrow \vec{V}_{\mu} = (-i\overset{*}{V}_{\mu} + W_{\mu})\hat{\varepsilon}_3$$

and $$\vec{G}_{\mu\upsilon} = \vec{G}_{\mu\upsilon} = (-iF_{\mu\upsilon} + H_{\mu\upsilon})\hat{\varepsilon}_3 \qquad \text{...(4.10)}$$

with $$H_{\mu\upsilon} = W_{\upsilon,\mu} - W_{\mu,\upsilon} \qquad \text{...(4.11)}$$

where W_μ may be identified as the potential of topological dyons in magnetic symmetry which is entirely fixed by $\hat{m}$ upto Abelian gauge degrees of freedom. Thus in the magnetic gauge, the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_μ by removing all non-essential gauge degrees of freedom and hence the topological structure of the theory may be brought into dynamics explicitly. It assures a non-trivial dual structure of the theory of dyons in magnetic gauge where dyons appear as point like Abelian ones. In this theory the gauge fields are expressible in terms of purely time like non-singular physical potentials V^*_μ and W_μ. Let us introduce a complex scalar field ϕ (Higg's field) to eliminate the point like behavior and to incorporate the extended structure of dyons. Then in the absence of quarks or any colored object, the RCD Lagrangian in magnetic gauge may be written as[1]

$$L = \frac{1}{4} H_{\mu\upsilon} H^{\mu\upsilon} + \frac{1}{2} | D_\mu \phi |^2 - V(\phi^* \phi) \qquad ...(4.12)$$

where $$D_\mu \phi = (\partial_\mu + i\ |q| W_\mu)\phi$$

and $V(\phi^*\phi)$ is the effective potential introduced to induce the dynamical breakdown of the magnetic symmetry. The Lagrangian (4.12) of RCD in magnetic gauge in the absence of quark or any colored object looks like Ginsburg- Landau Lagrangian for the theory of superconductivity if we identify the dyonic field as an order parameter and the generalized potential W_μ as the electric potential. The dynamical breaking of the magnetic symmetry, due to the effective potential $V(\phi^*\phi)$, induces the dyonic condensation of the vacuum. This gives rise to the dyonic super current, the real part of which (electric constituent) screens the electric flux which confines the magnetic color charge (through usual Meissner effect) and the imaginary part (*i.e.* magnetic constituent) of this super-current screens the magnetic flux that confines the electric color iso-charges (due to dual Meissner effect).

Lagrangian (4.12) has been obtained from the standard SU(2) Lagrangian and hence the desired dynamical breaking of magnetic symmetry is obtained by fixing the following form of the effective potential :

$$V(\phi^*\phi) = -\eta\ (|\phi|^2 - v^2)^2 \qquad ...(4.13)$$

where η is coupling constant of Higgs field and v is its vacuum expectation value *i.e.*

$$\upsilon = \langle\phi\rangle_0 \qquad ...(4.14)$$

In Prasad- Sommerfeld limit [43]

$$V(\phi) = 0$$

but $$\upsilon \neq 0.$$

In this limit, the dyons have lowest possible energy for given electric and magnetic charges e and g respectively. In this Abelian Higgs model of RCD in magnetic symmetry W_μ, defined by eqn. (4.11), is the dual gauge field with the mass of dual gauge boson given by

$$M_D = |q|\ v \qquad ...(4.15)$$

and ϕ is the dyonic field with charge q and mass

$$M_{\phi} = \sqrt{(8\eta)}\,v \qquad \ldots(4.16)$$

In the confinement phase of RCD the dyons are condensed under the condition (4.14). With these two mass scales the coherence length ε and the penetration length λ are given by

$$\varepsilon = \frac{1}{M_{\phi}} = \frac{1}{[\sqrt{(8\eta)}\,v]} \qquad \ldots(4.17)$$

and
$$\lambda = \frac{1}{M_D} = \frac{1}{(|q|v)}$$

The region in phase space, where $\varepsilon = \lambda$, constitutes the border between type-I and type-II super-conductors. The super-conductivity provides vivid model for the actual confinement mechanism and the color confinement is due to the generalized Meissner effect caused by dyonic condensation.

The Lagrangian of eqn. (4.12), with effective potential given by eqn. (4.13), yields the following field equations;

$$\partial_{\upsilon} H^{\mu\upsilon} - i\,|q|\,[\phi^{*} D^{\mu}\phi] = 0, \qquad \ldots(4.18)$$

and
$$D^{2}_{\mu}\phi - 4\eta\,[|\phi|^{2} - v^{2}]\,\phi = 0 \qquad \ldots(4.19)$$

Equation (3.6) may also be written as

$$\Box W_{\mu} - \partial^{\upsilon}\partial_{\mu} W_{\upsilon} = k_{\mu} \qquad \ldots(4.20)$$

where k_{μ}, the magnetic constituent of generalized dyonic current, is given as

$$k_{\mu} = |q|\,\mathrm{Im}\,[\phi^{*} D^{\mu}\phi] = |q|\,|\phi|^{2}\,[\partial_{\mu}\arg\phi + |q|\,W_{\mu}] \qquad \ldots(4.21)$$

In the Lorentz gauge, eqn. (4.20) reduces to

$$\Box W_{\mu} = i\,|q|\,\phi^{*}\phi\,[(\partial_{\mu}\phi)/\phi + i\,|q|\,W_{\mu}]$$

which furthur reduces to the following form for the small variation in ϕ;

$$\Box W_{\mu} + |q|^{2}\,|\phi|^{2}\,W_{\mu} = 0 \qquad \ldots(4.22)$$

which is a massive vector type equation where the equivalent mass of the vector particle state (condensed mode) may be identified as

$$M^{2} = |q|^{2}\,|\phi|^{2}$$

with its vacuum expectation value

$$\langle M \rangle = |q|\,v = M_D = \frac{1}{\lambda}, \qquad \ldots(4.23)$$

where λ is penetration length defined by eqn. (4.17).

In the confinement phase dyons are condensed and

$$|<\phi>| = v$$

Comparing the penetration length (*i.e.* screening length) λ of eqn. (4.23) with that of relativistic super conductor model *i.e.*

$$M_s = \sqrt{2}e\,|\phi| = \sqrt{2}ev = \frac{1}{\lambda_s},$$

where *e* is the electric charge of dyons, we get

$$\frac{M}{M_s} = \frac{\lambda_s}{\lambda} = \frac{|q|}{(e\sqrt{2})} = \left(\frac{1}{\sqrt{2}}\right)\frac{[(e^2+g^2)^{1/2}]}{e} \qquad ...(4.24)$$

In the representation of generalized charge of dyon in a two dimensional complex space [56], we have

$$\frac{g}{e} = -\tan\theta \qquad ...(4.25)$$

where θ is rotation parameter of the generalized charge space.

Then equation (4.24) gives

$$\frac{\lambda}{\lambda_s} = \sqrt{2}\cos\theta \qquad ...(4.26)$$

showing that for the rotation parameter $\theta \leq \pi/4$, we have

$$\lambda \geq \lambda_s \quad \text{and} \quad M \leq M_s \qquad ...(4.27)$$

On the other hand, for larger rotation in generalized charge space with $\theta > \pi/4$, we have

$$\lambda < \lambda_s \quad \text{and} \quad M > M_s \qquad ...(4.28)$$

Thus the optimum RCD generalized charge orientation is governed by rotation parameter value $\theta = \pi/4$. Equations (4.27) and (4.28) show that with the suitable choice of the generalized charge space parameter θ, the tubes of generalized confining flux can be made thin which gives rise to a higher degree of confinement of any generalized color flux by dynamically condensed vacuum. These equations demonstrate that the generalized charges lying on the cone of vertical angle $\theta = \pi/4$ in charge space give rise to thin tubes of confined color flux leading to strong confinement of the colored sources in RCD vacuum.

Dyonically condensed vacuum is characterized by the presence of two massive modes. The mass of the scalar mode, M_ϕ given by eqn. (4.16) determines how fast the perturbative vacuum around a colored source reaches condensation and the mass M_D of vector mode determines the penetration length of the colored flux. The masses of these generalized dyonic glueballs may be estimated [57] by evaluating string tension of the classical string solutions of quark pairs. For this let us examine the behavior of dyons around the RCD string. The classical field equations (4.20) and (4.21) contain a solution corresponding to the RCD string with a quark and an anti-quark at its ends. Let us consider the static solution, parallel to the third direction of reference frame, as

$$\phi(\rho) = vf(\rho)e^{i\psi} \qquad \text{... (4.29)}$$

and

$$W_1 = \frac{\hat{x}_2 h(\rho)}{(|q|\rho^2)},\ W_2 = \frac{\hat{x}_1}{(|q|\rho^2)h(\rho)},\ W_3 = 0,\ W_4 = 0, \qquad \text{...(4.30)}$$

where

$$\rho = (x_1^2 + x_2^2)^{1/2}$$

is the transverse distance to the string;

$$\Psi = \arg(x_1 + ix_2); \qquad \text{...(4.31)}$$

and

$$\lim_{\rho\to 0} f(\rho) = \lim_{\rho\to 0} h(\rho) = 0; \qquad \text{...(4.32)}$$

$$\lim_{\rho\to\infty} f(\rho) = \lim_{\rho\to\infty} h(\rho) = 1$$

From eqn. (4.31), we have

$$\frac{\partial\Psi}{\partial x_1} = -\frac{x_1}{\rho^2} \quad \text{and} \quad \frac{\partial\Psi}{\partial x_2} = \frac{x_2}{\rho^2} \qquad \text{...(4.33)}$$

Substituting relations (4.29), (4.30), (4.31) and (4.33) into eqn. (4.21), we get

$$k_1 = -\left(\frac{v^2 x_2}{\rho^2}\right)|q|\, f^2(\rho)[1 - h(\rho)]; \qquad \text{... (4.34)}$$

$$k_2 = \left(\frac{v^2 x_1}{\rho^2}\right)|q|\, f^2(\rho)[1 - h(\rho)];$$

$$k_3 = 0; \quad k_4 = 0.$$

Substituting relations (4.29), (4.30) and (4.33) into field equation (4.18), we have

$$f''(\rho) + \frac{f'(\rho)}{\rho} - \frac{f(\rho)}{\rho^2}[1 - h(\rho)^2] + \left(\frac{M_\phi^2}{2}\right)[1 - f^2(\rho)]f(\rho) = 0, \qquad \text{...(4.35)}$$

where dash devotes derivatives with respect to ρ. At large distance, in view of equations (4.32),

we may have $$f(\rho) = 1 - \varepsilon(\rho), \quad ...(4.36)$$

where ε (ρ) is infinitesimally small at large distance such that

$$\lim_{\rho \to \infty} (\rho) = 0.$$

Then eqn. (4.35) may be written as

$$\varepsilon''(\rho) + \frac{\varepsilon'(\rho)}{\rho} - M_\phi^2 \varepsilon(\rho) = 0$$

Substituting $r = M_\phi \rho$ into this equation, we get

$$\frac{d^2\varepsilon(r)}{dr^2} + \left(\frac{1}{r}\right)\frac{d\varepsilon(r)}{dr} - \varepsilon(r) = 0$$

which is modified Bessel's equation of zero order, with its solution given as

$$\varepsilon(r) = AI_0(r) = AI_0(M_0\rho), \quad ...(4.37)$$

where $I_0(r)$ is the modified Bessel's function of zero order, defined as

$$I_0(m_\phi \rho) = \sum_{n=0}^{\infty} \frac{(M_\phi \rho / 2)^{2n}}{(n!)^2} = J_0(iM_\phi \rho), \quad ...(4.38)$$

with $J_0(x)$ as the ordinary Bessel's function of zero order.

In the similar manner, the field equation (4.19) may be written into the following form by using relations (4.30) and (4.34);

$$h''(\rho) - \frac{h'(\rho)}{\rho} + M_D^2[1 - h(\rho)] f^2(\rho) = 0. \quad ...(4.39)$$

At large distance we may have

$$h(\rho) = 1 - \zeta(\rho), \quad ...(4.40)$$

where $$\lim_{\rho \to \infty} \zeta(\rho) = 0.$$

Then eqn. (4.39) reduces to

$$\frac{d^2\zeta(r)}{dr^2} - \frac{d\zeta(r)}{dr} - \zeta(r) = 0, \quad ...(4.41)$$

where $r = M_D \rho$. Let us substitute $\zeta(r) = r\chi(r)$ is to this equation. Then we have

$$\frac{rd^2\chi(r)}{dr^2}+\frac{d\chi(r)}{dr}-\chi(r)\left[1+\frac{1}{r^2}\right]=0 \qquad ...(4.42)$$

which is modified Bessel's equation of order-one with its solution given by

$$\chi(r)=\frac{\zeta(r)}{r}=BI_1(r) \qquad ...(4.43)$$

where $I_1(r)$ is modified Bessel's function of order one.

Thus we have

$$\zeta(\rho)=B(M_D\rho)I_1(M_D\rho) \qquad ...\ (4.44)$$

Substituting relations (4.37) and (4.44) into equations (4.36) and (4.40), we have, at large value of ρ,

$$f\ (\rho)=1-AI_0(M_\phi\rho) \qquad ...\ (4.45)$$

and

$$h(\rho)=1-B(M_D\rho)I_1(M_D\rho) \qquad ...\ (4.46)$$

Substituting these results into equation (4.29) and (4.30), we get the solution of classical field equation (4.18) and (4.19) corresponding to the RCD string with a quark and an anti-quark at its ends. The infinitely separated quark and anti-quark correspond to an axially symmetric solution of the string. For such a string solution with a lowest non-trivial flux the coefficient A in the solution (4.46) is always equal to one while the coefficient *B* is unity in the Bogomolnyi limit exactly on the border between the type I and type II superconductors [58] where $M_D = M_\phi$ *i.e.* coherence length and the penetration length coincide with each other. Thus in RCD close to border, we set $B = 1$ besides $A = 1$ and then we have

$$f\ (\rho)=1-I_0(M_\phi\rho)=-\sum_{n=1}^{\infty}\frac{(M_\phi\rho/2)^{2n}}{(n!)^2} \qquad ...(4.47)$$

and

$$h(\rho)=1-(M_\phi\rho)I_1(M_\rho)=1-\frac{(M_D\rho/2)^2}{2}-M_D\rho\sum_{n=1}^{\infty}\frac{(M_\phi\rho/2)^{2n+1}}{n!(n+1)!} \qquad ...(4.48)$$

The RCD string is well defined by these solutions. In view of conditions (4.32), the magnetic constituent of the dyonic current, given by eqn. (4.34), near the RCD string is zero at the centre of the string (*i.e.* for $x_1 = x_2 = 0$) and also zero at the points far from the string (where $h\ (\rho) \to 1$).

Substituting relations (4.47) and (4.48) into equations (4.29) and (4.30), the solutions of classical field equations (4.18) and (4.19), corresponding to the RCD string with a quark and antiquark at its ends, readily follows. The RCD string is well defined by solutions (4.47) and

(4.48) where the magnetic constituent of the dyonic current, given by eqn. (4.34) near the RCD string , is zero at the centre of the string and also zero at points far away from the string. Dyonic density in the absence of string has the contributions from monopole condensate [59, 60] and also from the perturbative fluctuations. According to eqns. (4.34), (4.47) and (4.48) the magnetic constituent of dyonic current at large transverse distance form the string should be controlled by the coherence length and the penetration length where the coherence length could be derived [31] directly from the measurement of dyonic density around a chromo-dyonic flux spanned between a static quark- anti- quark pair. In the maximal Abelian gauge, as used in RCD here, the penetration length and coherence length are almost the same and hence the vacuum is nearly the border between type I and type II dual superconductors. The solutions (4.47) and (4.48) define infinitely long RCD strings which cannot be terminated and hence behave like ANO vortices[61] and twisted superconducting semi-local strings[62] with conserved global current flowing through them. We expect that these solutions are stable in RCD mode.

SUPERCONDUCTIVITY DUE TO MONOPOLE DENSITY AROUND RCD STRING

For the case of pure monopoles, $q = g$, equation (4.21) reduces to

$$k_{\mu} = g\,\mathrm{Im}[\phi^{+} D_{\mu}\phi] = g\,|\phi|^{2}\,[\partial_{\mu}\arg\phi + gW_{\mu}] \qquad \text{... (5.1)}$$

and then equations (4.34) become

$$k_{i} = -\frac{(v^{2} \in_{ij} x_{j})}{\rho^{2}} g f^{2}(\rho)[1-h(\rho)]$$

$$k_{3} = 0 \quad \text{and} \quad k_{4} = 0 \qquad \text{... (5.2)}$$

where $\in_{12} = -\in_{21} = 1, \in_{11} = \in_{22} = 0$ and summation over repeated index is conventionally involved. Substituting relations (4.47) and (4.48) in to equations (5.2) we can find the monopole density in the vicinity of RCD string. To meet this end, let us use eqn (3.18) for partition function in Abelian Higgs Model (AHM) which incorporates dual superconductivity and hence confinement as the consequence of monopole condensation since the Higgs type mechanism arises here. With this partition function the quantum average of Wilson loop may be written as given by eqn. (3.23) where the expectation value is calculated in the form of eqn. (3.24) in AHM as the product of t' Hooft loop and the Wilson loop W^{c}; Then the effective electric and magnetic four-current density may be written as shown in equation (3.25). In equation (3.26) the operator $H_{q_e}^c$ creates the string spanned by the loop *C*, carrying the flux q_e . In AHM the monopoles are condensed and in its string representation the topological interaction exists in the expectation value of the Wilson loop W^c. In the centre of ANO string the Higgs field $\phi = |\phi| e^{i\theta}$ vanishes *i.e.*

$$\text{Re}\,\phi = \text{Im}\,\phi = 0,$$

and the phase is singular on the world sheets of ANO string. Then the measure of the integration over ϕ can be written as

$$D\phi = CD\,|\phi|^2\, D\theta$$

where *C* is a constant. The integral $\int D\theta$ contains the integration over functions which are singular on two dimensional manifolds. Let us divide the phase in to regular and singular parts as

$$\theta = \theta^r + \theta^s \qquad \text{...(5.3)}$$

where θ^s is defined by[3]

$$(\partial_\mu \partial_\nu - \partial_\nu \partial_\nu)\,\theta^s(x, \breve{x}) = 2\pi \varepsilon_{\mu\nu\alpha\beta} \sum\nolimits_{\alpha\beta}(x, \breve{x})$$

where

$$\sum\nolimits_{\alpha\beta}(x, \breve{x}) = \int_{\Sigma} d^2\sigma \varepsilon^{ab} \partial_a \breve{x}_\alpha \partial_b \breve{x}_\beta \delta^{(4)}[x - \breve{x}(\sigma)] \qquad \text{... (5.4)}$$

with $\partial_a = \dfrac{\partial}{\partial \sigma^a}, \breve{x}_\mu string - position$ and $\sum$ as the collection of all the closed surfaces.

$\sigma = (\sigma_1, \sigma_2)$ is the parameterization of string surface and *a*, *b* =1,2. Then the measure $D\theta$ can be decomposed as

$$D\theta = D\theta^r D\theta^s \qquad \text{...(5.5)}$$

Let us use these relations to find the monopole density in the vicinity of RCD string for vanishing and non-vanishing coherence lengths respectively in the following subsections:

(A) For Zero Coherence length

From equation (4.16), we find the vanishing coherence length in the limit $M_\phi \to \infty$ or $\eta \to \infty$ which corresponds to infinitely deep potential $V(\phi^*\phi)$ of equation (4.13). This limit is London limit. Then the RCD Lagrangian of equation (3.18) in AHM may be written as follows :

$$L_m = \frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{v^2}{2} (\partial_\mu \varphi + g W_\mu)^2$$

$$= L_m (W_\mu, \varphi) \qquad \text{... (5.6)}$$

where L_m denotes the Lagrangian density for monopoles. In terms of this Lagrangian, the partition function of eqn. (3.18) may be written as follows :

$$Z = \int_{-\pi}^{\pi} D\phi \left[\int_{-\infty}^{\infty} DW_\mu \exp\{-d^4 x L_m (W_\mu, \varphi)\} \right] \qquad \text{...(5.7)}$$

The string in RCD manifests itself as a singularity in the phase of the Higgs field according to eqns. (5.4).

Let us fix the unitary gauge as

$$\phi = 0$$

and make the consequent shift

$$W_\mu \to W_\mu - \frac{1}{g} \partial_\mu \phi \qquad \text{...(5.8)}$$

Then the shift in $H_{\mu\nu}$ will be

$$H\,\mu\nu \to H\,\mu\nu - \frac{2\pi}{g} \sum\nolimits_{\mu\nu}^{(d)} \qquad \text{... (5.9)}$$

where $$\Sigma_{\mu\nu}^{(d)}(x) = \frac{1}{2} \in_{\mu\nu\rho\sigma} \Sigma_{\rho\sigma}(x)$$

and we have used relations (5.4), (5.6), and (5.8). Substituting shifts (5.8) and (5.9) in to equation (5.7) and integrating over the field W_μ, we get

$$Z = \int_{\partial\Sigma = 0} d\Sigma \exp\{-A_{str}(\Sigma)\} \qquad \text{... (5.10)}$$

where A_{str} is the string action given as[3]

$$A_{str} = 2\pi^2 v^2 \int d^4 x \int d^4 y \Sigma_{\mu\nu} D_{M_B}(x-y) \Sigma_{\mu\nu}(y), \qquad \text{... (5.11)}$$

where D_{M_B} (x) is the scalar Yukawa propagator. It is the propagator of the scalar particle of mass $M_B = gv$ *i.e.*

$$(\Delta + m^2) D_{M_S}(x) = \delta^4(x) \qquad \text{... (5.12)}$$

with $m = gv$ as mass of dual gauge boson W_μ

For closed strings, we have

$$\partial_\nu \Sigma_{\mu\nu} = 0 \qquad ...(5.13)$$

On the other hand, when the strings are spanned on the current j_c, we have

$$\partial_\nu \Sigma_{\mu\nu} = J_\mu^c \qquad ... (5.14)$$

The action of the currents is given as follows by the short-ranged exchange of the dual gauge boson,

$$A_{curr}(j^c) = \frac{e^2}{2}\int d^4x \int d^4y j_\mu^{(c)}(x) D_{M_B}(x-y) j_y{}^c(y) \qquad ... (5.15)$$

where e is the electric charge of gluon, satisfying the quantization condition

$$eg = 2\pi \qquad ... (5.16)$$

The quantum average of Wilson loop can be written here as a sum over strings similar to equation (5.10),

$$\langle W_i^c \rangle = \frac{1}{Z}\int_{\delta\Sigma = jc} D\Sigma \exp\{-A_{str}(\Sigma) - A_{curr}(jc)\} \qquad ... (5.17)$$

where Z is given by equation (5.10). In this equation the sources of electric flux (*i.e.* quarks) running along the trajectory C are introduced with the help of W_i^c .

Let us place the static quarks at spatial infinites of the axis-x_3. Then the effects of quarks (*i.e.* boundary effects) are avoided and consequently the second term of the exponential in *rhs* of equation (5.17) may be ignored. In this case (*i.e.* infinite static string placed along the third direction) equation (5.4) reduces to the following form of string current

$$\Sigma_{\mu\nu} = (\delta_{\mu,3}\partial_{\nu,4} - \delta_{\mu,4}\partial_{\nu,3})\,\delta(x_1)\delta(x_2)$$

$$= \delta_{3,4}^{\mu,\nu}\delta(x_1)\delta(x_2) \qquad ... (5.18)$$

From equation (5.1), the monopole current may be written as follows in the London limit;

$$k_\mu = gv^2[\partial_\mu\varphi + gW_\mu] \qquad ... (5.19)$$

When the singular phase ϕ_Σ corresponds to the string position fixed by equation (5.4), the Lagrangian L_m in the exponential of equation (5.7) becomes

$$L_m(W_\mu, \varphi) = L_m(W_\mu, \varphi_\Sigma) \qquad ... (5.20)$$

and then the functional generating the partition function in equation (5.7) may be written as[3]

$$Z[\Sigma,C] = \int_{-\infty}^{\infty} DW_{\mu} \exp\left\{-\int d^4x \, [L_m(W_{\mu}, \varphi_{\Sigma}) - ik_{\mu}C_{\mu}]\right\} \quad \text{... (5.21)}$$

Then the monopole current in the presence of the string is given by the variational derivative[31]

$$\left\langle k_{\mu}(x) \right\rangle_{\Sigma} = \frac{g}{Z(\Sigma,0)} \left(\frac{\delta}{i\,\partial C_{\mu}(x)} \right)^2 Z[\Sigma,C]|_{C=0} \quad \text{... (5.22)}$$

and the squared monopole density is

$$\left\langle k_{\mu}^2(x) \right\rangle_{\Sigma} = \frac{g}{Z(\Sigma,0)} \left(\frac{\delta}{i\,\partial C_{\mu}(x)} \right)^2 Z[\Sigma,C]|_{C=0} \quad \text{... (5.23)}$$

In the manner analogous to equation (5.10) and equation (5.11), the generating functional (5.21) may be written as

$$Z[\Sigma,C] = \exp[-\int d^4x \int d^4y \{\frac{g^2 v^4}{2} C_{\mu}(x) D_{M_B}^{\mu\nu}(x-y) C_{\nu}(y)$$

$$-2\pi i v^2 C_{\mu}(x) D_{M_s}^{\mu\nu}(x-y) \partial_{\rho} \Sigma_{\rho\nu}^{(d)}(y) - A_{str}(\Sigma)] \quad \text{...(5.24)}$$

where string action $A_{\text{str}}(\Sigma)$ is given by equation (5.11).

Substituting this relation for generating functional in to equation (5.23) and evaluating the monopole density, we get the monopole current around the string as

$$k_{str}^{\mu} = \left\langle k^{\mu} \right\rangle_{\Sigma} = -2\pi g v^2 \int d^4y D_{M_B}^{\mu\nu}(x-y) \partial_{\rho} \Sigma_{\rho\nu}^{(d)}(y) \quad \text{… (5.25)}$$

For static string, this equation reduces to

$$k_{str}^{i} = -2\pi g v^2 \in^{ij} \frac{x_j}{\rho} \frac{\partial}{\partial\rho} D_{M_B}(\rho), i, j = 1, 2$$

$$k_{str}^3 = 0, \; k_{str}^4 = 0; \quad \text{...(5.26)}$$

where

$$D_{M_B}(\rho) = \frac{1}{2\pi} I_0(M_B \rho) \quad \text{... (5.27)}$$

with I_0 as modified Bessel's function of zero order. The function $D_{M_B}(\rho)$ is the propagator for a scalar massive particle in two space-time dimensions. Using equations (5.25) and (5.27), the explicit form of the non-zero component of the solenoidal current may be written as

$$k_{\theta}^{str} = v^2 g M_B I_1(M_B \rho) \quad \text{...(5.28)}$$

where $I_1(M_B \rho)$ is the modified Bessel's function of order one given in eqn. (4.44). Thus the monopoles form a solenoidal current which circulates around the string in transverse directions. This current gives rise to the following squared monopole density;

$$\left\langle k^2 \right\rangle_\Sigma = v^4 g^2 M_B^2 I_1^2 (M_B \rho) \qquad \text{... (5.29)}$$

Substituting the value of $I_1(M_B \rho)$ from equation (4.44) in to this relation, we find that the squared density of the monopole current, in London limit (where coherence length is zero), has a maximum at the distance of the order of the $\frac{1}{M_B}$ (*i.e.* the order of penetration length). Equation (5.19) gives the monopole current in London limit which corresponds to infinitely deep Higg's potential and leads to vanishing coherence length in the chromo magnetic superconductor.

(B) For Non-zero Coherence length

When the potential $V(\phi^*\phi)$ of equation (4.13) is of finite depth *i.e.* η is finite then M_ϕ is finite and hence coherence length ξ given by equation (4.17), is non-zero and finite. Then in the expression (5.29) for squared monopole density in the vicinity of RCD string a term corresponding to quantum vacuum correction is non-zero even in the absence of string. Thus the squared monopole density, in this case, is written as

$$(k_\mu^2)\Sigma = (k_\mu^{string})^2 + (k_\mu^{quan})^2 \qquad \text{... (5.30)}$$

where $(k_\mu^{string})^2$ is given by equation (5.29) and quantum vacuum correction $(k_\mu^{quan})^2$ is given by

$$(k_\mu^{quan})^2 = \left\langle k_\mu^2 \right\rangle_0 = g^2 v^4 D_{M_B}^2 (0)$$

$$= \frac{g^2 v^4 \wedge^2}{16\pi^2} \qquad \text{... (5.31)}$$

where we have used equation (5.27) and regularized the divergent expression by momentum cut off $\wedge$.

Replacing vacuum expectation value v of the Higgs field ϕ by $|\phi(\rho)|$ in relation (5.31) and then substituting it into equation (5.30), we get

$$(k_\mu^2)_\Sigma = (k_\mu string)^2 + g^2 \frac{|\varphi(\rho)|^4 \wedge^2}{16\pi^2} \qquad \text{... (5.32)}$$

$$= g^2 v^4 M_B^2 I_1^2(M_B \rho) + g^2 \frac{|\varphi(\rho)|^4 \wedge^2}{16\pi^2} \quad \text{... (5.33)}$$

For ρ of the order of coherence length ξ the quantum correction to the squared monopole density is much more than the vacuum expectation value measured far out side the string (ρ>>Σ). Thus the quantum corrections control the leading behavior of the total monopole density in the vicinity of the RCD string.

Using the asymptotic expansions of modified Bessel's functions in equation (4.47) and (4.48), and then introducing it into equation (4.29), we get

$$|\phi|^4 \approx \left[1 - 4\frac{\sqrt{\pi\xi}}{2\rho} e^{-\rho/\zeta}\right] \quad \text{... (5.34)}$$

Then equation (5.33) may be approximately written as follows [31] at large distances;

$$(k_\mu^2)_\Sigma \approx \frac{g^2 \wedge^2}{16\pi^2}\left[1 - 4\frac{\sqrt{\pi\xi}}{2\rho} e^{-\rho/\zeta}\right] \quad \text{... (5.35)}$$

which shows that the leading behaviors of the monopole density at large distances are controlled by coherence length ξ and not by penetration length λ.

SUPERCONDUCTIVITY IN RESTRICTED SU(3) GAUGE THEORY

Let us start with the construction of the restricted chromodynamics in SU(3) limit. The magnetic structure of this theory may be described by two internal Killing vectors which commute with each other and also with the gauge symmetry itself and are normalized to unity according to equation (4.3). These Killing vectors are a λ_3- like octet $\hat{m}$ and its symmetric product

$$\hat{m}' = \sqrt{3}(\hat{m} \times \hat{m}) \quad \text{... (6.1)}$$

which is λ_8 -like. The restricted theory (RCD) may be extracted from the full QCD by imposing the extra internal symmetries. Let us restrict the dynamical degrees of freedom of the theory (while keeping the full gauge degrees of freedom intact) by imposing the extra magnetic symmetry which restricts the generalized non-Abelian gauge potential $\vec{V}_\mu$ to satisfy the constraints given by

$$D_\mu \hat{m} = \partial_\mu \hat{m} + i\,|q|\,\vec{V}_\mu \times \hat{m} = 0 \quad \text{... (6.2)}$$

and $$D_\mu \hat{m}' = \partial_\mu \hat{m}' + i\,|q|\,\vec{V}_\mu \times \hat{m}' = 0$$

where D_μ is covariant derivative for the gauge group.

Introducing these magnetic structures, we obtain the following form[4] of the generalized restricted potential in the restricted SU(3) gauge theory:

$$\vec{V}_\mu = -iV^*_\mu \hat{m} - iV'^*_\mu \hat{m}' + \left(\frac{i}{|q|}\right)\hat{m} \times \partial_\mu \hat{m} + \left(\frac{i}{|q|}\right)\hat{m}' \times \partial_\mu \hat{m}' \quad \text{... (6.3)}$$

where $$\hat{m}.\vec{V}_\mu = -iV^*_\mu \quad \text{... (6.4)}$$

and $$\hat{m}'.\vec{V}_\mu = -iV'^*_\mu$$

are, respectively, λ_3 -like and λ_8 -like unrestricted Abelian components of the restricted potential. In the magnetic gauge $\hat{m}$ and $\hat{m}'$ become the space-time independent $\hat{\xi}_3$ and $\hat{\xi}_8$ respectively, where

$$\hat{\xi}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \hat{\xi}_8 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{... (6.5)}$$

Then the generalized potential of equation (4.10) may be written as

$$\vec{V}_\mu = (-iV^*_\mu + W_\mu)\hat{\xi}_3 + (-iV'^*_\mu + W'_\varpi)\hat{\xi}_8 \quad \text{... (6.6)}$$

where W_μ and W_μ may be identified as the potentials of topological dyons in magnetic symmetry of SU(3) gauge theory. These are entirely fixed by $\hat{m}$ and $\hat{m}'$, respectively, up to Abelian gauge degrees of freedom. The generalized field strength can, then, be constructed as

$$\vec{G}_{\mu\nu} = \vec{G}_{\mu\nu} + \frac{i}{|q|}[\vec{V}_\mu \times \vec{V}_\nu]$$

$$= (-iF_{\mu\nu} + H_{\mu\nu})\hat{\xi}_3 + (-iF'_{\mu\nu} + H'_{\mu\nu})\hat{\xi}'_8 \quad \text{... (6.7)}$$

where $F_{\mu\nu}$ is given by equation (4.6), $H_{\mu\nu}$ is given by equation (4.11) and

$$\vec{G}_{\mu\nu} = \partial_\nu \vec{V}_\mu - \partial_\mu \vec{V}_\nu ; \quad \text{... (6.8)}$$

$$F'_{\mu\nu} = \partial_\mu V_\nu^{/} - \partial_\nu V'^{*}_\mu ;$$

$$H'_{\mu\nu} = \partial_\mu W'_\nu - \partial_\nu W'_\mu$$

In this theory the gauge fields are expressible in terms of purely time-like non-singular potentials V^*_μ and V'^*_μ ,W_μ and W'_μ. Then in the absence of quarks or any colored object, the RCD Lagrangian of SU(3) theory in magnetic gauge may be written as

$$L = \frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{1}{4} H'_{\mu\nu} H'^{\mu\nu} + \frac{1}{4} [H_{\mu\nu} H'^{*\mu\nu} + H'_{\mu\nu} H^{*\mu\nu}] + \frac{1}{2} | D_\mu \varphi |^2$$

$$+ \frac{1}{2} | D'_\mu \varphi' |^2 - V(\varphi^* \varphi, \varphi'^* \varphi') \quad \text{...(6.9)}$$

where $D_\mu \phi = (\partial_\mu + i \ |q| W_\mu) \phi;$

$$D'_\mu \phi' = (\partial_\mu + i \ |q| W'_\mu) \phi'; \quad \text{... (6.10)}$$

and the dyonic field operators ϕ and ϕ' correspond to *m* and *m*' respectively. Here $V(\phi^* \phi, \phi'^* \phi')$ is the effective potential introduced to induce the dynamical breaking of the magnetic symmetry. This Lagrangian is a gauge extension of Lagrangian (4.12) and it leads to dyonic condensation, color confinement and the resulting dual superconductivity in SU(3) theory.

CONCLUSIONS

Starting with generalized field equations (2.3) and the corresponding Lagrangian (2.9) of the field associated with Abelian dyons, it has been demonstrated that topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by dyons which undergo condensation leading to confinement and consequently to superconducting model of QCD vacuum, where the Higgs fields play the role of a regulator only. It has also been demonstrated that for the self-dual fields the Abelian monopoles become the Abelian dyons and in low energy QCD the dyon interactions are saturated by duality when Abelian projection is described by the Abelian Higgs model where dyons are condensed leading to confinement and the state of dual superconductivity. Equations (2.20) and (2.27) for dyonic current correlations show that dyonic electric charge produces the screening effect for $A_\mu - propagator$ and anti-screening effect for $B_\mu - propagator$, while the dyonic magnetic charge produces screening effect for $B_\mu - propagator$ and anti-screening effect for $A_\mu - propagator$. This anti-screening effect maintains the asymptotic freedom of non-Abelian gauge theory (QCD) in its Abelian version. In QCD, because of asymptotic freedom, the Landau singularity (led by charged particles in ordinary electrodynamics) is in the infrared regime and hence the most convenient microscopic theory of low energy QCD is produced by the chromo-dynamic dyons. The correlations (2.27) give the generalized propagator associated with generalized field V_μ of dyons. In the Abelian projection of QCD with the simultaneous existence of electric charges and monopoles (but not dyons), the effective action is given by eqn. (2.31) and the current correlations are given by eqns. (2.32), (2.33) and (2.35) which demonstrate that any particle screens its own direct potential to which it minimally couples and anti-screens the dual potential (B_μ for electric charges and A_μ for monopoles). This dual anti-screening effect leads to dual superconductivity in accordance with generalized Miessner effect. This dual superconductivity is the Higgs phase of QCD in its Abelian projection. The anti-screening, described by eqns. (2.35), provides the prescription that the magnetic photon (B_μ) -charge particle vertex is identical to the $A_\mu -$ charge particle vertex with the constant e replaced by ie. Such prescription of coupling of a gauge particle to its dual charge must be used only when all dual charges appear in loops. The duality prescribed by these equations may be a strong guide to the description of confinement and interactions of chromo magnetic monopoles should be saturated by this duality, at least for low energy. The gauge depended part of the Lagrangian density, given by eqn. (3.4) for the fields associated with the non-Abelian dyons in the minimal gauge theory, is invariant under the linear transformation (3.7). Equations (3.13) and (3.15) demonstrate that the non-Abelian dyons give rise to Abelian dyons in the Abelian projection obtained by setting up conditions given by eqns. (3.14). The infrared properties of QCD in this Abelian projection can be described by the Abelian Higgs model with Lagrangian density given by eqn. (3.18) in which dyons are condensed. In this model the partition function in the Euclidean space-time is given by the first part of eqns. (3.18). This model incorporates dual superconductivity and confinement as the consequence of dyonic condensation. In the dyon theory, specified by the partition function given by eqn. (3.17) in terms of dyon Lagrangian (3.16), the quantum average of Wilson loop given by eqn. (3.20) corresponds to quark Wilson loop if we consider this partition function as an effective theory of QCD. In eqn. (5.2) this average is given in AHM with the effective electric and magnetic charges and the effective electric and magnetic four-current densities given by equations (5.3) and (5.4) respectively. t'Hooft loop is precisely given by eqn. (5.5) in terms of electromagnetic field tensor $H'_{\mu\nu}$ and the dual field tensor satisfies field equation (5.5a) which is identical to eqn. (2.7a) for the usual electromagnetic field tensor of field associated with Abelian dyons. It is what we expect in the Abelian projection of QCD in the present Abelian Higgs Model. In eqn. (5.2) this average is given in AHM with the effective electric and magnetic charges and the effective electric and magnetic four-current densities given by equations (3.24) and (3.25) respectively. t'Hooft loop is precisely given by eqn. (3.26) in terms of electromagnetic field tensor $H'_{\mu\nu}$ and the dual field tensor satisfies field equation (3.27) which is identical to the field equation for the usual electromagnetic field tensor of field associated with Abelian dyons. It is what we expect in the Abelian projection of QCD in the present Abelian Higgs Model.

The Lagrangian given by eqn. (4.12) for RCD in magnetic gauge, in the absence of quarks or any colored objects, establishes an analogy between super-conductivity and the dynamical breaking of magnetic symmetry which incorporates the confinement phase in RCD vacuum where the effective potential V $(\theta^{*}\theta)$ induces the dyonic condensation of vacuum. This gives rise to dyonic super-current. The electric constituent of this current (*i.e.* its real part) screens the electric flux and confines the magnetic charges due to usual Meissner effect while its imaginary part (*i.e.* its magnetic constituent) screens the magnetic flux and confines the color iso-charges as the result of dual Meissner effect. It dictates the mechanism for the confinement of the electric and magnetic fluxes associated with dyonic quarks in the present theory. This dyonic condensation mechanism of confinement implies that long-range physics is dominated by Abelian degrees of freedom (Abelian dominance) as depicted by eqns (4.10) and (4.11) which assure a non-trival dual structure of the theory of dyons in magnetic gauge, where these objects appear as point like Abelian ones. This idea of Abelian dominance has recently been verified by gauge fixing and Abelian projection [21] and also by constructing semilocal models in Extended Abelian Higgs model (EAH-model) [27,28]. The same idea has been used, more recently, in connection with the dual Meissner effect in local unitary gauges in SU(2) gluo-dynamics [29] and also with confining ensemble of dyons [30] and dual superconductivity in Yang-Mills theories [20].

In the confinement phase of RCD, the dyons are condensed under the condition (4.14) where the transition from $\langle\phi\rangle_0 = 0$ to $\langle\phi\rangle_0 = v \neq 0$ is of first order, which leads to the vacuum becoming a chromo-magnetic super-conductor in the analogy with Higgs-Ginsburg- Landau theory of super-conductivity. Dyonically condensed vacuum is characterized by the presence of two massive modes given by equations (4.15) and (4.16) respectively, where the mass of scalar mode M_ϕ determines how fast the perturbative vacuum around a color source reaches condensation and the mass M_D of vector mode determines the penetration length of the colored flux. With these two mass scales of dual gauge boson and dyonic field, the coherence length ε and the penetration length λ have been constructed by eqns. (4.17) in RCD theory.

These two lengths coincide at the border between type-I and type-II super-conductors. In general, the ratio of penetration length and coherence length distinguishes superconductors of type-I ($\lambda<\varepsilon$) from type II ($\lambda>\varepsilon$). Equation (4.23) gives the flux penetration depth in the dyonic model of RCD and shows that due to the dynamical breaking of magnetic symmetry, the vacuum acquires the properties similar to those of relativistic super-conductor where the quantum fields generate non-zero expectation values and induces screening currents. This penetration length excludes the generalized field in a manner similar to that in type II super-conductor where the appropriate screening currents are set up by the formation of Cooper's pairs giving rise to Meissner effect of magnetic flux confinement. Thus the generalized color flux is squeezed into flux tubes as a result of generalized Meissner effect caused by the coherence plasma of dyons in RCD vacuum which ultimately forces the quark (color) confinement in RCD. The generation of screening current and the finite range force field responsible for the confinement here are similar to those in the case of real electromagnetic super-conductor (*i.e.* relativistic superconductor). Equations (4.27) and (4.28) show that with the suitable choice of the generalized charge space parameter θ, the tubes of generalized confining flux can be made thin which gives rise to a higher degree of confinement of any generalized color flux by dynamically condensed vacuum. These equations demonstrate that the generalized charges lying on the cone of vertical angle $\theta = \pi/4$ in charge space give rise to thin tubes of confined color flux leading to strong confinement of the colored sources in RCD vacuum. On the other hand, the generalized charges lying outside such cone and still participating in the vacuum condensation, immediately after magnetic symmetry breaking, have weak confinement effects. The generalized charge space parameter θ associated with dyons has the remarkable ability to squeeze the color fluxes and to improve the confining properties of RCD vacuum. Thus a perfect confinement can be achieved with pure dyonic states participating in actual dyonic condensation of RCD vacuum as the result of magnetic symmetry breaking in strong coupling limit.

The RCD string is well defined by solutions (4.47) and (4.48) where the magnetic constituent of the dyonic current, given by eqn. (4.34) near the RCD string, is zero at the centre of the string and also zero at points far away from the string. This current is maximum at the transverse distance for which the conditions (4.48) are satisfied. The numerical value of this distance has been found to be about .2 fm corresponding to SU(2) gluon dynamics [31]. Dyonic density in the absence of string has the contributions from monopole condensate [59, 60] and also from the perturbative fluctuations. According to eqns. (4.34) and (4.47) the magnetic constituent of dyonic current at large transverse distance from the string should be controlled by the coherence length and the penetration length where the coherence length could be derived directly [25] from the measurement of dyonic density around a chromo-dyonic flux spanned between a static quark-anti quark pair. In the maximal Abelian gauge, as used in RCD here, the penetration length and coherence length are almost the same and hence the vacuum is nearly the border between type I and type II dual superconductors.The solutions (4.47) and (4.48) define infinitely long RCD strings which can not be terminated and hence behave like ANO vortices [42] and twisted superconducting semi-local strings [28] with conserved global current flowing through them. We expect that these solutions are stable in RCD mode.

It is clear from eqn (5.29) that the squared density of the monopole current, in London limit (where coherence length is zero), has a maximum at the distance of the order of the $\frac{1}{M_B}$ (*i.e.* the order of penetration length). Equation (5.19) gives the monopole current in London limit which corresponds to infinitely deep Higg's potential and leads to vanishing coherence length in the chromo magnetic superconductor. Equation (5.22) gives the monopole current in the presence of the string, which leads to squared monopole density given by equation (5.23). The monopole current given by equation (5.25) reduces to the components given by equation (5.26) in terms of propagator (5.27) for a scalar massive particle in two space-time dimensions.Equation (5.28) gives the explicit form of the non-zero component of the solenoidal current which circulates around the string in transverse directions. This current gives rise to the squared monopole current given by equation (5.29) in London limit (*i.e.* vanishing coherence length). This squared current has a maximum at the distance of the order of penetration length. Thus in London limit (zero coherence length) the monopole density around the string in RCD is governed by penetration length. Equation (5.30) shows that for non-zero finite coherence length, the monopole density is non-zero even in the absence of string. Equation (5.33) shows that the quantum correction to the squared monopole density is much more than the vacuum expectation value measured far outside the string. Thus the quantum corrections control the leading behavior of the total monopole density in the vicinity of the RCD string. Equation (5.34) shows that the leading behavior of the monopole density at large distances is controlled by the coherence length and not by the penetration length. This result is in agreement with the numerical result of Bornyakov *et al* [63,64].

Lagrangian given by eqn. (6.9) is a gauge extension of Lagrangian (4.12) and it leads to dyonic condensation, color confinement and the resulting dual superconductivity in SU(3) theory. In the light of the results of section–4, it is not difficult to guess the presence of two scalar modes and two vector modes as the consequence of the presence of two magnetic vectors $\hat{m}$ and $\hat{m}'$ in SU(3) theory. Equation (6.2) give the magnetic structure of restricted chromo-dynamics in SU(3) theory where two internal Killing vectors λ_{3-} like octet and λ_{8-} octet given by equation (6.5) have been introduced keeping in view the facts that any system possessing a SU(3) symmetry suffers with a non-Abelian magnetic instability for the 4-7th gluons [65] and the 8th gluon corresponds to the diagonal generator in color space [66].

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