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A CURIOUS CONNECTION BETWEEN FERMAT'S NUMBER AND MULTIPLE FACTORIANGULAR NUMBERS

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In the seventeenth century Fermat defined a sequence of numbers $F_n = 2^{2^n} + 1$ for $n \geq 0$ known as Fermat's number . If F_n happens to be prime then F_n is called Fermat prime. All the Fermat's number are of the form $n!^k + \Sigma n^k$ for some fixed value of k and n. Further we will prove that after F_4 no other Fermat prime exist upto 10^{50}.

Keywords : Fermat's Number, prime number, multiple factoriangular numbers, Fermat prime.

INTRODUCTION

Fermat Number : A positive number of the form $F_n = 2^{2n} + 1$ where n is non negative integer.

First few Fermat's number are 3, 5, 17, 257, 65537,…

Pierre de Fermat conjectured that all numbers

$$F_n = 2^{2^n} + 1 \quad \text{for } m = 0, 1, 2, \ldots \qquad \ldots(1.1)$$

are prime. Nowadays we know that the first five members of this sequence are prime and that (see [2])

$$F_n \text{ is composite for } 5 \leq m \leq 32. \qquad \ldots(1.2)$$

The status of F_{33} is for the time being unknown, *i.e.*, we do not know yet whether it is prime or composite.

The numbers F_n are called Fermat numbers. If F_n is prime, we say that it is a Fermat prime.

Fermat numbers were most likely a mathematical interest before 1796. When C. F. Gauss mentioned that there is a remarkable relation between the Euclidean construction (*i.e.*, by ruler and compass) of regular polygons and the Fermat numbers, interest in the Fermat primes skyrocketed. In particular, he proved that if the number of sides of a regular polygonal shape is of the form $2^k F_{m1} \ldots F_{mr}$, where $k \geq 0$, $r \geq 0$, where F_{mi} are distinct Fermat primes, then this polygonal shape can be made by using compass ruler. The converse statement was proved later by Wantzel in [8].

There exist many necessary and sufficient conditions concerning the primality of F_n. For instance, the number F_n ($n > 0$) is a prime if and only if it can be written as a sum of two squares in essentially only one way, namely $F_n = (2^{2^{n-1}})^2 + 1^2$.

Recall also further necessary and sufficient conditions: the well-known Pepin's test, Wilson's Theorem, Lucas's Theorem for primality, etc., see [4].

Multiple Factoriangular number [7] : A generalization of factoriangular number is known as multiple factoriangular numbers and are defined as

$$F_t(n,k) = (n!)^k + \Sigma n^k$$

where $$T_n(k) = \Sigma n^k = 1^k + 2^k \ldots + n^k.$$

In this paper, we establish a connection between multiple factoriangular numbers and Fermat number .

n	$F_t(2, 2^n - 1)$	**Prime factorization of F_t (*n*,15)**	**Number of digits**	**Sum of squares of prime, integer, natural numbers**
0	3	Prime	1	
1	5	Prime	1	$2^2 + 1^2$
2	17	Prime	2	$4^2 + 1^2$
3	257	Prime	3	$16^2 + 1^2$
4	65537	Prime	5	$256^2 + 1^2$
5	4294 967297	641 × 6 700417	10	$65536^2 + 1^2$
6	18 446744 073709 551617	274177 × 67 280421 310721	20	$4046\,803256^2 + 1438\,793759^2$
7	340 282366 920938 463463 374607 431768 211457	59649 589127 497217 × 5704 689200 685129 054721	39	$18\,446744\,073709\,551616^2 + 1^2$
8	115792 089237 316195 423570 985008 687907 853269 984665 640564 039457 584007 913129 639937	238 926361 552897 × 93 461639 715357 977769 163558 199606 896584 051237 541638 188580 280321	78	$339\,840244\,399005\,511779\,394711\,120340\,266111^2 + 17\,340632\,172455\,487023\,654788\,790090\,010704^2$

By the common observation we see that the sequence of number so formed is well known Fermat Number Sequence and it follow the properties described in [2],[4].

Now $F_t(2, 2^n - 1) = (2!)^{2^n - 1} + \Sigma\, 2^{2^n - 1} = 2^{2^n} + 1$.

COROLLARY

All the Fermat prime are multiple factoriangular primes.

CONCLUSION

We end up with the conclusion that the only primes we get in different sequences of multiple factoriangular numbers till 10^{50} are the Fermat Prime F_0, F_1, F_2, F_3, F_4. Also Sequence of Fermat Number are a special case of multiple Factoriangular number by fixing $n = 2$, $k = 2^n - 1$.

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