HEAT AND MASS TRANSFER EFFECTS IN MHD FLOW PAST A VERTICAL WALL THROUGH POROUS MEDIUM

N. P. SINGH, A.K. SINGH, R. R. SINGH

Department of Mathematics, C. L. Jain (PG) College, Firozabad-283203 (India)

AND

KHILANAND GUPTA

Department of Physical Sciences, MGCG, Vishwavidyalaya, Chitrakoot-485780 (India)

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The present paper deals with heat and mass transfer effects in steady MHD flow of an incompressible, electrically conducting viscous fluid along a continuously moving vertical non-conducting porous surface with slip velocity heat source and radiative heat flux in presence of uniform transverse magnetic field. The surface temperature is raised at a uniform rate and also the mass is diffused from the plate to the fluid at uniform rate. The radiative heat flux term is used following Rosseland approximation. The expressions for the velocity, temperature distribution and mass transfer are derived analytically and are shown graphically for different numerical values of the parameters.

KEYWORDS: Radiative heat flux, Heat source, MHD, Incompressible.

NOMENCLATURE

A : thermal diffusion parameter,

B_0 : magnetic field component along y-axis,

C' : concentration in dimensional form,

C : concentration in non-dimensional form,

C'_∞ : concentration in the equilibrium state,

Cp : specific heat at constant pressure,

D_C : concentration diffusivity,

D_T : thermal diffusivity,

g : acceleration due to gravity,

Gc : mass Grashoff number,

Gr : thermal Grashoff number,

K_T : thermal conductivity of the fluid,

K : the porosity parameter,

K_0 : the permeability of the porous medium,

k_1 : the absorption coefficient,

k_s : slip parameter,

M : the Hartmann number,

m : mass flux per unit area,

N : heat radiation parameter,

Pr : Prandtl number,

Q' : heat generation/absorption coefficient,

Q : heat flux per unit area,

q'_r : the radiative heat flux,

S : heat source parameter,

Sc : Schmidt number,

T' : the fluid temperature in dimensional form,

T : the fluid temperature in non-dimensional form,

T'_∞ : initial temperature of the plate and the fluid,

u', v' : the dimensional velocity components along x', y' direction,

u, v : the non-dimensional velocity components along x, y direction,

u_s : the uniform velocity of the fluid issuing from the slit,

v_0 : suction velocity,

x', y' : dimensional coordinates,

x, y : non-dimensional coordinates,

Greek symbols

β^* : the coefficient of concentration expansion,

β' : the coefficient of sliding friction,

β : the coefficient of volume expansion,

ρ : the density of the fluid,

σ : electrical conductivity of the fluid,

σ_1 : the Stefan-Boltzmann constant,

μ : viscosity of the fluid,

υ : kinematic viscosity of the fluid,

Introduction

Examples of moving continuous surface are observed in a polymer or metal sheet extruded continuously from a long fiber or filament traveling between a feed roller and a take-up roller. It is identified that the boundary layer develops in the direction opposite to the direction of the motion on a moving surface of finite length, while on a continuously moving surface such as a long polymer sheet or fiber extruded from a slit and taken up by a wind up roller at a finite distance away, the boundary layer on the sheet or fiber originates at the slit and produces in the direction of motion of the body. The transpiration of cooling is a very effective process to create certain structural element such as combustion chamber walls, exhaust nozzle walls or gas turbine blades or turbo jets and rocket engines. In many industrial processes, the cooling of the threads or sheets or some polymer materials gets it importance in the production line. In the presence of an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled successfully to the finishing product of desired characteristics by drawing outfit etc.

Rees and Pop [1] have discussed free convection flow of an incompressible fluid. In addition, along a vertical wavy surface in a porous medium. Rees and Pop have [2] examined free convection induced by a vertical wavy surface with uniform heat flux. The magnetohydrodynamic boundary layer flow with heat transfer on a continuously moving wave surface was studied by Hossain *et al.* [3]. Hossain *et al.* [4] investigated natural convection flow of with temperature dependent viscosity from heated vertical wavy surface. Saxena and Dubey [5] investigated the effects on moving isothermal vertical surface through porous medium with uniform mass flux and transpiration. Ahmed and Hazarika [6] investigated MHD free and forced convection mass transfer flow past a porous vertical plate. Recently Adhikari [7] studied heat and mass transfer effects of steady MHD flow past a vertical surface with slip velocity through porous medium. The present paper extension of adhikari [7] considering heat source and radiative heat flux.

Formulation of the Problem

Consider a two-dimensional steady incompressible flow of an electrically conducting viscous fluid past a continuous vertical surface through porous medium. The fluid is issuing from a slit and moving with uniform velocity u_s in the fluid at rest, in presence of a uniform transverse magnetic field of strength B_0. Let the *x*-axis be taken along the direction of motion of the sheet in the upward direction and the *y*-axis is taken normal to it. The velocity components *u* and *v* are directed along their axes respectively. The surface temperature is raised uniformly and concentration level near the surface is also raised at a uniform rate.

The governing equations are follows :

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0\,, \qquad \ldots (1)$$

$$u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}=g\beta\left(T'-T'_{\infty}\right)+g\beta^{*}\left(C'-C'_{\infty}\right)$$

$$+\upsilon\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma}{\rho}B_0^2u'-\frac{\upsilon}{K_0}u',, \qquad \ldots (2)$$

$$u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\frac{K_T}{\rho Cp}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho Cp}\frac{\partial q_r'}{\partial y'}-\frac{Q'}{\rho Cp}\left(T'-T_\infty'\right), \quad \dots (3)$$

$$u'\frac{\partial C'}{\partial x'}+v'\frac{\partial C'}{\partial y'}=D_C\frac{\partial^2 C'}{\partial y'^2}+D_T\frac{\partial^2 T'}{\partial y'^2}. \quad \dots (4)$$

The radiative heat flux term by using the Rosseland approximation is given by:

$$q_r'=-\frac{4\sigma_1}{3k_1}\frac{\partial T'^4}{\partial y'}, \quad \dots (5)$$

we assume that the temperature difference within the fluid flow is sufficiently small, so that T'^4 may be expanded in Taylor's series about the temperature T_∞' as follows:

$$T'^4=\left(T_\infty'+T'-T_\infty'\right)^4$$

$$=T_\infty'^4+\left(T'-T_\infty'\right)4T_\infty'^3+\frac{\left(T'-T_\infty'\right)^2}{2!}12T_\infty'^2+\dots\dots\dots$$

Neglecting higher order terms in the above series beyond the first degree in $\left(T'-T_\infty'\right)$, we get:

$$T'^4\cong 4T_\infty'^3T'-3T_\infty'^4. \quad \dots (6)$$

The boundary conditions relevant to the problem are:

$$\mu\frac{\partial u'}{\partial y'}=-\beta' u',\ v'=-v_0\ \text{(constant)},$$

$$\frac{\partial T'}{\partial y'}=-\frac{Q}{K_T},\quad \frac{\partial C'}{\partial y'}=-\frac{m}{D_C}\qquad \text{at } y'=0,$$

$$u'\to 0,\quad T'\to T_\infty',\quad C'\to C_\infty'\qquad \text{as}\quad y'\to\infty. \quad \dots (7)$$

Making use of assumptions that the velocity, temperature and concentration field are independent of the distance parallel to the surface and Boussinesq's approximations, equations (1) to (4) and boundary conditions (7) are given by:

$$-v_0\frac{\partial u'}{\partial y'}=g\beta\left(T'-T_\infty'\right)+g\beta^*\left(C'-C_\infty'\right)+\upsilon\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma}{\rho}B_0^2u'-\frac{\upsilon}{K_0}u',, \quad \dots (8)$$

$$-v_0\frac{\partial T'}{\partial y'}=\frac{K_T}{\rho Cp}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho Cp}\frac{\partial q_r'}{\partial y'}-\frac{Q'}{\rho Cp}\left(T'-T_\infty'\right), \quad \dots (9)$$

$$-v_0\frac{\partial C'}{\partial y'}=D_C\frac{\partial^2 C'}{\partial y'^2}+D_T\frac{\partial^2 T'}{\partial y'^2}. \quad \dots (10)$$

The boundary conditions (7) reduce to:

$$\mu \frac{\partial u'}{\partial y'} = -\beta' u', \qquad \frac{\partial T'}{\partial y'} = -\frac{Q}{K_T}, \quad \frac{\partial C'}{\partial y'} = -\frac{m}{D_C} \qquad \text{at} \quad y' = 0,$$

$$u' \to 0, \; T' \to T'_\infty, \; C' \to C'_\infty \quad \text{as} \quad y' \to \infty. \qquad \ldots (11)$$

The symbols are defined in nomenclature.

We introduce the following non-dimensional quantities and parameters:

$$y = \frac{y' v_0}{\upsilon}, \quad u = \frac{u'}{u_s}, \quad T = \frac{T' - T'_\infty}{\frac{Q\upsilon}{K_T v_0}}, \quad C = \frac{C' - C'_\infty}{\frac{m\upsilon}{D_C v_0}}, \quad Gr = \frac{\upsilon g \beta}{u_s v_0^2}\left(\frac{Q\upsilon}{K_T v_0}\right),$$

$$Gc = \frac{\upsilon g \beta^*}{u_s v_0^2}\left(\frac{m\upsilon}{D_C v_0}\right), \quad Pr = \frac{\mu Cp}{K_T}, \quad M = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2}, \quad Sc = \frac{\upsilon}{D_C}, \quad K = \frac{K_0 v_0^2}{\upsilon^2},$$

$$A = \frac{D_T}{\upsilon} \frac{\left(Q\upsilon / K_T v_0\right)}{\left(m\upsilon / D_C v_0\right)}, \quad S = \frac{Q' \upsilon^2}{K_T v_0^2}, \quad N = \frac{4\sigma_1 T'^3_\infty}{k_1 K_T}.$$

Substituting q'_r and above mentioned non-dimensional quantities and physical parameters, the equations (8), (9) and (10) transform to:

$$\frac{d^2 u}{dy^2} + \frac{du}{dy} + Gr\,T + Gc\,C - M_1 u = 0, \qquad \ldots (12)$$

$$\left(3 + 4N\right)\frac{d^2 T}{dy^2} + 3Pr\frac{dT}{dy} - 3S\,T = 0, \qquad \ldots (13)$$

$$\frac{d^2 C}{dy^2} + ASc\frac{d^2 T}{dy^2} + Sc\frac{dC}{dy} = 0, \qquad \ldots (14)$$

where $\qquad M_1 = M + \frac{1}{K}.$

The boundary conditions (11) reduce to:

$$\frac{du}{dy} = -k_s u, \quad \frac{dT}{dy} = -1, \quad \frac{dC}{dy} = -1 \;\text{ at }\; y = 0,$$

$$u \to 0, \qquad T \to 0, \quad C \to 0 \qquad \text{as } y \to \infty,, \qquad \ldots (15)$$

where $\qquad k_s = \frac{\beta' \upsilon}{\mu v_0}.$

Method of Solution

Solving equations (12) to (14) and using the boundary conditions (15), we get:

$$u = K_2 e^{-K_1 y} - \frac{Gr}{\alpha\left(\alpha^2 - \alpha - M_1\right)} e^{-\alpha y} - \left(\frac{1}{Sc} - \frac{A\alpha}{Sc - \alpha}\right) \frac{Gc}{\left(Sc^2 - Sc - M_1\right)} e^{-Sc\, y}$$

$$- \frac{A\,Gc\,Sc}{\left(\alpha^2 - \alpha - M_1\right)\left(Sc - \alpha\right)} e^{-\alpha y}, \qquad \ldots (16)$$

$$T = \frac{1}{\alpha} e^{-\alpha y}, \qquad \ldots (17)$$

$$C = \left(\frac{1}{Sc} - \frac{A\alpha}{Sc - \alpha}\right) e^{-Sc\, y} + \frac{A\,Sc}{Sc - \alpha} e^{-\alpha y}, \qquad \ldots (18)$$

where $K_1 = \frac{1}{2}\left(1 + \sqrt{1 + 4M_1}\right)$,

$$K_2 = \frac{1}{\left(K_1 - k_s\right)} \left[\frac{Gr\left(1 - \frac{k_s}{\alpha}\right)}{\alpha^2 - \alpha - M_1} + \left(\frac{1}{Sc} - \frac{A\alpha}{Sc - \alpha}\right) \right.$$

$$\left. \frac{Gc\left(Sc - k_s\right)}{\left(Sc^2 - Sc - M_1\right)} + \frac{A\,Gc\,Sc\left(\alpha - k_s\right)}{\left(Sc - \alpha\right)\left(\alpha^2 - \alpha - M_1\right)} \right],$$

$$\alpha = \frac{3Pr + \sqrt{9Pr^2 + 12S\left(3 + 4N\right)}}{2\left(3 + 4N\right)}.$$

Skin -Friction at the Surface

The skin-friction (τ) at the surface in the direction of flow is given by:

$$\tau = \left(\frac{du}{dy}\right)_{y=0}$$

$$= -K_1 K_2 + \frac{Gr\alpha}{\alpha\left(\alpha^2 - \alpha - M_1\right)} \left(\frac{1}{Sc} - \frac{A\alpha}{Sc - \alpha}\right)$$

$$\frac{Gc\,Sc}{\left(Sc^2 - Sc - M_1\right)} + \frac{A\,Gc\,Sc\,\alpha}{\left(Sc - \alpha\right)\left(\alpha^2 - \alpha - M_1\right)}. \qquad \ldots (19)$$

Verification of the Problem

If heat source/sink parameter $S = 0$ and heat radiation parameter $N = 0$, the results of the present study are exactly the same to those obtained by Adhikari [7] exact notations.

RESULTS AND DISCUSSION

In order to get insight of the problem, the effects of the physical parameters calculated, presented graphically and discussed.

Fig. 1 is illustrated to demonstrsates the velocity profiles against *y* for different numerical values of *M*, *Pr* and k_s. It is observed that the velocity decreases with an increasing *M* and k_s. We also noted that the velocity increases with increase in Prandtl number.

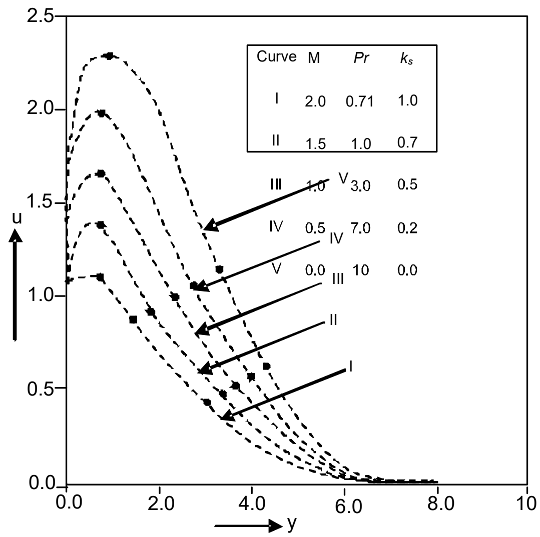

Fig. 1. Variation in velocity profile against *y* for different values of *M*, *Pr*, and k_s (t = 1.0, S = 5.0, K = 0.1, N = 1.0, Sc = 0.40, Gr = 5.0, Gc = 5.0, A = 2.0).

Fig. 2 is intended to show the variations in velocity profiles against *y* for different values of *S*, *K* and *Gr*. As expected, it is observed that an increase in heat source (negative values of *S*) increase the velocity, whereas increase in heat sink (positive values of S) decrease the velocity . In fact, heat source reduces the frictional force, which increase which decreases the velocity. We noted that the velocity increases with increasing *Gr*. The velocity decreases with increasing *K*.

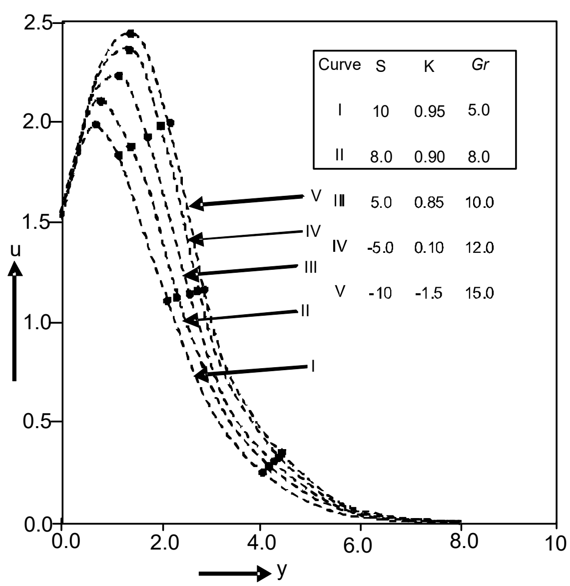

Fig. 2. Variation in velocity profile against *y* for different values of *S*, *K*, and *Gr* (t = 1.0, M = 0.5, Pr = 7.0, N = 1.0, Sc = 0.40, k_s = 0.2, Gc = 5.0, A = 2.0).

Fig. 3 is intended to show the variations in temperature profiles against y for different values of S, N and Pr. It is observed that an increases in heat source parameter (S) increase the temperature, whereas in increase in heat sink parameter decreases the temperature due to heat generation and heat obsorption respectively. We noted that decreases the temperature an increase in N and Pr.

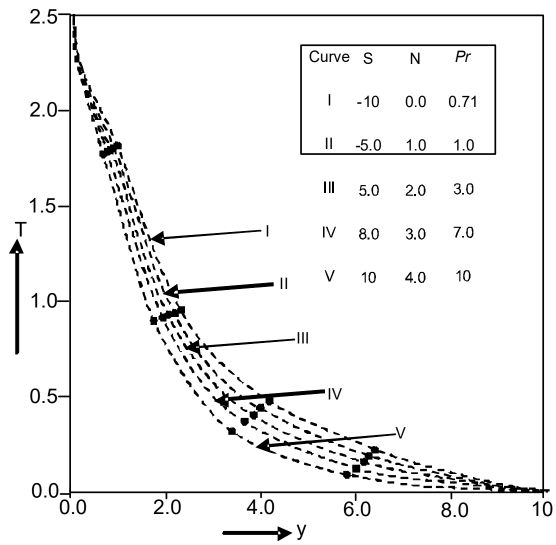

Fig. 3. Variation in temperature profile against y for different values of S, N and Pr (t = 1.0, M = 0.5, K = 0.1, Sc = 0.40, k_s = 0.2, Gc = 5.0, Gr = 5.0, A = 2.0).

Fig. 4 is intended to show that the variations in concentration profiles against y for different values of A, Pr and Sc. It is observed that an increases concentration with increasing A. The concentration decreases with increasing Pr. We also note that the concentration decreases with increasing Sc.

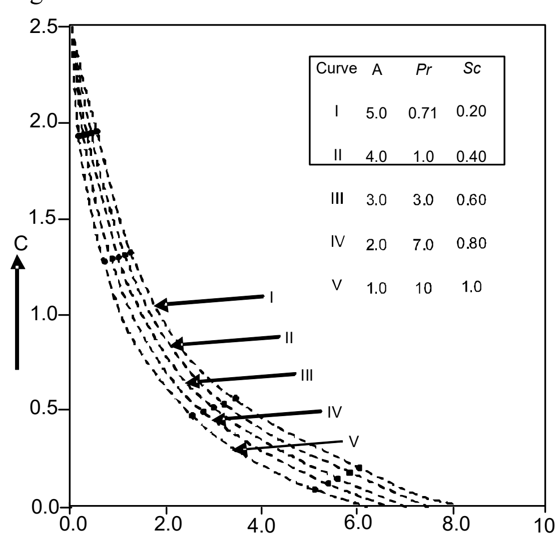

Fig. 4. Variation in concentration profile against y for different values of A, Pr, and Sc (t = 1.0, M = 0.5, K = 0.1, S = 5.0, k_s = 0.2, Gc = 5.0, Gr = 5.0, N = 1.0).

Fig. 5 is intended to show the variations in skin-friction profiles against *M* for different values of *Gr* and *K*. It is observed that an increase *Gr* leads to a decrease skin-friction. It is also noted that an increases the *K* with increasing skin-friction.

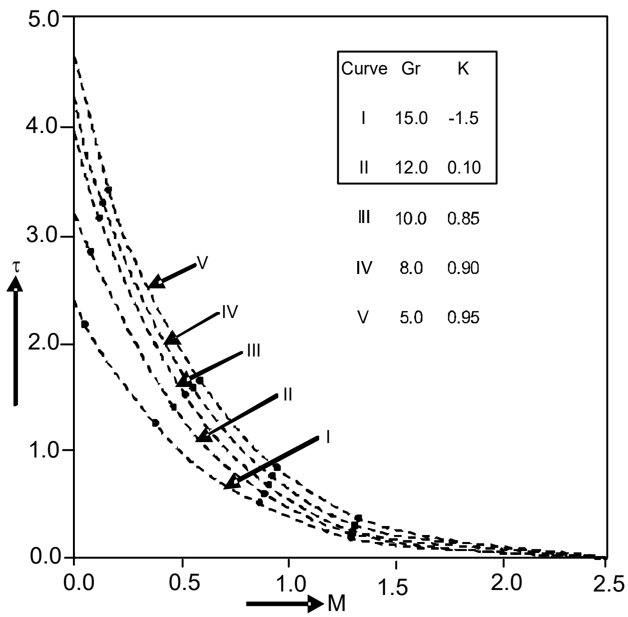

Fig. 5. Coefficient of Skin friction profile against *M* for different values of *Gr* and *K* (t = 1.0, Pr = 7.0, N = 1.0, Sc = 0.40, k_s = 0.2, Gr = 5.0, A = 2.0, S = 5.0).

Conclusions

1. The fluid velocity decreases with increase in the Hartmann number and the slip parameter but an increasing the velocity with increases in Prandtl number.

2. The velocity increases with increase in heat source but decreases with increase in heat sink and the porosity parameter.

3. The fluid temperature increases with increase in source parameter, whereas it decreases with increase in Prandtl number, radiation parameter or heat sink parameter.

4. The concentration increases with increasing thermal diffusion parameter, whereas it decreases with increase in Prandtl number and Schmidt number.

5. The skin-friction decreases due to increase thermal Grashoff number but increases with increase in the porosity parameter.

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