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SYMMETRIC & RECURRENT–P–SASAKIAN MANIFOLD

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In this paper we are interested in obtaining some theorems by dealing affine motion on recurrent & symmetric *p*-sasakian manifold

PRELIMINARIES

Let M be an n-dimensional almost para contact manifold there exists a recurrent metric $g_{\beta\alpha}$ box, which is called an associated recurrent metric.

$$\eta_\lambda \xi^\lambda = 1, \qquad \ldots (1.1)$$

$$\phi^x{}_\alpha \, \eta_\lambda = 0, \qquad \ldots (1.2)$$

$$\phi^\eta_\lambda \xi^\lambda = 0, \qquad \ldots (1.3)$$

$$\phi^\lambda_\alpha \theta^\eta_\lambda = \delta^\eta_\alpha - \eta_\alpha \xi^\eta \qquad \ldots (1.4)$$

$$\eta_\alpha = g_{\alpha\lambda} \xi^\lambda \qquad \ldots (1.5)$$

$$g_{\tau t} \, \phi^{\tau t} \, \phi^t_\alpha = g_{\beta\alpha} - \eta_\beta \eta_\alpha \qquad \ldots (1.6)$$

$$\text{Rank } \phi^\eta_\alpha = \eta - 1 \qquad \ldots (1.7)$$

The set $\left(\phi^\eta_\alpha, \xi^\lambda, \eta_\alpha, g_{\beta\alpha}\right)$ is called an almost para-contact Riemannian structure & manifold with this structure is called almost para-contact Riemannian manifold.

Also
$$\nabla_j \eta_i - \nabla_i \eta_j = 0 \qquad \ldots (1.8)$$

$$\nabla_k \nabla_j \eta_i = \left(-g_{kj} + \eta_k \eta_j\right) \eta_j + \left(-g_{kj} + \eta_k \eta_j\right) \eta_i \qquad \ldots (1.9)$$

The following relation hold in *p*-sasakian manifold.

$$R^\lambda_{\gamma\beta\alpha} \eta_\lambda = g_{\gamma\alpha} \eta_\beta - g_{\beta\alpha} \eta_r \qquad \ldots (1.10)$$

$$R^\lambda_\alpha \eta_\lambda = -(\eta - 1) \eta^\alpha \qquad \ldots (1.11)$$

$$R^\alpha_\beta \xi^\beta = -(\eta - 1) \xi^\alpha \qquad \ldots (1.12)$$

Recurrent *p*-sasakian:

A *p*-sasakian manifold whose curvature tensor is recurrent is termed as recurrent *p*-sasakian manifold.

then $$\nabla_l R^h{}_{\gamma\beta\alpha} = K_l R^h{}_{\gamma\beta\alpha} \quad \ldots (1.13)$$

K_l is recurrent vector

The contraction with $R_n^{\gamma\beta\alpha}$ given

$$= g^{vd} g^{bc} g^{ab} R^a{}_{dcb} \text{ gives}$$

$$\frac{1}{2}\nabla_l \left(R_{dcba}.R^{dcba}\right) = \left(R_{dcba}R^{dcba}\right).k_l \quad \ldots (1.14)$$

from which it follows

$$\left(R_{dcba}.R^{dcba}\right)\left(\nabla_{mkl} - \nabla_{lkm}\right) = 0$$

Thus a *p*-sasakian manifold has positive different metric & we have

$$R^{\;h}_{\gamma\,\beta\alpha} = 0 \quad \ldots (1.15)$$

$$\nabla^{\,kl}_{\,m} = \nabla^{\,km}_{\,l} \quad \ldots (1.16)$$

SYMMETRIC *p*-SASAKIAN MANIFOLD

A *p*-sasakian manifold whose curvature tensor satisfies the following termed as symmetric

$$\nabla_l R^{\;h}_{\gamma\,\beta\alpha} = 0 \quad \ldots (2.1)$$

or $$\nabla_m \nabla_l R^{\;h}_{\gamma\beta\alpha} - \nabla_l \nabla_m R^{\;h}_{\gamma\beta\alpha} = 0 \quad \ldots (2.2)$$

applying Ricci surtainty to above equation we obtain

$$R^{\;\lambda}_{\gamma\beta\alpha} R^{\;h}_{mlx} - R^{\;h}_{\gamma\beta\alpha} R^{\;\lambda}_{mlx} - R^{\;h}_{\gamma\lambda\alpha} R^{\;\lambda}_{ml\beta} - R^{\;h}_{\gamma\beta\alpha} R^{\;\lambda}_{ml\gamma} = 0 \quad \ldots (2.3)$$

translucing 2.2 with $3^l \nabla$ using 1.1 and 1.2 we get

$$R_{\gamma\beta\alpha m} = -\left(g_{\beta\alpha} g_{\gamma m} - g_{\gamma\alpha} g_{\beta m}\right) \quad \ldots (2.14)$$

Sp-PARA SASAKIAN

If in para sasakian manifold also we have obtained [4]

$$g\left(X, \xi\right) = \eta\left(X\right), \quad \ldots (3.1)$$

$$\eta\left(\xi\right) = 1 \quad \ldots (3.2)$$

$$\nabla_X \xi = \phi(X), \qquad \dots (3.3)$$

Here the manifold is called *Sp*-para sasakian manifold

In a *Sp* Para Sasakian manifold, we have

$$\phi\xi = 0, \qquad \dots (3.4)$$

$$\phi^2 X = X - \eta(X)\xi, \qquad \dots (3.5)$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X) - \eta(Y) \qquad \dots (3.6)$$

$$S(X,\xi) = -(n-1)\eta(X) \qquad \dots (3.7)$$

$$\eta\left[\tilde{R}(X,Y)Z\right] = g(X,Z)\eta(Y) - g(Y,Z)\eta \qquad \dots (3.8)$$

$$\tilde{R}(\xi,X)Y = \eta(Y)X - g(X,Y)\xi \qquad \dots (3.9)$$

$$\tilde{R}(\xi,X)Y\xi = \eta(X)Y - \eta(Y)X \qquad \dots (3.10)$$

$$\tilde{R}(\xi X, Y\xi) = \eta(Y).\eta(X) - g(X,Y) \qquad \dots (3.11)$$

$$\tilde{R}(\xi,X,Y,Z) = g(X,Z)\eta(Y) - g(X,Y)\eta(Z) \qquad \dots (3.12)$$

where $$\tilde{R}(X,Y,Z,u) = g\left[\tilde{R}(X,Y)Z,u\right] \qquad \dots (3.13)$$

and *S* is Ricci tensor (0,2) tensor.

Main result

Birecurrent p-sasakian manifold:

$$\nabla_l \nabla_m R^{\ \ \ h}_{\gamma\beta\alpha} = k_{lm} R^{\ \ \ h}_{\gamma\beta\alpha^2} \qquad \dots (4.1)$$

K_{lm} is a birecurrent tensor contracting this with

$$R_h^{\beta\alpha\gamma} = g^{\gamma h} g^{\beta c} g^{\alpha b} R^{\ a}_{cb}, \qquad \dots (4.2)$$

We get

$$1/2\,\nabla_l.\nabla_m R_{dcba} R^{dcba} = k_{lm} R_{dcba} R^{dcba}$$

from which

or $$1/2\,\nabla_m.\nabla_l R_{dcba} R^{dcba} = k_{lm} R_{dcba} R^{dcba} \qquad \dots (4.3)$$

the projective curvature tensor W^h_{ijk}

conformal curvature tensor $C^{\ h}_{ijk}, \dots\ \dots, \dots\ \dots,\ jk$

The concircular harmonic curvature tensor. L^h_{ijk} & the conformal curvature tensor $M^{\ h}_{ijk}$ are respectively Singh & Kothari [6]:

$$W^h_{ijk} = R^h_{ijk} + \left(1/(n-1)\right)\left(R_{ik}\delta^h_j\right) - R_{jk}\delta^h_i \quad \dots (4.4)$$

$$W^h_{ijk} = R^h_{ijk} + \left(1/(n-2)\right)\left(R_{ik}\delta^h\right) - R_{jk}\delta^h_i + g_{ik}R^h_j - \left(g_{ik}\delta^h\right) - g_{ik}R^h_i - \frac{R}{(n-1)(n-2)} \quad \dots (4.5)$$

$$M^h_{ijk} = R^h_{ijk} + R/n(n-1)\left[g_{ik}\delta^h_j - g_{jk}\delta^h_i\right] \quad \dots (4.6)$$

$$M^h_{ijk} = R^h_{ijk} + R/(n-2)\left[g_{jk}R^h_i - g_{ik}R^h_j + R_{ik}\delta^h_j - R_{jk}\delta^h_i\right]$$

$$W^h_{ijk}, lm = R^h_{ijk}, lm + 1/n - 1\left[R_{ik}; lm.\delta^h_j - \delta^h_i R_{jk}, lm\right] \quad \dots (4.7)$$

$$C^h_{ik}, lm = R^h_{ijk}, lm + 1/n - 2\left[R_{ik}.lm.\delta^h_i - R_{jk}lm + g_{ik}R^h_j, lm - g_{jk}R^h{}_i, lm\right]$$
$$+R, lm/(n-1)(n-2)\left[g_{ik}\delta^h_j - g_{jk}\delta^h_j\right]\delta^h_j \quad \dots (4.8)$$

$$L^h_{ijk}.lm = R^h_{ijk}, lm + R,\ lm/n(n-1)\left[g_{ik}\delta^h_j - g_{jk}\delta^h_j\right] \quad \dots (4.9)$$

$$M^h_{jk}, lm = R^h_{ijk}, lm + R,\ lm/(n-2)\left[g_{ik}R^h_j - g_{jk}R^h_i + R_{ik}\delta^h{}_j - R_{jk}\delta^h_i\right]$$
$$+R/n - 2\left[g_{ik}R^h_j lm - g_{jk}R^h_i lm + R_{ik}, lm\delta^h_j - R_{jk}, lm\delta^h_i\right] \quad \dots (4.10)$$

Thus

$$W^h_{ijk}, lm = k_{lm}.R^h_{ijk} + k_{lm}/n - 1\left[R_{ik}\delta^h_{ji} - R_{jk}\delta^h_{ji}\right]$$
$$= k_{lm}\left\{R^h_{ijk} + 1/n - 1\left(R_{jk}\delta^h\right) - R_{jk}\delta^h_i\right\} \quad \dots (4.11)$$

$$W^h_{ijk\ lm} = k_{lm}.W^h_{ijk} \quad \dots (4.12)$$

since $C^h_{ijk,\ lm} = k_{lm}.C^h_{ijk}$, ... (4.13)

$$L^h_{ijk,\ lm} = k_{lm}.L^h_{ijk}, \quad \dots (4.14)$$

and $M^h_{ijk,\ lm} = k_{lm}.M^h_{ijk}$ ... (4.15)

(a) A para sasakian mani fold is projectively birecurrent if it is birecurrent manifold.

(b) If the para sasakian manifold satisfies any two of the following then IIIrd must hold.

(1) It is birecurrent.

(2) It is projective birecurrent

(3) It is Ricci birecurrent.

(c) (1) It is Ricci birecurrent.

(2) It is birecurrent birecurrent.

(3) It is conformal birecurrent.

(d) (1) It is concircular birecurrent.

(2) It is Ricci birecurrent.

(3) It is birecurrent.

Theorem 4.2. If a *p*-para same manifold's.

(I) Projectively symmetric if it is symmetric.

(II) If the p-para sasakian manifold satisfy any two of these then it satisfies-III

(1) It is bi symmetric.

(2) It is projective bisymmetric.

(3) It is Ricci bi symmetric.

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