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# EFFECT OF HALL CURRENT ON VISCOUS FLUID FLOW OVER A ROTATING POROUS DISK

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The effect of Hall Current on viscous fluid flow over a rotating porous disk is discussed. The governing equations are solved analytically using the Laplace transform technique. The effects of Hall parameter on the velocity profile are presented through graphs. The numerical computations are done by MATLAB R2009a.

**KEYWORDS :** Hall Effect, Magnetic field, Viscous Fluid, Rotating Porous disk

## INTRODUCTION

In many MHD problems it is assumed that the electrical conductivity of the fluid is isotropic and such as a scalar quantity. However, this need not be always in nature and the conductivity of the medium is an anisotropic if the medium is rarefied if a strong magnetic field is present. In the presence of a strong magnetic field, the charged particles are tied to the lines of force, and this prevents their motion transverse to the magnetic field. Then the tendency of the current flow in a direction is normal to both the electrical and magnetic field is known as Hall Current. The Hall Effect is the production of a voltage difference (Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. It was discovered by Edwin Hall in 1879. Thus the Hall Effect rotates the current vector away from the direction of the electric field generally reduces the level of the Lorenz force on the flow.

Hall Effects gained widespread interest in fluid dynamics due to their applications in many geophysical and astrophysical situations as well as in engineering problems such as Hall accelerators, Hall Effect sensors, constructions of turbines and centrifugal machines.

The Hall coefficient is defined as the ratio of the induced electric field to the product of the current density and applied magnetic field. It is a characteristic of the material from which the conductor is made, since its value depends on the type, number, and properties of the charge carries that constitute the current.

Hall probes are often used as magnetometers (*i.e.*) to measure magnetic fields, or inspect materials (such as tubing or pipelines) using the principles of magnetic fluid leakage. Hall Effect devices produce a very low signal level and thus require amplification. It was only with the development of the low cost integrated circuit that the Hall Effect sensor became suitable for mass application. Many devices now sold as Hall Effect sensors in fact contain both the sensor as described above plus a high gain integrated circuit (IC) amplifier in a single package. The flow of incompressible fluid due to non – coaxial rotations of a disk and a fluid at infinity was studied by many researchers.

An exact solution of this type of problem of was obtained Berker (1963). Coirier (1972) studied the flow due to a disk and a fluid at infinity which is rotating non-coaxially at a slightly different angular velocity. An exact solution of the three dimensional Navier-stokes equation for the flow due to non – coaxial rotations of a porous disk and a fluid at infinity was studied by Erdogan (1976, 1977). Ram and Murty (1978) have studied the magneto hydrodynamic flow and heat transfer due to eccentric rotations of a porous disk and a fluid at infinity. The unsteady flow due to non coaxial rotations of a disk, oscillating its own plane and a fluid at infinity was studied by Kasiviswanathan and Rao (1987).

The influence of an external magnetic field on the flow due to a rotating disk was studied by many researchers Attia (1988); EI-Mistikiway and Attia (1990) without considering the Hall Effect. Aboul – Hassan and Attia (1997) studied the steady hydromagnetic problem taking the Hall Current into consideration. In recent years, considerable interest has been shown in mass addition to boundary layer flows, especially in connection with the cooling of the turbine blades and the sting of high speed aero vehicles.

Rotating disk flows of conducting fluids have practical applications in many areas such as rotating machinery, lubrication, computer storage devices, viscometry and crystal growth process. In most cases the Hall term was ignored in applying Ohm’s law as it has no marked effect for small and moderate values of the magnetic field. However, the magnetic field, so that the influence of electromagnetic force is noticeable. Under these conditions the Hall Current is important and it has marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. Aboul-Hassan and Attia (1997) studied the steady hydro magnetic problem taking the Hall Current into consideration.

In recent years, considerable interest has been shown in mass addition to boundary layer flows, especially in connection with the cooling of the turbine blades and the sting of high speed aero vehicles. Hassan and Attia (1997) investigated the steady magneto hydrodynamics boundary layer flow due to an infinite disk rotating with uniform angular velocity in the presence of an axial magnetic field. They neglected induced magnetic field but considered Hall Current and accordingly solved steady state equation numerically using finite difference approximations.

Magneto hydrodynamic which rises as a theory of the macroscopic interaction of electrically conducting fluids and electromagnetic fields has many practical applications in astronomy, space physics and geophysics as well as in many engineering fields. Some interesting results on the effects of the magnetic field on the steady flow due to the rotation of a disk of infinite or finite extent was pointed out by EI-Mistikawy et al. (1990) and Attia and Hassan (2004). In special, the study of hydro magnetic flows with Hall Effects has important in engineering applications in problems of magneto hydrodynamics generators and of Hall accelerators as well as in flight magneto hydrodynamics. T. Hayat, R. Ellahi and S. Asghar (2008) are studied the Hall Effects on unsteady flow due to non-coaxially rotating disk and a fluid at infinity.

The aim of this paper is to present hydro magnetic flow of a viscous electrically conducting fluid due to the rotation of a porous disk in a magnetic field is studied considering the Hall Effect.

## MATHEMATICAL FORMULATION

An incompressible viscous fluid conducting which occupies the space $z > 0$ and is in contact with an infinite porous disk making oscillations in its own plane is considered. The axis of rotation of both the disk and the fluid, are assumed to be in the plane $x = 0$. The Cartesian coordinate system with the *z*-axis normal to the disk, which lies in the plane, $z = 0$ is considered. The distance between the axes is being assumed to be *l*. Initially, the disk and the fluid at infinity are rotating with the same angular velocity $\Omega$ about the $z^1$ axis and at time *t = 0*. The disk start to oscillate suddenly along the *x* – axis and to rotate impulsively about the z – axis with $\Omega$ and the fluid continues to rotate with $\Omega$ about the $z^1$-axis. A uniform magnetic field $B_0$ is applied in the positive *z*-direction.

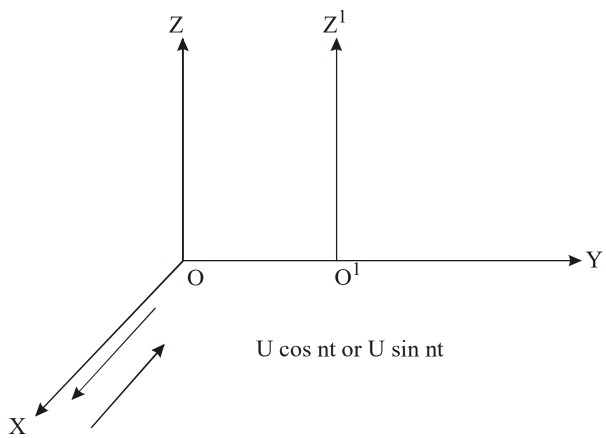


**Fig. 1. Flow Geometry**

Under the above assumptions, the equations governing the unsteady motion of the conducting viscous incompressible fluid are those pertaining to the conservation of momentum and of mass which are

$$\frac{dv}{dt} = -\frac{1}{\rho}\nabla p_1 + v\nabla^2 V + \frac{1}{\rho}(J \times B)$$

$$div\ V = 0 \qquad \ldots (1)$$

The corresponding initial and boundary conditions are

$$u = -\Omega y + U\cos nt \quad \text{(or)}$$

$$u = -\Omega y + U\sin nt ; v = \Omega x \ \text{ at } z = 0 \ for\ t > 0$$

$$u = -\Omega(y-1), v = \Omega x \text{ as } z \to \infty \text{ for all } t, \qquad \ldots (2)$$

$$u = -\Omega(y-1), v = \Omega x \text{ at } t = 0 \text{ for } z > 0,$$

where *u*, *v*, *w* are the velocity components along *x*, *y* directions respectively and *n* being the frequency of the non-torsional oscillations.

The equations governing the flow consists of the Maxwell equations and a generalized Ohm's law which after neglecting the displacement currents are

$$div\ B = 0 \qquad \ldots (3)$$

$$curl\, B = \mu_m J \qquad \ldots (4)$$

$$curl\ E = \frac{-\partial B}{\partial t} \qquad \ldots (5)$$

The geometry of the problem suggests that the velocity field in the flow is of the form

$$u = -\,\Omega\, y + f\,(z,t)\,, v = \Omega x + g\,(z,t) \qquad \ldots (6)$$

Let us consider the uniform porous disk is of the form

$$w = -w_0 \qquad \ldots (7)$$

The generalised Ohm's law, on taking Hall Currents into account (Cowling, 1957) and neglecting ion-slip and thermo-electric effect is given by

$$J + \frac{w_e \tau_e}{B_0}(J \times B) = \sigma\left[E + V \times B + \frac{1}{en_e}\nabla p_e\right] \qquad \ldots (8)$$

where *J* is the current density vector, *B* is the magnetic induction vector, *E* is the electric field vector, $w_e$ is the cyclotron frequency and $\tau_e$ is the collision time of electron. In the absence of an external applied electric field and with negligible effects of polarization of the ionized gas, *E* is taken as zero $(E = 0)$. The induced magnetic field is negligible which is a valid consideration is on the laboratory scale. Further, it is assumed that $w_e\, \tau_e \approx 0(1)$, and $w_e\, \tau_e << 1$, where $w_e$ and $\tau_e$ are cyclotron frequency and collision time for ions respectively.

With the help of the equations (1), (6) & (7) and in view of the above assumptions, the flow with Hall Effects is governed by the following scalar equations:

$$\frac{\partial f}{\partial t} - \Omega g - w_0 = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} + \frac{v\partial^2 f}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1-im)}(f - \Omega y) \qquad \ldots (9)$$

$$\frac{\partial g}{\partial t} + \Omega f - w_0 \frac{\partial g}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial y} + \frac{v\partial^2 g}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1-im)}(g - \Omega x) \qquad \ldots (10)$$

$$\frac{1}{\rho}\frac{\partial \hat{p}}{\partial z} = \frac{\sigma B_0^2}{\rho(1-im)} w_0 \qquad \ldots (11)$$

where $m = \omega_e \tau_e$ the Hall is parameter and the modified pressure $\hat{p}$ is

$$\hat{p} = p_1 - \frac{\rho r^2 \Omega^2}{2}, \quad r^2 = x^2 + y^2 \qquad \ldots (12)$$

In view of equations (6) and (1),

$$\begin{aligned} &f(0,t) = U\cos nt \text{ (or) } f(0,t) = U\sin nt\,; g(o,t) = 0 \text{ for } t > 0 \\ &f(z,t) = \Omega l\,; g(z,t) = 0 \ as\, z \to \infty \text{ for all } t, \\ &f(z,0) = \Omega l;\ g(z,0) = 0 \text{ for } z > 0 \end{aligned} \qquad \ldots (13)$$

Eliminating $\hat{p}$ from (9) to (11), and using the boundary conditions (13) and combining the resulting equations, the problem can be written as

$$\frac{\partial f}{\partial t}+i\frac{\partial g}{\partial t}-i\Omega g+\Omega f-w_0\frac{\partial f}{\partial z}-iw_0\frac{\partial g}{\partial z}$$

$$=\frac{-1}{\rho}\frac{\partial \hat{p}}{\partial x}-i\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x}-i\frac{1}{\rho}\frac{\partial \hat{p}}{\partial y}+v\left[\left[\frac{\partial^2 f}{\partial z^2}+i\frac{\partial^2 g}{\partial z^2}\right]-\sigma\frac{B_0^2}{1-im}\right](f+ig)$$

$$(-(\Omega y+i\Omega x))$$

$$\frac{-1}{\rho}\left[\frac{\partial \hat{p}}{\partial x}-i\frac{1}{\rho}\frac{\partial \hat{p}}{\partial y}\right]+v\left[\frac{\partial^2 f}{\partial z^2}+i\frac{\partial^2 g}{\partial z^2}\right]-\sigma\left[\frac{B_0^2}{1-im}\right](f+ig) \quad \ldots (14)$$

$$[-\Omega y-i\Omega x]-\frac{\partial}{\partial t}(f+ig)-\Omega(f+ig)+w_0\left[\frac{\partial}{\partial z}(f+ig)\right]$$

$$v\frac{\partial^2 G}{\partial z^2}-\frac{\partial G}{\partial t}+w_0\frac{\partial G}{\partial z}-(N+i\Omega)G=0$$

The corresponding boundary conditions are given by

$$G(0,t)=\frac{U}{\Omega l}\cos nt-1 \text{ (Or) } G(0,t)=\frac{U}{\Omega l}\sin nt-1;\ t>0$$

$$\begin{aligned}&G(0,t)=0, as\ z\to\infty \text{ for all } t,\\ &G(z,0)=0 \text{ for } z>0\end{aligned} \quad \ldots (15)$$

in which

$$G=\frac{f}{\Omega l}+i\frac{g}{\Omega l}-1 \quad \ldots (16)$$

$$N=\frac{\sigma B_0^2(1+im)}{\rho(1+m^2)} \quad \ldots (17)$$

and introducing

$$H=Ge^{i\Omega t} \quad \ldots (18)$$

Therefore, the above governing problem becomes

$$v\frac{\partial^2 H}{\partial z^2}-\frac{\partial H}{\partial t}+w_0\frac{\partial H}{\partial z}-NH=0 \quad \ldots (19)$$

with the boundary conditions

$$H(0,t)=-1+\frac{U}{\Omega l}\cos nt \quad \text{or} \quad H(0,t)=-1+\frac{U}{\Omega l}\sin nt;\ t>0 \quad \ldots (20)$$

The auxiliary equation of (19) is given by

$$vm^2+w_0m-N=0$$

$$m=\frac{-w_0\pm\sqrt{w_0^2+4vN}}{2v}$$

$$m = \frac{-w_0 \pm \left[\sqrt{\frac{w_0^2}{4v^2} + \frac{N}{v}}\right] 4v^2}{2v}$$

$$m = \frac{-w_0}{2v} \pm \sqrt{\frac{w_0^2}{4v^2} + \frac{N}{v}}$$

$$\bar{H}(z,s) = c_1 e^{+\left[\frac{w_0}{2v} + \sqrt{\left(\frac{w_0}{2v}\right)^2 + \frac{N}{v}}\right] z} + c_2 \, e^{\left[-\frac{w_0}{2v} + \sqrt{\left(\frac{w_0}{2v}\right)^2 + \frac{N}{v}}\right] z}$$

Now consider the equation (20) in terms of $s$,

$$\bar{H}(0,s) = -1 + \frac{U}{\Omega l} \cos nt \quad \text{(or)} \quad \bar{H}(0,s) = -1 + \frac{U}{\Omega l} \sin nt$$

Substituting $z = 0$ in the above results,

$$\bar{H}(0,s) = c_1 + c_2$$

$$c_1 + c_2 = -1 + \frac{U}{\Omega l} \cos nt \qquad \dots (21)$$

and

$$c_1 + c_2 = -1 + \frac{U}{\Omega l} \sin nt \qquad \dots (22)$$

adding (21) and (22),

$$c_1 + c_2 = -1 + \frac{U}{2\Omega l} [\sin nt + \cos nt]$$

Therefore,

$$\bar{H}(z,s) = -1 + \frac{U}{2\Omega l} [\sin nt + \cos nt] \, e_1 \qquad \dots (23)$$

in which

$$e_1 = e^{-\left[\frac{w_0}{2v} + \sqrt{\left(\frac{w_0}{2v}\right)^2 + \frac{N}{v}}\right] z} \qquad \dots (24)$$

Using the Laplace transform technique, the solution for the above resulting transformed problems for $U \cos nt$ is given by

$$\bar{H}(z,s) = \left[-\frac{1}{s - i\Omega} + \frac{U}{2\Omega l}\left\{\frac{1}{s + i(n - \Omega)} + \frac{1}{s - i(n + \Omega)}\right\}\right] e_1 \, ; \, n > \Omega \qquad \dots (25)$$

$$\bar{H}(z,s) = \left[-\frac{1}{s - i\Omega} + \frac{U}{2\Omega l}\left\{\frac{1}{s - i(n - \Omega)} + \frac{1}{s - i(n - \Omega)}\right\}\right] e_1 \, ; \, n < \Omega \qquad \dots (26)$$

and for $U \sin nt$

$$\bar{H}(z,s)=\left[-\frac{1}{s-i\Omega}+\frac{iU}{2\Omega l}\left\{\frac{1}{s+i(n-\Omega)}+\frac{1}{s-i(n+\Omega)}\right\}\right]e_1\,;\; n>\Omega \qquad \ldots (27)$$

$$\bar{H}(z,s)=\left[-\frac{1}{s-i\Omega}+\frac{iU}{2\Omega l}\left\{\frac{1}{s-i(n-\Omega)}-\frac{1}{s-i(n+\Omega)}\right\}\right]e_1\,;\; n<\Omega \qquad \ldots (28)$$

after taking inverse Laplace transform, and using the equations (25) to (28), the suction solutions for $U\cos nt, n>\Omega$ **,** is given by

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=1+e^{-\sqrt{2}w\xi}\left(\begin{array}{l}-\frac{1}{2}\left(\begin{array}{l}e^{(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\+\frac{U}{2\Omega l}e^{-ik\tau}\left(\begin{array}{l}e^{(x_3+iy_3)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_3+iy_3)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_3+iy_3)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_3+iy_3)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\-\frac{U}{2\Omega l}e^{ik\tau}\left(\begin{array}{l}e^{(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\end{array}\right) \qquad \ldots (29)$$

and for $n<\Omega$

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=1+e^{-\sqrt{2}w\xi}\left(\begin{array}{l}-\frac{1}{2}\left(\begin{array}{l}e^{(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\+\frac{U}{2\Omega l}e^{-ik\tau}\left(\begin{array}{l}e^{(x_3+iy_3)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_3+iy_3)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_3+iy_3)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_3+iy_3)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\-\frac{U}{2\Omega l}e^{ik\tau}\left(\begin{array}{l}e^{(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\end{array}\right) \qquad \ldots (30)$$

for $U\sin nt, n>\Omega$

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=1+e^{-\sqrt{2}\,w\xi}\left(\begin{array}{l}-\frac{1}{2}\left(\begin{array}{l}e^{(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\+i\frac{U}{2\Omega l}e^{-ik\tau}\left(\begin{array}{l}e^{(x_3+iy_3)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_3+iy_3)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_3+iy_3)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_3+iy_3)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\-i\frac{U}{2\Omega l}e^{ik\tau}\left(\begin{array}{l}e^{(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\end{array}\right) \quad \ldots(31)$$

and for $n<\Omega$

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=1+e^{-\sqrt{2}\,w\xi}\left(\begin{array}{l}-\frac{1}{2}\left(\begin{array}{l}e^{(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_2+iy_2)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\+i\frac{U}{2\Omega l}e^{-ik\tau}\left(\begin{array}{l}e^{(x_5+iy_5)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_5+iy_5)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_5+iy_5)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_5+iy_5)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\-i\frac{U}{2\Omega l}e^{ik\tau}\left(\begin{array}{l}e^{(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_4+iy_4)\xi}\,erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_4+iy_4)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\end{array}\right) \quad \ldots(32)$$

and introducing $\xi=\sqrt{\frac{\Omega}{2v}}z,\ k=\frac{n}{\Omega},\ \tau=\Omega t,\ N_1=\frac{\sigma B_0^2}{\rho\Omega},\ w=\frac{w_0}{2\sqrt{v\Omega}}$, the following expressions can be obtained.

$$x_2=\left[\sqrt{\left(w^2+\frac{N_1 m}{1+m^2}\right)^2+\left(1+\frac{N_1 m}{1+m^2}\right)^2}+\left(w^2+\frac{N_1}{1+m^2}\right)\right]^{\frac{1}{2}}$$

$$x_3 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k - 1 - \frac{N_1 m}{1+m^2}\right)^2} + \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

$$x_4 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k + 1 + \frac{N_1 m}{1+m^2}\right)^2} + \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

$$x_5 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2} - k\right)^2} + \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

$$y_2 = \left[ \sqrt{\left(w^2 - \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

$$y_3 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k - 1 - \frac{N_1 m}{1+m^2}\right)^2} - \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

$$y_4 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(k - 1 - \frac{N_1 m}{1+m^2}\right)^2} - \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

$$y_5 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2} - k\right)^2} - \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

The solutions (30) to (32) are unsteady and valid for suction case. For blowing, the unsteady solutions can be obtained from equations (30) to (32) by replacing $w$ by $-w_1 (w_1 > 0)$.

**Resonant case** $\left(\frac{n}{\Omega} = 1\right)$

Employing the same methodology of solution as in the above case, the unsteady suction solutions for $U \cos nt$ can be written as

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=1+e^{-\sqrt{2}\,w\xi}\left(\begin{array}{l}-\frac{1}{2}\left(\begin{array}{l}e^{(x_2+iy_2)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_2+iy_2)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\+\frac{U}{2\Omega l}e^{-ik\tau}\left(\begin{array}{l}e^{(x_6+iy_6)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_6+iy_6)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_6+iy_6)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_6+iy_6)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\+\frac{U}{2\Omega l}e^{ik\tau}\left(\begin{array}{l}e^{(x_7+iy_7)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_7+iy_7)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_7+iy_7)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_7+iy_7)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\end{array}\right) \quad \ldots (33)$$

$$\frac{f}{\Omega l}+i\frac{g}{\Omega l}=1+e^{-\sqrt{2}\,w\xi}\left(\begin{array}{l}-\frac{1}{2}\left(\begin{array}{l}e^{(x_2+iy_2)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_2+iy_2)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_2+iy_2)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\+i\frac{U}{2\Omega l}e^{-ik\tau}\left(\begin{array}{l}e^{(x_6+iy_6)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_6+iy_6)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_5+iy_5)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_6+iy_6)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\\-i\frac{U}{2\Omega l}e^{ik\tau}\left(\begin{array}{l}e^{(x_7+iy_7)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}+(x_7+iy_7)\sqrt{\frac{\tau}{2}}\right)\\+e^{-(x_7+iy_7)\xi}\;erf\left(\frac{\xi}{\sqrt{2\tau}}-(x_7+iy_7)\sqrt{\frac{\tau}{2}}\right)\end{array}\right)\end{array}\right) \quad \ldots (34)$$

in which

$$x_6=\left[\sqrt{\left(w^2+\frac{N_1}{1+m^2}\right)^2+\left(1+\frac{N_1 m}{1+m^2}\right)^2}+\left(w^2+\frac{N_1}{1+m^2}\right)\right]^{\frac{1}{2}}$$

$$x_7=\left[\sqrt{\left(w^2+\frac{N_1}{1+m^2}\right)^2+\left(2+\frac{N_1 m}{1+m^2}\right)^2}+\left(w^2+\frac{N_1}{1+m^2}\right)\right]^{\frac{1}{2}}$$

$$y_6 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(1 + \frac{N_1 m}{1+m^2}\right)^2} - \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

$$y_7 = \left[ \sqrt{\left(w^2 + \frac{N_1}{1+m^2}\right)^2 + \left(\frac{N_1 m}{1+m^2}\right)^2} - \left(w^2 + \frac{N_1}{1+m^2}\right) \right]^{\frac{1}{2}}$$

For blowing replacing $w$ by $-w_1 (w_1 > 0)$ and the respective unsteady solutions for $U \cos nt$ and $U \sin nt$ are given by

$$\frac{f}{\Omega l} + i\frac{g}{\Omega l} = 1 + e^{-\sqrt{2}\,w\xi} \left( \begin{array}{l} -\frac{1}{2}\left( \begin{array}{l} e^{(\tilde{x}_2 + i\tilde{y}_2)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_2 + i\tilde{y}_2)\sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_2 + i\tilde{y}_2)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_2 + i\tilde{y}_2)\sqrt{\frac{\tau}{2}} \right) \end{array} \right) \\ + i\frac{U}{2\Omega l} e^{-ik\tau} \left( \begin{array}{l} e^{(\tilde{x}_3 + i\tilde{y}_3)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_3 + i\tilde{y}_3)\sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_3 + i\tilde{y}_3)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_3 + i\tilde{y}_3)\sqrt{\frac{\tau}{2}} \right) \end{array} \right) \\ - \frac{U}{2\Omega l} e^{ik\tau} \left( \begin{array}{l} e^{(\tilde{x}_4 + i\tilde{y}_4)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_4 + i\tilde{y}_4)\sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_4 + i\tilde{y}_4)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_4 + i\tilde{y}_4)\sqrt{\frac{\tau}{2}} \right) \end{array} \right) \end{array} \right) \quad \ldots (35)$$

$$\frac{f}{\Omega l} + i\frac{g}{\Omega l} = 1 + e^{-\sqrt{2}\,w\xi} \left( \begin{array}{l} -\frac{1}{2}\left( \begin{array}{l} e^{(\tilde{x}_2 + \tilde{y}_2)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_2 + i\tilde{y}_2)\sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_2 + \tilde{y}_2)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_2 + i\tilde{y}_2)\sqrt{\frac{\tau}{2}} \right) \end{array} \right) \\ + i\frac{U}{2\Omega l} e^{-ik\tau} \left( \begin{array}{l} e^{(\tilde{x}_3 + \tilde{y}_3)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_3 + i\tilde{y}_3)\sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_3 + \tilde{y}_3)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_3 + i\tilde{y}_3)\sqrt{\frac{\tau}{2}} \right) \end{array} \right) \\ - \frac{U}{2\Omega l} e^{ik\tau} \left( \begin{array}{l} e^{(\tilde{x}_4 + \tilde{y}_4)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} + (\tilde{x}_4 + i\tilde{y}_4)\sqrt{\frac{\tau}{2}} \right) \\ + e^{-(\tilde{x}_4 + \tilde{y}_4)\xi}\, erf\left( \frac{\xi}{\sqrt{2\tau}} - (\tilde{x}_4 + i\tilde{y}_4)\sqrt{\frac{\tau}{2}} \right) \end{array} \right) \end{array} \right) \quad \ldots (36)$$

where $$\tilde{x}_3 = \left[\sqrt{\left(w_1^2 + \frac{N_1}{1+m^2}\right)^2 + \left(\frac{N_1 m}{1+m^2}\right)^2} + \left(w_1^2 + \frac{N_1}{1+m^2}\right)\right]^{\frac{1}{2}}$$

$$\tilde{x}_4 = \left[\sqrt{\left(w_1^2 + \frac{N_1}{1+m^2}\right)^2 + \left(2 + \frac{N_1 m}{1+m^2}\right)^2} + \left(w_1^2 + \frac{N_1}{1+m^2}\right)\right]^{\frac{1}{2}}$$

$$\tilde{y}_3 = \left[\sqrt{\left(w_1^2 + \frac{N_1}{1+m^2}\right)^2 + \left(\frac{N_1 m}{1+m^2}\right)^2} - \left(w_1^2 + \frac{N_1}{1+m^2}\right)\right]^{\frac{1}{2}}$$

$$\tilde{y}_4 = \left[\sqrt{\left(w_1^2 + \frac{N_1}{1+m^2}\right)^2 + \left(2 + \frac{N_1 m}{1+m^2}\right)^2} - \left(w_1^2 + \frac{N_1}{1+m^2}\right)\right]^{\frac{1}{2}}$$

Using the following asymptotic values of the complementary error function

$$erfc\left(\frac{\xi}{\sqrt{2\tau}} \pm \left(x_j + y_j\right)\sqrt{\frac{\tau}{2}}\right) -- \rightarrow (0, 2), \quad j = 1 \text{ to } 4 \qquad \ldots (37)$$

The following steady state solutions in the respective case are obtained.

For $U\cos nt$,

$$\frac{f}{\Omega l} + i\frac{g}{\Omega l} = 1 + e^{\sqrt{2}} w_1 \xi \left\{ -e^{\left(\tilde{x}_2 + i\,\tilde{y}_2\right)\xi} + \frac{U}{\Omega l} e^{-i\tau} e^{\left(\tilde{x}_3 + i\,\tilde{y}_3\right)} + \frac{U}{\Omega l} e^{i\tau} e^{\left(\tilde{x}_4 + i\,\tilde{y}_4\right)\xi} \right\} \qquad \ldots(38)$$

# RESULTS AND DISCUSSION

The illustration is made for how the Hall Effect modifies the structure of flow, and the profiles of velocity for both cosine and sine oscillations. The effect of Hall Parameter *m* = 1.5, 2, 3 and *w* = 0 & *k* = – 4, 1, 4 on the velocity profiles for cosine oscillations are shown in Figure 2 (i), (ii), & (iii). It is observed that, the magnitudes of $f/\Omega l$ increases and $g/\Omega l$ decreases with the increase of m.

The effect of Hall parameter when *m* = 1.5, 2, 3 and the cosine oscillations *w* = 1 & the values of *k* = – 4, 1, 4 are displayed in Figure 3 (i), (ii), & (iii). Here, the boundary layer thickness *k* > 1 & *k* < 1 shows that, the values of $f/\Omega l$ increases and $g/\Omega l$ also increased and in the case of *k* = 1, the boundary layer thickness is minimum and the velocity profiles are maximum.

Finally, Figure 4 (i), (ii), & (iii) indicate that the same values as Figure 2 (i), (ii), & (iii) for $m = 1.5, 2, 3$, $w = -1$ and $k = -4, 1, 4$. From the results of these figures it is observed that, the magnitudes of $f/\Omega l$ increases and $g/\Omega l$ decreases with the increase of $m$.

# Graphs

$(a)\ W = 0, N_1 = 4, \tau = 0.2, \frac{n}{\Omega} = 4$

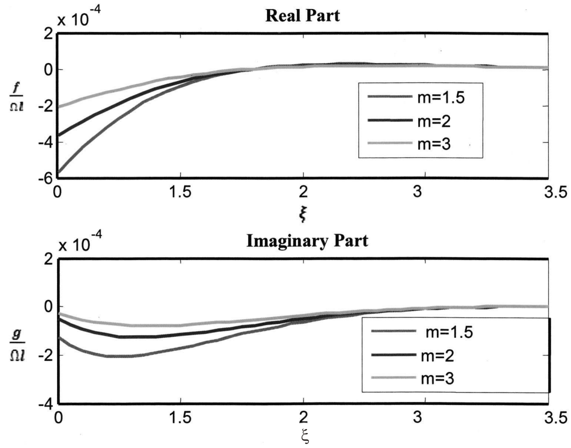


**Fig. 2 (i). The effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the absence of suction and blowing at $\left(\frac{U}{4\Omega l} = 1\right)$**

$(b)\ W = 0, N_1 = 4, \tau = 0.2, \frac{n}{\Omega} = -4$

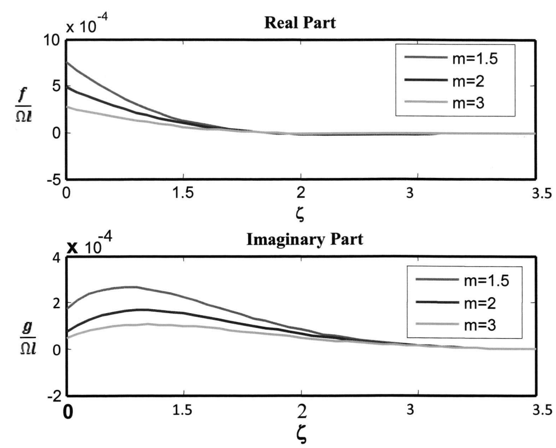


**Figure 2 (ii). The effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the absence of suction and blowing at $\left(\frac{U}{4\Omega l} = 1\right)$**

$(c)\ W = 0, N_1 = 4, \tau = 0.2, \dfrac{n}{\Omega} = 1$

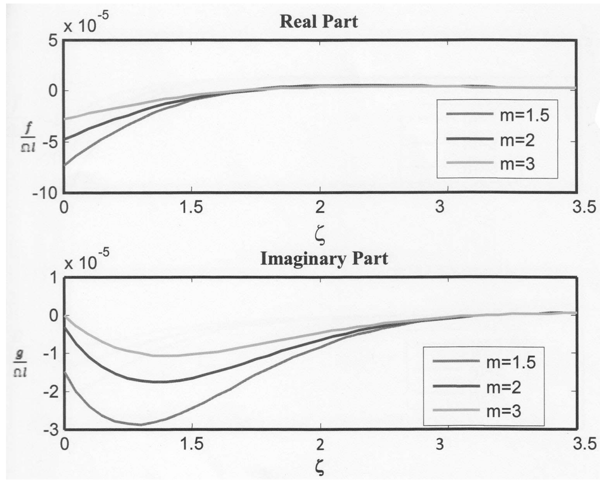


**Figure 2 (iii). The effect of Hall parameter on $\dfrac{f}{\Omega l}$ and $\dfrac{g}{\Omega l}$ for Cosine oscillation in the absence of suction and blowing at $\left(\dfrac{U}{4\Omega l} = 1\right)$**

$(a)\ W = 1, N_1 = 4, \tau = 0.2, \dfrac{n}{\Omega} = 4$

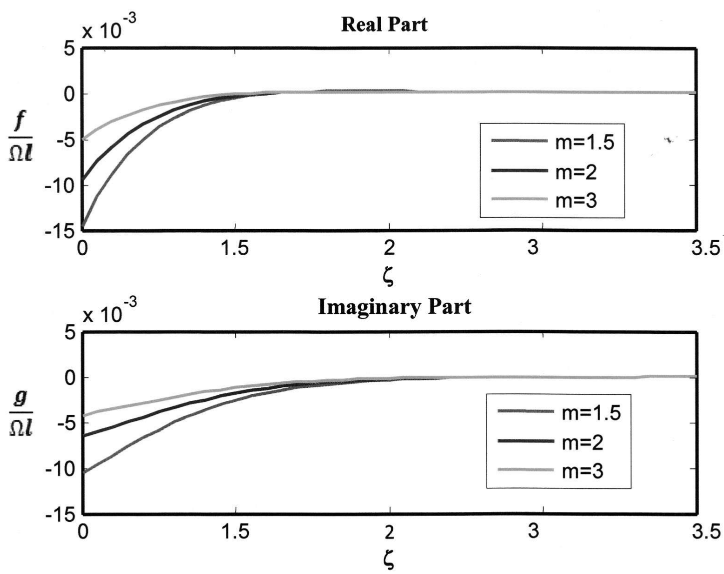


**Figure 3 (i). The effect of Hall parameter on $\dfrac{f}{\Omega l}$ and $\dfrac{g}{\Omega l}$ for Cosine oscillation in the presence of suction at $\left(\dfrac{U}{4\Omega l} = 1\right)$**

$(b)\ W = 1, N_1 = 4, \tau = 0.2, \dfrac{n}{\Omega} = -4$

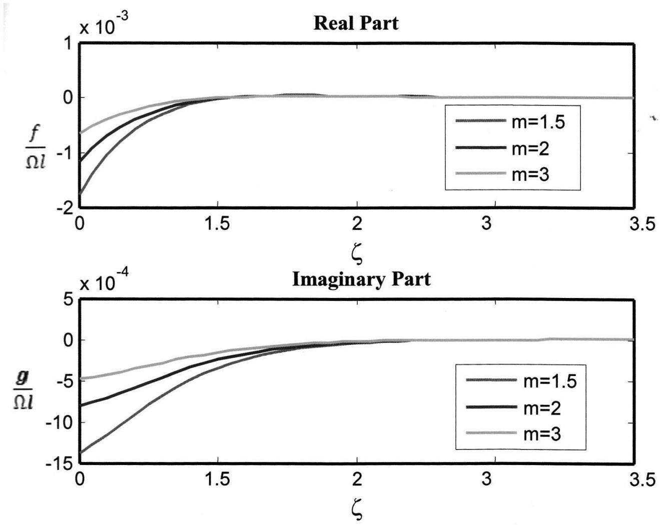


**Figure 3 (ii). The effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the presence of suction at $\left(\frac{U}{4\Omega l} = 1\right)$**

$(c)\ W = 1, N_1 = 4, \tau = 0.2, \dfrac{n}{\Omega} = 1$

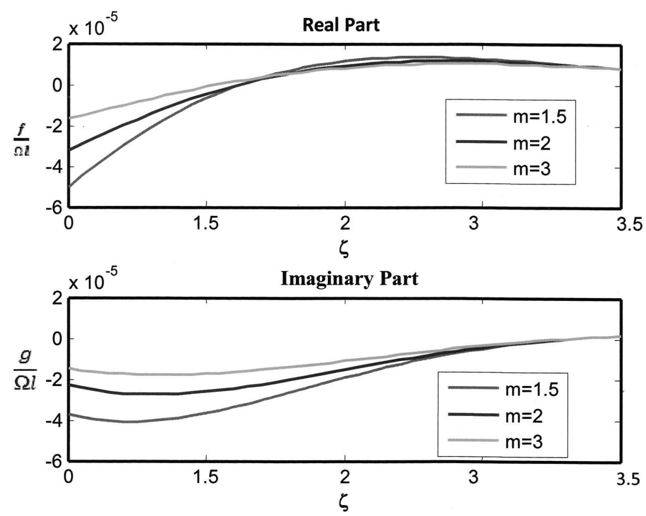


**Figure 3 (iii). The effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the presence of suction at $\left(\frac{U}{4\Omega l} = 1\right)$**

$(a)\ W = -1, N_1 = 4, \tau = 0.2, \dfrac{n}{\Omega} = 4$

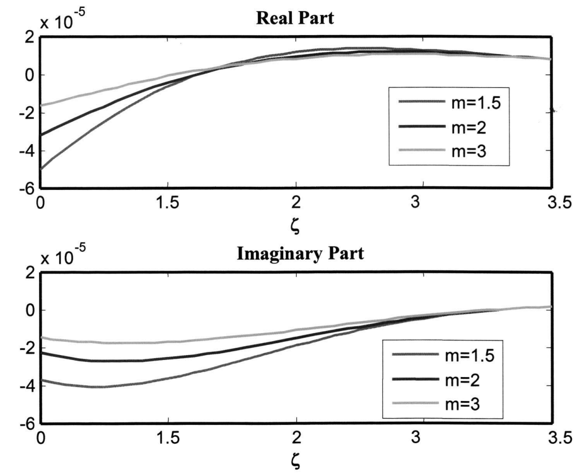


**Figure 4 (i). The effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the presence of blowing at $\left(\frac{U}{4\Omega l} = 1\right)$**

$(b)\ W = -1, N_1 = 4, \tau = 0.2, \dfrac{n}{\Omega} = -4$

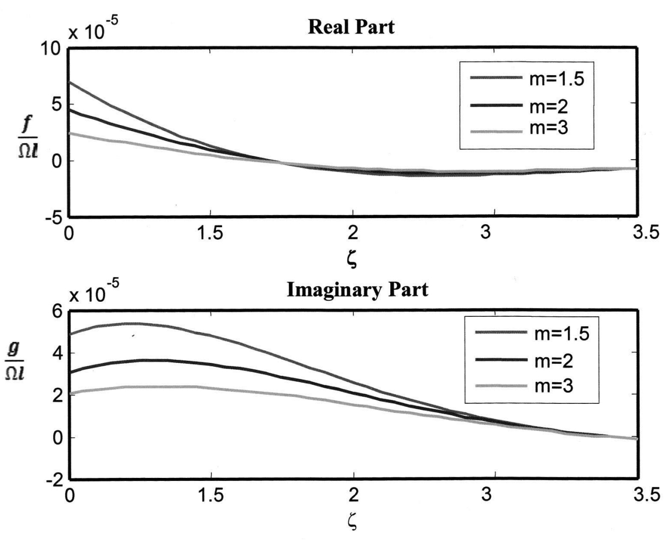


**Figure 4 (ii). The effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the presence of blowing at $\left(\frac{U}{4\Omega l} = 1\right)$**

$(c)\ W = -1, N_1 = 4, \tau = 0.2, \dfrac{n}{\Omega} = 1$

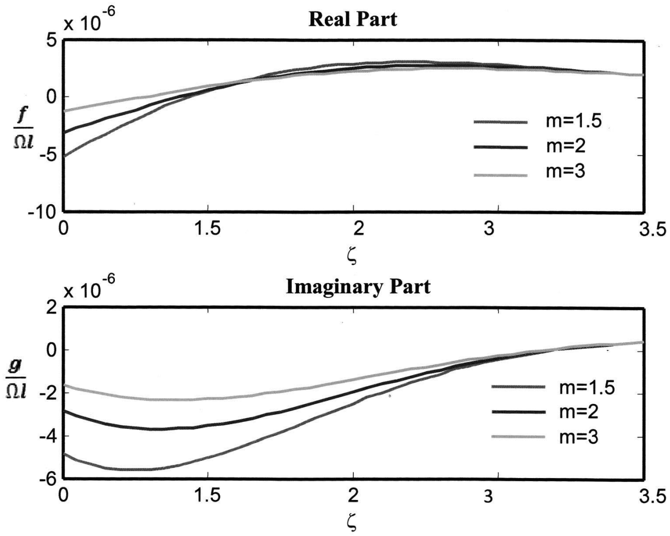


**Figure 4 (iii). The effect of Hall parameter on $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ for Cosine oscillation in the presence of blowing at**

$$\left(\frac{U}{4\Omega l} = 1\right)$$

# CONCLUSION

The effect of Hall Current on the viscous fluid flow due to rotation of an oscillating porous disk at infinity is examined. For the cosine oscillation, it is observed that, when the applied magnetic field is strong, both $f/\Omega l$ and $g/\Omega l$ depend strongly on the Hall parameter *m*.

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