### sg\*- CLOSED SETS AND sTa-SPACES IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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The aim of this paper is to introduce and study different properties of  $sg^*$ -closed sets and  $sT_a$ -spaces in intuitionistic fuzzy topological space.

**KEY WORDS :** Intuitionistic Fuzzy (IF) sets, IF semi closed sets, *sg*-closed sets, gs-closed sets,  $sg^*$ - closed sets and  $sT_a$ -separation axiom, etc.

# INTRODUCTION

Atanassov introduced the theory of intuitionistic fuzzy sets in 1983 [1, 2] and intuitionistic fuzzy topology by Coker in 1997 [5]. In this paper we introduce the concept intuitionistic fuzzy  $sg^*$ - closed sets. Intuitionistic fuzzy  $sT_a$ - space is also introduced with the help of intuitionistic fuzzy  $sg^*$ - closed sets. Throughout this paper we denote  $(X, \tau_1)$ ,  $(Y, \tau_2)$ (or simply X, Y) as intuitionistic fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated.

## Preliminaries

**Definition 2.1** [1, 2]. Let X be a nonempty fixed set. An intuitionistic fuzzy (IF for short) set A in X is given by a set of ordered triples  $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ , where  $\mu_A(x), \nu_A(x) : X \to [0, 1]$  are functions such that  $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$ . The numbers  $\mu_A(x)$  and  $\nu_A(x)$  represent the degree of membership and degree of non-membership for each element  $x \in X$  to  $A \subset X$ , respectively.

**Definition 2.2** [5]. Let  $\alpha, \beta \in [0, 1]$  and  $\alpha + \beta \le 1$ . An IF point  $x_{(\alpha, \beta)}$  in X is an IF set in X defined by

 $x_{(\alpha,\beta)}(y) = (\alpha,\beta), \text{ if } y = x \text{ and } (0,1) \text{ if } y \neq x.$ 

In this case, *x* is called the support of  $x_{(\alpha, \beta)}$  and  $\alpha$  and  $\beta$  are called the value and the non value of  $x_{(\alpha, \beta)}$ , respectively.

**Definition 2.3** [5]. Let  $x_{(\alpha,\beta)}$  be an IF point in X such that  $\alpha, \beta \in (0, 1)$  and  $A = \{ < x, \mu_A (x), \nu_A (x) > : x \in X \}$  be an IF set in X,  $x_{(\alpha,\beta)}$  is said to be properly contained in A  $(x_{(\alpha,\beta)} \in A \text{ for short})$  if and only. If  $\alpha < \mu_A (x)$  and  $\beta > \nu_A (x)$ .

**Definition 2.4.** An IF set A of an IF topological space  $(X, \tau)$  is said to be

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(a) IF semi closed if there exists a IF closed set U such that int  $(U) \subseteq A \subseteq U[6]$ .

(b) IF g-closed if cl  $(A) \subseteq O$  whenever  $A \subseteq O$  and O is an IF open set [8].

(c) IF gs-closed if scl  $(A) \subseteq O$  whenever  $A \subseteq O$  and O is an IF open set [7].

(d) IF sg-closed if scl  $(A) \subseteq O$  whenever  $A \subseteq O$  and O is an IF semi open set [9].

(e) IF  $g^*$ -closed set if cl (A)  $\subseteq O$  whenever  $A \subseteq O$  and O is an IF g-open set [3].

**Result 2.1.** Every IF closed (open) set is IF semi closed [6], IF-g closed [8] (respectively semi open, g-open) set but the converse may not be true [9].

**Definition 2.5** [10]. Two IF sets A and B in an IF topological space  $(X, \tau)$  are called q-separated if cl  $(A) \cap B = 0_{\sim} = A \cap cl (B)$ .

# IF sg<sup>\*</sup>-CLOSED SETS

In this section, the concept  $sg^*$ -closed set in IF topological space is introduced and its different properties are studied.

**Definition 3.1:** An IF set A of an IF topological space  $(X, \tau)$  is said to be an IF  $sg^*$ -closed set if scl  $(A) \subseteq O$  whenever  $A \subseteq O$  and O is an IF sg-open set.

Every IF semi closed set is IF  $sg^*$ - closed but the converse is not true as shown in following example.

**Example 3.1.** Let  $X = \{a, b\}$  be an non empty set and  $A = \langle x, (a/0.5, b/0.3), (a/0.5, b/0.7) \rangle$ ,  $B = \langle x, (a/0.6, b/0.4), (a/0.4, b/0.6) \rangle$  are two IF sets of X. Then the family  $\tau = \{0_{\sim}, 1_{\sim}, A, B\}$  is an IF topology on X. Then the IF set C defined by  $C = \langle x, (a/0.2, b/0.4), (a/0.8, b/0.6) \rangle$  is IF sg<sup>\*</sup> closed but is not IF semi closed as  $C \subseteq A^c$ , where  $A^c = \langle x, (a/.5, b/.7), (a/.5, b/.3) \rangle$  is IF semi closed but int  $(A^c) = A \not\subset C$ .

**Theorem 3.1.** Every IF *sg*<sup>\*</sup>- closed is IF semi *g*-closed.

**Proof:** Let A be an IF  $sg^*$ -closed set and O an IF open set in an IF topological space  $(X, \tau)$  such that  $A \subseteq O$ . Since O is IF open, O is IF g-open. A being  $g^*$ -closed set, Cl  $(A) \subseteq O$ , whenever  $A \subseteq O$  and O is an IF g-open. Hence, Cl  $(A) \subseteq O$ , whenever  $A \subseteq O$  and O is an IF g-open. Hence, Cl  $(A) \subseteq O$ , whenever  $A \subseteq O$  and O is an IF open set. Thus A is an IF semi g-closed set.

But the converse is not true as shown in the following example.

**Example 3.2.** Let  $X = \{a, b\}$  be an non empty set and  $A = \langle x, (a/0.4, b/0.5), (a/0.6, b/0.5) \rangle$ . Then the family  $\tau = \{0_{-1}, A\}$  is an IF topological on X. Then the IF set  $B = \langle x, (a/0.7, b/0.5), (a/0.3, b/0.5) \rangle$  is IF semi g-closed. But B is not IF sg<sup>\*</sup>-closed.

**Remark 3.1.** The relationship of IF  $sg^*$ -closed set with other types of closed sets are as follows



**Remark 3.2.** Union of two IF  $sg^*$ - closed sets may not be IF  $sg^*$ -closed which can be shown from the following example.

**Example 3.3.** In example 3.1 let  $D = \langle x, (a/0.4, b/0.2), (a/0.5, b/0.8) \rangle$ . Then D is also IF  $sg^*$ -closed. Now  $C \cup D = \langle x, (a/0.4, b/0.4), (a/0.5, b/0.6) \rangle$  is not IF semi  $g^*$ - closed. Since  $C \cup D \subseteq B$ , where B is an IF sg-open set but scl  $(C \cup D) \not\subset B$ .

**Theorem 3.2.** Union of two IF  $sg^*$ -closed set is IF  $sg^*$ -closed iff Union of two IF semi closed sets is IF  $sg^*$ - closed.

**Proof**: Let union of two IF  $sg^*$ -closed set is IF  $sg^*$ -closed, A and B are two IF semi closed sets then A and B are IF  $sg^*$ -closed and so  $A \cup B$  is IF  $sg^*$ -closed.

Conversely, let union of two IF semi closed sets is IF  $sg^*$ -closed set. Let A and B are two IF  $sg^*$ - closed sets such that scl (A) = C, scl (B) = D, where C, D are IF semi closed sets. Then scl  $(A) = C \subseteq O$ , scl  $(B) = D \subseteq O$  whenever  $A \subseteq O$ ,  $B \subseteq O$  and O is IF sg-open. Now  $C \cup D \subseteq O$ . Since  $C \cup D$  is IF  $sg^*$ -closed, therefore scl  $(C \cup D) \subseteq O$  and this is true for all IF sg-open sets O containing  $C \cup D$ . Hence the theorem is proved.

**Theorem 3.3.** Let A is IF  $sg^*$ -closed and  $A \subseteq B \subseteq scl(A)$ , then B is IF  $sg^*$ -closed.

**Proof**: Let  $B \subseteq O$  and O is IF sg-open. Since  $A \subseteq B$  so  $A \subseteq O$ , now since A is IF sg<sup>\*</sup>closed scl  $(A) \subseteq O$ . By hypothesis  $B \subseteq$  scl (A), so scl  $(B) \subseteq$  scl (A). Hence scl  $(B) \subseteq O$ . Thus Bis IF sg<sup>\*</sup>-closed.

**Definition 3.2.** An IF set A of an IF topological space  $(X, \tau)$  is said to be an IF  $sg^*$ - open set iff  $A^c$  is IF semi  $g^*$ -closed.

**Theorem 3.4.** An IF set A is IF  $sg^*$ -open iff  $F \subseteq sint(A)$ , whenever F is IF semi g-closed and  $F \subseteq A$ .

**Proof :** Let A is IF sg<sup>\*</sup>-open and F is IF semi g-closed such that  $F \subseteq A$ . Then  $A^c$  is IF sg<sup>\*</sup>closed and contained in the IF semi g-open set  $F^c$ . Since  $A^c$  is IF sg<sup>\*</sup>-closed therefore scl  $(A^c)$  $\subseteq F^c$ . Now scl  $(A^c) = (\text{sint } (A))^c$ , where sint (A) is IF semi-open and  $(\text{sint } (A))^c$  is IF semiclosed. Hence  $(\text{sint } (A))^c \subseteq F^c$ , *i.e.*  $F \subseteq \text{sint } (A)$ .

Conversely, if F is IF semi g-closed and  $F \subseteq$  sint (A), whenever  $F \subseteq A$ . It follows that  $A^c \subseteq F^c$  and  $(sint (A))^c = scl (A^c) \subseteq F^c$ , where  $F^c$  is IF semi g-open. Hence  $A^c$  is IF  $sg^*$ -closed and A is IF  $sg^*$ -open.

**Theorem 3.5.** Let A is IF  $sg^*$ -open and sint  $(A) \subseteq B \subseteq A$ , then B is IF  $sg^*$ -open.

**Proof**: Let  $F \subseteq B$  and F is IF sg-closed. Since  $B \subseteq A$  so  $F \subseteq B$ , now since A is IF sg<sup>\*</sup>open  $F \subseteq \text{sint}(A)$ . By hypothesis sint  $(A) \subseteq B$ , so  $F \subseteq \text{sint}(A) \subseteq \text{sint}(B)$ . Thus B is IF sg<sup>\*</sup>open.

**Definition 3.3.** Two IF sets A and B in an IF topological space  $(X, \tau)$  are called semi separated if scl  $(A) \cap B = 0_{\tau} = A \cap \text{scl}(B)$ .

**Theorem 3.6.** Union of two semi separated IF sg<sup>\*</sup>-open sets is IF sg<sup>\*</sup>-open.

**Proof**: Let A and B be semi separated IF  $sg^*$ -open sets. Then we have scl  $(A) \cap B = 0$ ~ =  $A \cap$  scl (B). If F is an IF semi closed set such that  $F \subseteq A \cup B$ , then  $F \cap$  scl  $(A) = (A \cup B) \cap$ scl  $(A) = (A \cap$  scl  $(A)) \cup (B \cap$  scl  $(A)) = A \cup 0$ ~ = A. Similarly  $F \cap$  scl (B) = B. Now by theorem 3.4  $F \cap$  scl (A) = sint (A) and  $F \cap$  scl (B) = sint (B). Hence  $F = F \cap (A \cup B)$  =  $(F \cap A) \cup (F \cap B) \subseteq (F \cap \text{scl}(A)) \cup (F \cap \text{scl}(B)) \subseteq \text{sint}(A) \cup \text{sint}(B) \subseteq \text{sint}(A \cup B)$ . Hence by theorem 3.4  $A \cup B$  is IF semi  $g^*$ -open.

**Theorem 3.7.** An IF set A is IF  $sg^*$ -closed iff scl  $(A) \cap A^c$  does not contain any non-null IF sg-closed set.

**Proof :** Let A is IF  $sg^*$ -closed and F is IF semi g-closed such that  $F \subseteq (scl(A) \cap A^c)$ , then  $F^c$  is IF semi g-open and  $A \subseteq F^c$ , it follows from definition 3.1 that  $scl(A) \subseteq F^c$ , implies  $F \subseteq (scl(A))^c$ , So  $F \subseteq (((scl(A))^c \cap (scl(A) \cap A^c)) = 0_{\sim}$ .

Conversely, let the given condition be satisfied. Let  $A \subseteq O$ , where O is IF sg-open set. If scl (A) is not an IF sub set of O, then scl (A)  $\cap O = 0_{\sim}$ . But scl (A)  $\cap O^c$  is a non-null IF sg-closed set contained in scl (A)  $\cap A^c$ , a contradiction. So A is IF sg<sup>\*</sup>-closed.

**Theorem 3.8.** An IF set A is IF  $sg^*$ -closed iff scl  $(A) \cap A^c$  is IF  $sg^*$ -open.

**Proof**: Let A is IF  $sg^*$ -closed and F is IF semi g-closed such that  $F \subseteq (scl(A) \cap A^c)$ , then by theorem 3.7  $F = 0_{\sim}$ . Hence  $F \subseteq sint(scl(A) \cap A^c)$  and by theorem 3.4 scl(A)  $\cap A^c$  is IF  $sg^*$ -open.

Conversely, let scl  $(A) \cap A^c$  be IF  $sg^*$ -open and  $A \subseteq O$ , where O is IF sg-open set. Then  $O^c \subseteq A^c$  and  $(scl (A)) \cap O^c \subseteq (scl (A)) \cap A^c$ . Thus is an IF sg-closed sub set of scl  $(A) \cap A^c$ , since  $scl (A) \cap A^c$  is IF  $sg^*$ -open, therefore by theorem 3.4  $(scl (A) \cap O^c) \subseteq sint (scl (A) \cap A^c)$ = 0... This is possible if  $scl (A) \subseteq O$ . Thus A is IF  $sg^*$ -closed.

**Notation 3.1.** Let  $(X, \tau)$  be an IF topological space and IFSGC (X) (respectively IFSGO (X)) be the family of all IF semi *g*-closed (respectively IF semi *g*-open) sets of *X*.

**Theorem 3.9.** In an IF topological space  $(X, \tau)$  if IFSGC (X) = IFSGO (X) and every IF sg-closed is IF semi closed then every IF subset of X is IF sg<sup>\*</sup>-closed.

**Proof**: Let A is an IF sub set of X and  $A \subseteq O$  and O is an IF sg-open. Since IFSGC (X) = IFSGO (X), O is IF sg-closed also. According to hypothesis O is IF semi closed. Hence scl  $(A) \subseteq$  scl (O) = O and A is an IF sg<sup>\*</sup>-closed set.

**Theorem 3.10.** In an IF topological space  $(X, \tau)$  if every IF subset of X is IF  $sg^*$ -closed then IFSGC (X) = IFSGO(X).

**Proof**: Suppose that every IF subset of X is IF  $sg^*$ -closed. Let  $A \in IFSGO(X)$ , now since every IF  $sg^*$ -closed set is IF semi g-closed set,  $A \in IFSGC(X)$ . Thus IFSGO  $(X) \subseteq IFSGC(X)$ . Again if  $A \in IFSGC(X)$  then  $A^c \in IFSGO(X) \subseteq IFSGC(X)$  and hence  $A \in IFSGO(X)$ . Consequently, IFSGC  $(X) \subseteq IFSGO(X)$ . Hence IFSGO (X) = IFSGC(X).

### F SEMI T<sub>a</sub>- SPACES

In this section, a new notion, called IF semi  $T_a$  – space with the help of IF  $sg^*$ -closed sets is defined and some of its properties are studied.

**Definition 4.1.** An IF topological space  $(X, \tau)$  is said to be an IF semi  $T_a$ - space iff every IF  $sg^*$ -closed set is IF semi closed.

**Theorem 4.1.** For every IF point  $x_{(\alpha, \beta)}$  in X, either  $x_{(\alpha, \beta)}$  is IF semi g-closed or its complement  $\{x_{(\alpha, \beta)}\}^c$  is  $sg^*$ -closed in IF topological space  $(X, \tau)$ .

**Proof :** Let  $x_{(\alpha,\beta)}$  is not IF semi *g*-closed in  $(X, \tau)$ . Then  $\{x_{(\alpha,\beta)}\}^c$  is not IF semi *g*-open and  $1_{\sim}$  is the only IF semi *g*-open set containing  $\{x_{(\alpha,\beta)}\}^c$ . Therefore scl $\{x_{(\alpha,\beta)}\}^c \subseteq 1_{\sim}$  holds and so  $\{x_{(\alpha,\beta)}\}^c$  is IF semi  $g^*$ -closed.

**Definition 4.2.** An IF semi  $g^*$ -closure operator of an IF set A in an IF topological space  $(X, \tau)$  is defined as scl<sup>\*</sup> $(A) = \cap \{F : A \subseteq F, F \text{ is IF } sg^*\text{-closed in } X\}.$ 

If A is IF  $sg^*$ -closed set then  $scl^*(A) = A$ .

**Theorem 4.2.** In an IF topological space  $(X, \tau)$ , if  $\{x_{(\alpha, \beta)}\} \neq \{y_{(\chi, \delta)}\}$  then scl<sup>\*</sup>  $\{x_{(\alpha, \beta)}\} \neq scl^* \{y_{(\chi, \delta)}\}$ .

**Proof :** By theorem 4.1 in the IF topological space  $(X, \tau)$  IF point  $\{x_{(\alpha, \beta)}\}$  is either IF semi *g*-closed or its complement  $\{x_{(\alpha, \beta)}\}^c$  is IF *sg*<sup>\*</sup>-closed. Hence the proof will be complete if we consider the following two cases.

If IF point  $\{x_{(\alpha, \beta)}\}$  is IF semi g-closed then  $\{x_{(\alpha, \beta)}\}$  is also IF  $sg^*$ -closed, since an IF semi g-closed set is an IF  $sg^*$ -closed set. Hence scl<sup>\*</sup>  $\{x_{(\alpha, \beta)}\} = \{x_{(\alpha, \beta)}\}$ . Now  $\{y_{(\chi, \delta)}\} \notin \{x_{(\alpha, \beta)}\}$ , therefore scl<sup>\*</sup>  $\{x_{(\alpha, \beta)}\} \neq$  scl<sup>\*</sup>  $\{y_{(\chi, \delta)}\}$ .

Now let  $\{x_{(\alpha, \beta)}\}^c$  be IF semi  $g^*$ -closed. Since  $\{y_{(\chi, \delta)}\} \subset \{x_{(\alpha, \beta)}\}^c$ ,  $\{y_{(\chi, \delta)}\} \in \operatorname{scl}^* \{y_{(\chi, \delta)}\} \subset \{x_{(\alpha, \beta)}\}^c$  and hence  $\operatorname{scl}^* \{x_{(\alpha, \beta)}\} \neq \operatorname{scl}^* \{y_{(\chi, \delta)}\}$ .

By IFSO ( $\tau$ ) we mean the collection of all IF semi open sets in the space (X,  $\tau$ ) and similarly, we define IFSO<sup>\*</sup>( $\tau$ ) = {A : scl<sup>\*</sup>(A<sup>c</sup>) = A<sup>c</sup>}.

**Theorem 4.3.** In an IF topological space  $(X, \tau)$ , IFSO  $(\tau) \subset$  IFSO<sup>\*</sup> $(\tau)$ .

**Proof**: Let  $A \in \text{IFSO}(\tau)$ . Then  $A^c$  is IF semi closed and  $A^c = \text{scl}(A^c)$ . Since  $A^c$  is IF semi closed, therefore  $A^c$  is also IF  $sg^*$ -closed, so  $\text{scl}^*(A^c) = A^c$ . Hence  $A \in \text{IFSO}^*(\tau)$ .

**Theorem 4.4.** An IF topological space  $(X, \tau)$  is IF semi  $T_a$ -space iff IFSO  $(\tau) = \text{IFSO}^*(\tau)$ .

**Proof**: Let  $(X, \tau)$  be an IF semi  $T_a$ -space. Then scl  $(A) = \text{scl}^*(A)$  holds for every IF subset A, since IF semi closed sets and IF  $sg^*$ -closed sets coincide in IF semi  $T_a$ - space. Therefore we have IFSO  $(\tau) = \text{IFSO}^*(\tau)$ .

Conversely, let A be IF sg<sup>\*</sup>-closed set of  $(X, \tau)$ . Then  $A = \operatorname{scl}^*(A)$  and hence  $A^c \in \operatorname{IFSO}(\tau)$ . Thus A is IF semi closed. Therefore  $(X, \tau)$  is IF semi  $T_a$ -space.

**Theorem 4.5.** An IF topological space  $(X, \tau)$  is IF semi  $T_a$ - space iff for each  $x_{(\alpha, \beta)} \in X$ ,

 $\{x_{(\alpha,\beta)}\}$  is IF semi open or IF semi *g*-closed.

**Proof**: Let  $x_{(\alpha, \beta)} \in X$  and  $\{x_{(\alpha, \beta)}\}$  is not IF semi *g*-closed. Then  $\{x_{(\alpha, \beta)}\}^c$  is not IF semi *g*-open. This implies that  $1_{\sim}$  is the only IF semi *g*-open set containing  $\{x_{(\alpha, \beta)}\}^c$ . scl  $\{x_{(\alpha, \beta)}\}^c \subseteq 1_{\sim}$ . So  $\{x_{(\alpha, \beta)}\}^c$  is IF *sg*<sup>\*</sup>-closed set of  $(X, \tau)$ . Since  $(X, \tau)$  is IF semi *T<sub>a</sub>*-space,  $\{x_{(\alpha, \beta)}\}^c$  is IF semi closed. Therefore  $\{x_{(\alpha, \beta)}\}$  is IF semi open.

For the converse part it is enough to prove that IFSO<sup>\*</sup> ( $\tau$ )  $\subset$  IFSO ( $\tau$ ). Let  $A^c \in$  IFSO<sup>\*</sup> ( $\tau$ ) and  $A \notin$  IFSO ( $\tau$ ). Then scl<sup>\*</sup> ( $A^c$ ) =  $A^c$  and scl ( $A^c$ )  $\neq A^c$ . Then there exist a point  $x_{(\alpha,\beta)}$  of X such that  $x_{(\alpha,\beta)} \in$  scl ( $A^c$ ) and  $x_{(\alpha,\beta)} \notin$  scl<sup>\*</sup> ( $A^c$ ) =  $A^c$ . Since  $x_{(\alpha,\beta)} \notin$  scl<sup>\*</sup> ( $A^c$ ) there exists an IF semi  $g^*$ -closed set F such that  $x_{(\alpha,\beta)} \notin F$  and  $A^c \subset F$ . By the hypothesis, { $x_{(\alpha,\beta)}$ } is IF semi open or IF semi g-closed. **Case I.** Let  $\{x_{(\alpha,\beta)}\}$  is IF semi open. Since  $\{x_{(\alpha,\beta)}\}^c$  is IF semi closed and  $A^c \subset \{x_{(\alpha,\beta)}\}^c$ , we have scl  $(A^c) \subset \{x_{(\alpha,\beta)}\}^c$ , *i.e.*  $x_{(\alpha,\beta)} \notin$  scl  $(A^c)$ . This contradicts the fact that  $x_{(\alpha,\beta)} \in$  scl  $(A^c)$ . Therefore  $A \in$  IFSO ( $\tau$ ).

**Case II.** Let  $\{x_{(\alpha,\beta)}\}$  is IF semi *g*-closed. Since  $\{x_{(\alpha,\beta)}\}^c$  is IF semi *g*-open set containing the IF  $sg^*$ -closed set  $F \supset A^c$ , we have  $\{x_{(\alpha,\beta)}\}^c \supset \text{scl}(F) \supset \text{scl}(A^c)$ . Therefore  $\{x_{(\alpha,\beta)}\} \notin \text{scl}(A^c)$ . This is a contradiction. Hence  $A \in \text{IFSO}(\tau)$ . Hence in both the cases  $A \in \text{IFSO}(\tau)$ . Hence IFSO<sup>\*</sup>  $(\tau) \subset \text{IFSO}(\tau)$ .

**Definition 4.3.** An IF topological space  $(X, \tau)$  is said to be an IF semi  $T_0$ - space iff for any pair of distinct points  $x_{(\alpha, \beta)}$  and  $y_{(\chi, \delta)}$  in X, either  $x_{(\alpha, \beta)} \notin \text{scl} \{y_{(\chi, \delta)}\}$  or  $y_{(\chi, \delta)} \notin \text{scl} \{x_{(\alpha, \beta)}\}$ , *i.e.* scl  $\{x_{(\alpha, \beta)}\} \neq \text{scl} \{y_{(\chi, \delta)}\}$ .

**Theorem 4.6.** Every IF semi  $T_a$ - space is IF semi  $T_0$ - space.

**Proof**: Let  $(X, \tau)$  be an IF semi  $T_a$ - space but not IF semi  $T_0$ -space. Then there exists two distinct points  $x_{(\alpha, \beta)}$  and  $y_{(\chi, \delta)}$  in X, such that scl  $\{x_{(\alpha, \beta)}\} = \text{scl }\{y_{(\chi, \delta)}\}$ . Let  $A = \{x_{(\alpha, \beta)}\}^c$ . Clearly  $\{x_{(\alpha, \beta)}\}$  is not IF semi closed, otherwise scl  $\{x_{(\alpha, \beta)}\} = \{x_{(\alpha, \beta)}\} \neq \text{scl }\{y_{(\chi, \delta)}\}$ . By the theorem 4.1, A is IF sg<sup>\*</sup>-closed. But A is not IF semi closed, otherwise  $y_{(\chi, \delta)} \in \{x_{(\alpha, \beta)}\}^c = A$ implies scl  $\{y_{(\chi, \delta)}\} \subseteq \{x_{(\alpha, \beta)}\}^c$  and scl  $\{x_{(\alpha, \beta)}\} \neq \text{scl }\{y_{(\chi, \delta)}\}$ , contradicting our hypothesis. Hence  $(X, \tau)$  is IF semi  $T_0$ - space.

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## References

- 1. Atanasov, K., Intuituioistic Fuzzy Sets, VII, ITKR's Session, Sofia, Bulgaria (1983).
- 2. Atanasov, K., Intuituioistic Fuzzy Sets, Fuzzy Sets and Systems, 20, 87-96 (1986).
- Bhattacharjee, A. and Bhaumik, R.N., g<sup>\*</sup>-Closed Sets and T<sub>a</sub> Separation Axiom in Intuitionistic Fuzzy Topological Spaces, *International Journal of Innovative Science, Engineering & Technology*, 2 (5), 851-855 (May 2015).
- 4. Coker, D., An introduction to IFT spaces, Fuzzy Sets and Systems, 88(1), 81-89 (1997).
- 5. Coker, D. and Demirci, M., On IF points, Notes on IFS, 1(2), 79-84 (1995).
- Gurcay, H., Coker, D. and Es, A.H., On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math., Vol. 5, No. 2, 365-378 (1997).
- 7. Sakthivel, K., Intuituioistic fuzzy Alpha generalized continuous mappings and intuitionistic fuzzy Alpha generalized irresolute mappings, *Applied Mathematical Sciences*, **4(37)**, 1831-1842 (2010).
- 8. Thakur, S.S. and Chaturvedi, Rekha, Genaralised Closed sets in intuitionistic fuzzy topology, *The Journal of Fuzzy Mathematics*, **16(3)**, 559-572 (2008).
- Thakur, S.S. and Bajpai, J.P., Intuitionistic fuzzy W-closed sets and Intuitionistic fuzzy W-continuity In Intuitionistic Fuzzy Topological Spaces, *International Journal of Contemporary* Advanced Mathematics, 1(1), 1-15 (2011).
- 10. Turnali, N. and Coker, D., Fuzzy Connected ness in Intuitionistic Fuzzy Topological Spaces, *Fuzzy* Sets and Systems, **116 (3)**, 369-375 (2000).