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A VACATION MODELS (M/G/1) (G, MV) WITH NON- EXHAUSTIVE SERVICE

SANDEEP DIXIT

Department of Mathematics, V.S.S.D. College, Kanpur (U.P.), India

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In this paper we discuss a gated service system with multiple vacations, when the server returns from a vacation, it accepts and serves only those customers present in the system. If no customers are in the system, the server starts another vacation and keeps taking vacation until it finds some customers waiting in the system. This system, called a gated service multiple vacation model.

KEYWORDS : Gated Service, Multiple vacation, Single Server.

INTRODUCTION

Let W_q be the number of customers present in the system at the starting of a service period, and Let S_q be the length of service period. It is clear that a number of customers arriving during S_q and the number of customers arriving during V are independent.

For convenience in analysis, we assumed that there is always a zero-length service period between two consecutive vacations. Therefore the service period and the vacation occurs alternatively. The number of customers in the system at the starting of a zero-length service period W_q is 0.

Lemma 1: In a steady state ($M/G/1$) (G, MV), the *p.g.f.* $S_q(Z)$ of satisfies the equation

$$W_q(Z) = W_q\{B^*\lambda(1-Z)\}\, V^*(\lambda(1-Z)) \qquad \ldots (1)$$

and expected value L_q is

$$E(W_q) = \frac{\lambda\, E(V)}{(1-\ell)} \qquad \ldots (2)$$

Proof : Let $W_q^{(n)}$ and $L_q^{(n)}$ be the number of customers present at the starting of the n^{th} service period and the length of the n^{th} service period, respectively. By gated service rule, $W_q^{(n+1)}$ equal to the sum of the number of customers arriving during $S_q^{(n)}$, therefore

$$W_q^{(n+1)}(Z) = S_q^{(n)*}\left[\lambda\left(1-Z\right)\right] V^*\left[\lambda\left(1-Z\right)\right] \qquad \ldots (3)$$

Due to the gated service rule, the number of customers served in a service period is equal to the number of customers present at the starting of the service period, thus we have

$$S_q^{(n)*}(s) = W_q^{(n)}\left[B^*(s)\right] \qquad \ldots (4)$$

from equation (3) & (4) we have

$$W_q^{(n+1)}(Z) = W_q^{(n)}\left[B^*\left(\lambda\,(1-Z)\right)\right] V^*\left(\lambda\,(1-Z)\right)$$

Because it is easy to see that in steady state, the p.g.f. does not depend on n. Taking the derivative equation (1) both side with respect to Z. at $Z = 1$.

$$E(W_q) = W_q\left[B^*\left(\lambda\,(1-Z)\right)\right] V^*(-\lambda) + V^*\left(\lambda\,(1-\lambda)\right) q_E\, /(Z)$$

Lemma 2 : For $\ell < 1$, in an ($M/G/1$) (G, MV) system, the stationary queue length L_q can be decomposed into the sum of three independence random variables

$$L_q = L + L_i + L_j \tag{4}$$

where L is the queue length of a classical $M/G/1$ queue without vacations and L_i and L_j is the additional queue length due to vacation effect, with *P.g.f.* are given by

$$L_i(Z) = \frac{1 - V^*\left[\lambda(1-Z)\right]}{\lambda E(V)(1-Z)}, \qquad L_j(Z) = W_q\left[B^*\left(\lambda(1-Z)\right)\right] \qquad \ldots(5)$$

Proof : From the gated service rule that the number of customers served during a service period ϕ equals W_q. Let $\underline{L_n}$ be the number of customers at the n^{th} customers departure instant in this service period. We have

$$L_n = W_q - n + \sum_{k=1}^{n} A_K, \quad n = 1, 2, 3........ W_q$$

where A_K is the number of customers arriving during the K^{th} customer service. A_k's are i.i.d. random variable with the p.g.f. $A(Z) = B^*\left[\lambda(1-Z)\right]$. . Then

$$E\left\{\sum_{n=1}^{\phi} Z^{L_n}\right\} = E\left[\sum_{n=1}^{W_q} Z^{W_q - n}\{B^*\left(\lambda(1-Z)\right)\}^n\right]$$

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