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HARMONIC MEAN LABELING OF H-SUPER SUBDIVISION OF CYCLE GRAPHS

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A graph *G* with *p* vertex node and *q* edges is called a harmonic mean graph if it is possible to label the vertex node $x \in V$ with distinct labels *f*(*x*) from {1,2,…..*q*+1} in such a way that each edge *e* = *uv* is labeled with $f(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove that some families of graphs such as H- super subdivision of cycle HSS(C_n), HSS($C_n \odot K_1$), HSS($C_n \odot \overline{K_2}$), HSS($C_n \odot K_2$) are harmonic mean graphs.

Harmonic mean graph,H- super subdivision of cycle HSS(C_n),HSS($C_n \odot K_1$), HSS($C_n \odot \overline{K_2}$), HSS($C_n \odot K_2$)

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INTRODUCTION

Let G=(V,E) be a (p,q) graph with p = |V(G)| vertices and q =|E(G)| edges, where V(G) and E(G) respectively denote the vertex set and edge set of the graph G. In this paper, we consider the graphs which are simple, finite and undirected. For graph theoretic terminology and notations we refer to S. Arumugam [1]

The concept of graph labeling was introduced by Rosa in 1967. A detailed survey of graph labeling is available in Gallian [4]. The concept of Harmonic mean labeling of graph was introduced by S.Somasundaram, R.Ponraj and S.S.Sandhya and they investigated the existence of harmonic mean labeling of several family of graphs such as this concept was then studied by several authors. We have proved Harmonic mean labeling of subdivision graphs such as $P_n \odot K_1$, $P_n \odot \overline{K_2}$, H-graph,crown, $C_n \odot K_1$,$C_n \odot \overline{K_2}$, quadrilateral snake, Triangular snake and also proved Harmonic mean labeling of some graphs such as Triple triangular snake T(T_n), Alternate Triple triangular snake A[T(T_n)], Triple quadrilateral snake T(Q_n), Alternate Triple quadrilateral snake A[T(Q_n)], Twig graph T(n), balloon triangular snake $T_n(C_m)$, and key graph Ky(m, n). The following definitions are useful for the present investigation.

Definition 1.1 [8]

A Graph $G = (V, E)$ with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertex node $v \in V$ with distinct labels $f(v)$ from $\{1,2,\ldots,q+1\}$ in such a way that when each edge $e = uv$ is labeled with $f(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition 1.2 [2]

Let G be a (p,q) graph. A graph obtained from G by replacing each line e_i by a H-graph in such a way that the ends e_i are merged with a pendent vertex in P_2 and a pendent vertex $P_2{}'$ is called H-Super Subdivision of G and it is denoted by HSS(G) where the H-graph is a tree on 6 vertices in which exactly two vertices of degree 3.

Definition 1.4 [2]

A closed path is said to be cycle and cycle of length n is denoted by C_n

In this paper we prove that H- super subdivision of cycle HSS(C_n), HSS($C_n \odot K_1$), HSS($C_n \odot \overline{K_2}$), HSS($C_n \odot K_2$) are harmonic mean graphs.

II. Harmonic mean labeling of graphs

Theorem 2.1

The H- super subdivision of cycle HSS(C_n) is a harmonic mean graphs

Proof: Let HSS(C_n), $n \geq 3$ be the H- super subdivision of cycle graph whose vertex set

V (G) $= \{ u_i, v_i, x_i, y_i, w_i \,/ 1 \leq i \leq n-1\} \cup \{ u_n, v_n, x_n, y_n, w_n\}$ and the edge set

E (G) $= \{u_iv_i, v_ix_i, y_iw_i, v_iw_i, w_iu_{i+1} \,/\, 1 \leq i \leq n-1\} \cup \{u_nv_n, v_nx_n, y_nw_n, v_nw_n, w_nu_1\}$.

Define a distinct labels f : $V(G) \rightarrow \{1,2,\ldots,q+1\}$ by

$$f(u_1) = 3$$
$$f(u_i) = 5i-4 \quad \text{for } 2 \leq i \leq n$$
$$f(v_i) = 5i-3 \quad \text{for } 1 \leq i \leq n$$
$$f(w_i) = 5i \quad \text{for } 1 \leq i \leq n$$
$$f(x_1) = 1$$
$$f(x_i) = 5i-2 \quad \text{for } 2 \leq i \leq n$$
$$f(v_i) = 5i-1 \quad \text{for } 1 \leq i \leq n$$

Then the resulting edge labels are distinct.

$$f(x_1v_1) = 1$$
$$f(x_iv_i) = 5i-2 \quad \text{for } 2 \leq i \leq n$$

$f(y_1w_1)$ = 4

$f(y_iw_i)$ = 5i for $2\leq i \leq n$

$f(v_iu_i)$ = 5i – 3 for $1\leq i \leq n$

$f(w_1u_2)$ = 5

$f(w_iu_{i+1})$ = 5i + 1 for $2\leq i \leq n$

$f(w_nu_1)$ = 6

$f(v_1w_1)$ = 3

$f(v_iw_i)$ = 5i –1 for $2\leq i \leq n$

Thus f provides a harmonic mean labeling of graph *G*.

Hence G is a harmonic mean graph.

Example 2.1.1 : A harmonic mean labeling of graph *G* obtained by H- super subdivision of cycle HSS(C_7) are given in fig 2.1.1

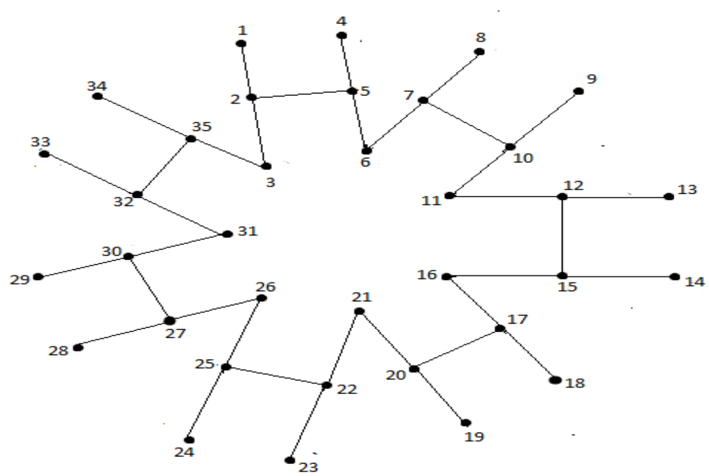

fig. 2.1.1.

Theorem 2.2 : The H- super subdivision of cycle HSS ($C_n \odot K_1$) is a harmonic mean graph.

Proof: Let HSS($C_n \odot K_1$), n ≥ 3 be the H- super subdivision of cycle graph whose vertex set

$V(G)$ ={ $u_i, v_i, x_i, \mathrm{y_i}, w_i$ /1≤ $i \leq n$ -1} ∪ { $u_n, v_n, x_n, \mathrm{y_n}, w_n$ } ∪ { u_i, z_i/1≤ $i \leq n$}and the edge set

$E(G)$ = {u_iv_i,v_ix_i,y_iw_i, w_iv_i,w_iu_{i+1} / 1≤ $i \leq n$-1}∪ {u_nv_n,v_nx_n,y_nw_n,w_nv_n,w_nu_1}
∪ { u_i, z_i/1≤ $i \leq n$}.

Define a distinct labels $f: V(G) \rightarrow \{1,2,\ldots, q+1\}$ by

$f(u_1)$ = 4

$f(u_i)$ = 6i – 5 for $2 \leq i \leq n$

$f(v_1)$ = 3

$f(v_i)$ = 6i – 2 for $2 \leq i \leq n$

$f(w_i)$ = 6i for $1 \leq i \leq n$

$f(x_1)$ = 2

$f(x_i)$ = 6i – 3 for $2 \leq i \leq n$

$f(y_i)$ = 6i – 1 for $1 \leq i \leq n$

$f(z_1)$ = 1

$f(z_i)$ = 6i – 4 for $2 \leq i \leq n$

Then the resulting edge labels are distinct.

$f(u_i v_i)$ = 6i – 3 for $1 \leq i \leq n$

$f(w_1 u_2)$ = 6

$f(w_i u_{i+1})$ = 6i + 1 for $2 \leq i \leq n$

$f(w_n u_1)$ = 7

$f(v_1 x_1)$ = 2

$f(v_i x_i)$ = 6i – 2 for $2 \leq i \leq n$

$f(w_1 y_1)$ = 5

$f(w_i y_i)$ = 6i for $2 \leq i \leq n$

$f(v_1 w_1)$ = 4

$f(v_i w_i)$ = 6i – 1 for $2 \leq i \leq n$

$f(u_1 z_1)$ = 1

$f(u_i z_i)$ = 6i – 4 for $2 \leq i \leq n$

Thus f provides a harmonic mean labeling of graph *G*.

Hence *G* is a harmonic mean graph.

Example 2.2.1 : A harmonic mean labeling of graph *G* obtained by *H*- super subdivision of cycle HSS($C_6 \odot K_1$) are given in fig 2.2.1

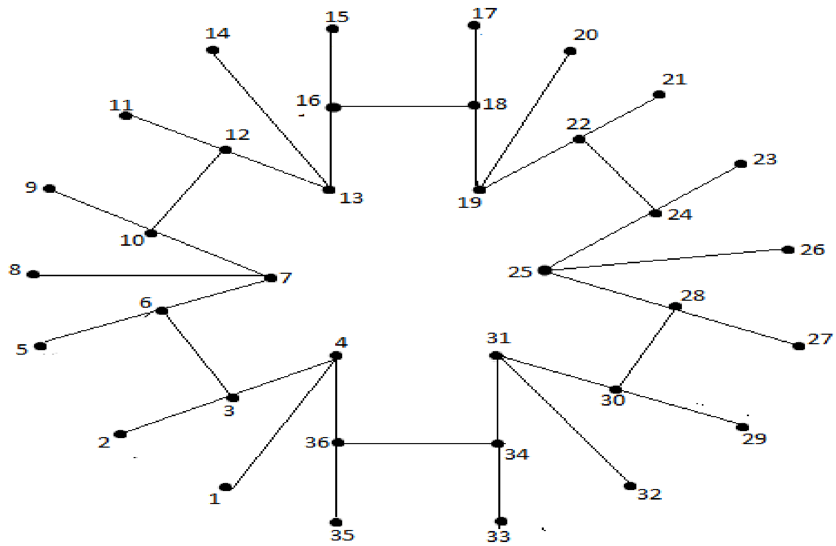

Fig 2.2.1

Theorem 2.3 : The *H*- super subdivision of cycle HSS($C_n \odot \overline{K_2}$) is a harmonic mean graph.

Proof: Let HSS($C_n \odot \overline{K_2}$), $n \geq 3$ be the H- super subdivision of cycle graph whose vertex set

$V(G) = \{ u_i, v_i, x_i, y_i, w_i \ /1 \leq i \leq n\ \text{-}1\} \cup \{ u_n, v_n, x_n, y_n, w_n \} \cup \{u_i, s_i, t_i/1 \leq i \leq n\}$and the edge set

$E(G) = \{u_iv_i, v_ix_i, y_iw_i, w_iv_i, w_iu_{i+1} \ / \ 1 \leq i \leq n\text{-}1\} \cup \{u_nv_n, v_nx_n, y_nw_n, w_nv_n, w_nu_1\} \cup \{ u_is_i, u_it_i/1 \leq i \leq n\}$.

Define a distinct labels $f: V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(u_i)$ = 7i – 4 for $1 \leq i \leq n$

$f(v_i)$ = 7i–3 for $1 \leq i \leq n$

$f(w_i)$ = 7i for $1 \leq i \leq n$

$f(x_i)$ = 7i-2 for $1 \leq i \leq n$

$f(y_i)$ = 7i –1 for $1 \leq i \leq n$

$f(s_i)$ = 7i –6 for $1 \leq i \leq n$

$f(t_i)$ = 7i –5 for $1 \leq i \leq n$

Then the resulting edge labels are distinct.

$f(u_1v_1)$ = 3

$f(u_iv_i)$ = 7i –3for $2 \leq i \leq n$

$f(w_iu_{i+1})$ = 7i+1 for $1 \leq i \leq n$

$f(w_n u_1)$ = 6

$f(v_1 w_1)$ = 5

$f(v_i w_i)$ = 7i –1 for $2\leq i \leq n$

$f(v_1 x_1)$ = 4

$f(v_i x_i)$ = 7i –2 for $2\leq i \leq n$

$f(w_i y_i)$ = 7i for $1\leq i \leq n$

$f(u_1 s_1)$ = 1

$f(u_i s_i)$ = 7i –5 for $2\leq i \leq n$

$f(u_1 t_1)$ = 2

$f(u_i t_i)$ = 7i –4 for $2\leq i \leq n$

Thus f provides a harmonic mean labeling of graph *G*.

Hence *G* is a harmonic mean graph.

Example 2.3.1 : A harmonic mean labeling of graph *G* obtained by H- super subdivision of cycle HSS($C_6 \odot \overline{K_2}$) are given in fig 2.3.1

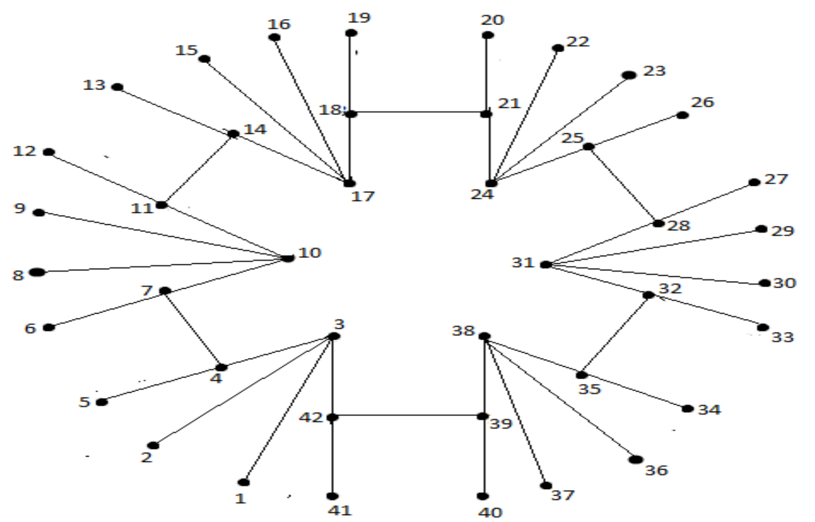

Fig 2.3.1

Theorem:2.4 : The *H*-super subdivision of cycle HSS($C_n \odot K_2$) is a harmonic mean graph.

Proof: Let HSS($C_n \odot K_2$), n ≥ 3 be the *H*- super subdivision of cycle graph whose vertex set

$V(G)$ ={ u_i, v_i, x_i, y_i, w_i /1≤ $i \leq n$ -1} ∪ { u_n, v_n, x_n, y_n, w_n } ∪ {u_i, s_i, t_i/1≤ $i \leq n$}and the edge set

E (G) = $\{u_i v_i, v_i x_i, y_i w_i, w_i v_i, w_i u_{i+1}$ / 1$\leq i \leq n$-1$\} \cup \{u_n v_n, v_n x_n, y_n w_n, w_n v_n, w_n u_1\}$ $\cup$ $\{ u_i s_i, s_i t_i, t_i u_i$/1$\leq i \leq n\}$.

Define a distinct labels f: $V(G) \rightarrow \{1,2,\ldots,q+1\}$ by

$f(u_i)$ = 8i – 4 for $1\leq i \leq n$

$f(v_i)$ = 8i – 3 for $1\leq i \leq n$

$f(w_i)$ = 8i for $1\leq i \leq n$

$f(x_i)$ = 8i – 2 for $1\leq i \leq n$

$f(y_i)$ = 8i – 1 for $1\leq i \leq n$

$f(s_1)$ = 1

$f(s_i)$ = 8i – 5 for $2\leq i \leq n$

$f(t_i)$ = 8i – 6 for $1\leq i \leq n$

Then the resulting edge labels are distinct.

$f(u_1 v_1)$ = 4

$f(u_i v_i)$ = 8i – 3 for $2\leq i \leq n$

$f(w_i u_{i+1})$ = 8i + 1 for $1\leq i \leq n$

$f(w_n u_1)$ = 8

$f(v_1 x_1)$ = 5

$f(v_i x_i)$ = 8i – 2 for $2\leq i \leq n$

$f(w_1 y_1)$ = 7

$f(w_i y_i)$ = 8i for $2\leq i \leq n$

$f(w_1 v_1)$ = 6

$f(w_i v_i)$ = 8i – 1 for $2\leq i \leq n$

$f(u_i t_i)$ = 8i – 5 for $1\leq i \leq n$

$f(t_1 s_1)$ = 1

$f(t_i s_i)$ = 8i – 6 for $2\leq i \leq n$

$f(u_1 s_1)$ = 2

$f(u_i s_i)$ = 8i -4 for $2\leq i \leq n$

Thus f provides a harmonic mean labeling of graph *G*.

Hence *G* is a harmonic mean graph.

Example 2.4.1 :A harmonic mean labeling of graph *G* obtained by *H*- super subdivision of cycle HSS($C_8 \odot K_2$) are given in fig 2.4.1

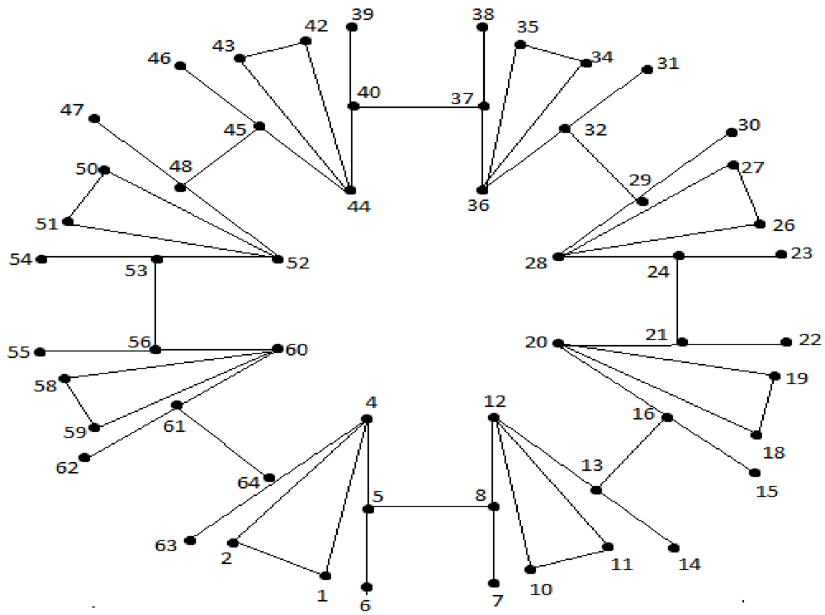

Fig 2.4.1

CONCLUSION

We have proved four results on Harmonic mean labeling of graphs related to cycle such as H-super subdivision of cycle HSS(C_n), HSS($C_n \odot K_1$), HSS($C_n \odot \overline{K_2}$), HSS($C_n \odot K_2$). Similar work can be carried out for other families and in the context of different types of graph labeling techniques.

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