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# TWO-DIMENSIONAL FREE CONVECTIVE VORTEX FLOW OF VISCOUS INCOMPRESSIBLE FLUID THROUGH POROUS MEDIA VOIDS

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In several situations fluid has to flow through pores of some naturally occurring or manmade porous materials. The characteristics of such porous media voids have a profound effect on flow parameters. Fluids such as water, petroleum, natural gas etc. flow through the soil; and in chemical industries certain chemicals are allowed to flow through various porous environments. Underground water is recharged by its flow through the soil pores during rainy season. Such a vast range of applications for the flow of viscous incompressible fluids through porous media voids is a great motivation towards their scientific studies. In this paper, we have studied two-dimensional laminar free convective vortex flow of an incompressible viscous fluid through porous media voids. It is shown that the motion of fluid is of free vortex type under the condition of Reynold's number being low. Vorticity in this type of flow is calculated and is found to be zero.

**Key Words:** Porosity, Vortex flow, Vorticity, Pharmaceutics, Bio-remediation.

## INTRODUCTION

Fluid flows through porous media have gained considerable interest not only in scientific applications but also in engineering field due to their wide ranging applications. Porous materials possess empty spaces (*i.e.* pores) in their surfaces. The characteristics of these pores are responsible in determining the parameters of the flow under consideration. A number of studies concerning movements of oil, water and natural gas through the oil reservoirs can be easily found in literature. An important area of application is the petroleum industry. The advancements and introduction of new techniques in petroleum industry are dependent on findings of these studies. Another important application of such fluid flows is in the exploration of underground water resources and to study seepage of water through soil, riverbeds etc to recharge water table. Porous materials are also finding applications in catalysis, chemical separation, tissue engineering etc. Materials such as bricks, ceramics, concrete etc are porous to varying degrees. The skeletal portion of the material is termed as matrix whereas the network of pores forms the pore space.

The porosity, pore size, pore shape, and specific surface area are some of the characteristics that influence the flow of fluids through the porous materials. Those materials which have porous voids in their structure play a significant role in the increase of specific surface area and hence in increasing photocatalytic activity. Porous materials have been found to be suitable for their use in energy management, vibration suppression, and heat insulation. They are being used in chemical engineering for the purification and filtration of water. Biological tissues such as bones, wood and cork are important naturally occurring porous materials. Fluid flow studies concerning porous media are useful for several disciplines of science such as acoustics, soil mechanics, bio-remediation, material science etc.

The most important characteristics of porous materials *i.e.* porosity (a number lying between 0.0 and 1.0) is very significant in applications such as in pharmaceutics, ceramics, manufacturing etc. Various kinds of nanomaterials such as liposomes, dendrimers, niosomes etc are also finding applications in pharmaceutics as drug delivery vehicles to cure diseases like cancer, tuberculosis etc. Porous flows have applications in inkjet printing and nuclear waste disposal technologies. Porosity has significant influence on the mechanical, physical and chemical properties of the materials.

Vortex flows of liquid and gas have considerable importance for several hydrodynamic processes such as in ecology, atmospheric turbulence, ocean currents etc. Such flows can be ordered as well as chaotic. V. V. Denisenko *et al.* have made numerical simulation about the vortex motion in the tube flow of ideal media [1]. Jiin-Yuh Jang and Jiing-Lin Chen studied effects of variable porosity on the vortex instability of a horizontal mixed convection flow in a porous medium which was saturated [2]. The porosity was assumed to vary exponentially with the distance from the wall. Their numerical calculations based on local similarity method suggests that the effect of variable porosity causes an increase in the heat transfer rate and gives rise to the vortex mode of instability in the flow. Hassanipour Fatemeh has made a study regarding numerical analysis of a 2D flow propagating through a porous media [3]. According to this study, formation of vortices and flow pattern in porous media strongly depend on the permeability (i.e. the ability of a porous material to allow the fluids to pass through it) of the porous media but only has a weak dependence on the porosity and Reynold's number (which is the measure of relative strength of inertial and viscous forces). Further, the average vorticity is not affected by porosity and permeability of the porous media for low Reynold's number. However, for high Reynold's number, average vorticity is affected by permeability but not by porosity. A. M. Vaishman has studied self similar vortex motion in porous half space [4]. They investigated effect of various physical parameters such as Prandtl number, Grashoff number, permeability parameter and ratio of free stream velocity to parallel wall parameter on the velocity in boundary layer and skin friction coefficient. Their findings indicate that fluid velocity increases with increase in either Grashoff number or permeability parameter or ratio of free stream velocity parameter to parallel wall parameter. A decrease in fluid temperature also occurs with decrease in either of the number, ratio of free stream velocity parameter to parallel wall motion of a fluid in bend or entrance of pump.

The vortex motion in porous media voids is of purely circulatory (*i.e.* free vortex type) as the fluid flows due to its own natural effect without the need of external energy [5]. Being an irrotational flow, streamlines are curved rather than being straight lines. Examples include whirlpool in a river and flow in centrifugal pump casing or around the circular bend in a pipe. The other type of vortex motion is forced vortex type which requires aid of external energy for the flow of fluid and this flow is rotational. The constant vorticity associated with this flow is twice the angular velocity. An increase in the average empty space in the solid constituent (*i.e.* porosity) leads to an increase in the overall vorticity. As per Fatemeh Hassanipur's study, pumping creates vortex flows in the porous underground layers. Hassanipour Fatemeh & Jose L. Lage studied the persistence and decay of vortex flows in porous media by allowing a pair of vortices to impinge on a permeable wall [6]. It resulted into interaction of vortices with the porous medium. Their findings show that permeability of the porous medium has a more important effect on this interaction compared with either porosity or the vortex transport velocity.

Changwoo Kang and Parisa Mirbod examined the influence of the porosity on the fully developed laminar flow in a channel partially replaced with a porous material [7]. They found that for high permeability, the higher porosity induces the increase of driving force and accelerates the flow while it decelerates the flow for low permeability by causing stronger viscous drag of the porous medium. Xudong An *et al.* carried out an experimental investigation about air vortex interaction with porous screens [8]. In this study they examined the effect of parameters such as porosity and air injection velocity on the air flow behaviour and also on the transport phenomena of vortex flow while interacting with porous screens. As per their observations the porosity values dominantly affect the vortex ring expansion, contraction and splitting during interaction with porous screens. Further the most notable behaviour of vortices occurs for interaction of vortex ring with fine porous screen. R.V Goldstein et al. undertook the problem of rotation of a porous disk in a viscous incompressible fluid which fills the half space above the disk [9]. They analyzed the problem by combining the motions of free fluid and the fluid contained inside the porous disk. This coupling of two motions makes the whole analysis very simple and using that they established that the boundary condition associated with the continuity of the tangential strains and tangential velocity components is satisfied at the fluid-disk interface.

## FORMULATION OF THE PROBLEM

In this study we have considered two- dimensional laminar free convective vortex flow of an incompressible fluid through porous media voids. The vortex motion involves curved streamlines which gives rise to the setting up of centrifugal forces within the body of the fluid flow. Obviously, there must also be forces due to difference in pressure along the pipe through which the fluid is flowing. The two kinds of forces have opposite directions and hence produce balancing effect on each other and result into the flow of fluid on curved path with curved streamlines instead of being straight ones. The geometry of the flow under consideration is sketched in fig. 1. We consider the equilibrium of the fluid element enclosed between two concentric streamlines *MN* & *RS* as shown there. The streamline *MN* is at a radial distance r from the centre of curvature C and fluid pressure on it is *p*, say. Let the other streamline *RS* be at a distance $(r + dr)$ from C, where $dr \ll r$. Suppose the fluid pressure at this streamline be $(p + dp)$, with *dp* being a small quantity. For keeping things simple, let us assume that the depth of the fluid be unity. The arc *MN* is $rd\theta$ and the arc *RS* is $(r + dr)\, d\theta$.

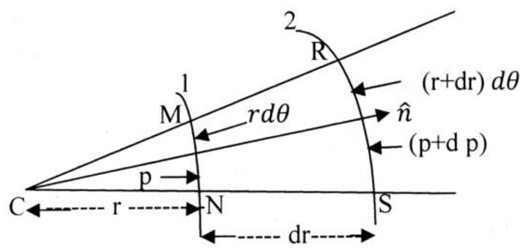


**Fig. 1 Curved Streamlines Enclosing Fluid Element**

## ANALYSIS

Forces on the fluid element MRSN due to difference in pressure act radially inwards and are equal to $rdpd\theta$ when terms which contain squares or products of negligibly small quantities, are ignored. Centrifugal force acting outwards from the centre of curvature *C* is $\frac{mv^2}{r}$, where *m* be the mass of the fluid element contained between *MN* and *RS* and is the product of the volume of element $(= rd\theta dr)$ and density of the fluid, say $\rho$. Thus the centrifugal force is equal to $\frac{(\rho r d\theta dr)v^2}{r}$ *i.e.* $\rho v^2 r d\theta dr$. If the fluid element is under the equilibrium condition, then the two kinds of forces balance each other *i.e.*

$$rdp\, d\theta = \rho v^2 dr\, d\theta$$

$$\frac{dp}{dr} = \frac{\rho v^2}{r}$$

But the fluid pressure, $p = p(r, \alpha)$

i.e. $$dp = \frac{dp}{dr} dr + \frac{dp}{d\alpha} d\alpha$$

or
$$dp = \left(\frac{\rho v^2}{r}\right) dr + (-\rho g) d\alpha \qquad ...(3.1)$$

For free vortex type motion, total energy $\left(E = p + \frac{\rho v^2}{2}\right)$ remains constant *i.e.* in the free vortex plane, the energy is independent of *r i.e.* we hav $\frac{dE}{dr} = 0$, which gives rise to

$$\frac{dv}{dr} + \frac{v}{r} = 0$$

Integration of which leads to the result

$$v = \frac{A}{r} \qquad ...(3.2)$$

where *A*, is any constant. In vector notation,

$$\vec{v} = \frac{A}{r}\hat{r}$$

where $\hat{r}$ is the unit vector along the axis of the flow.

Using above value of *v* in equation (3.1) and then integrating between streamlines $1(r_1, \alpha_1)$ & $2(r_2, \alpha_2)$, we obtain

$$(p_2 - p_1) = \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(\alpha_1 - \alpha_2) \qquad ...(3.3)$$

Equation (3.3) is the equation of conservation of energy and represents free vortex flow. Hence the flow of fluid under consideration is free vortex type. The vorticity in this fluid flow is given by

$$\vec{\Omega} = \vec{\nabla} \times \vec{v} = \vec{\nabla} \times \frac{A}{r}\hat{r} = 0 \qquad ...(3.4)$$

## RESULTS AND DISCUSSION

Governing equations for two-dimensional fluid flow through porous media have been considered and solved for determining the nature of fluid flow. The result $\frac{dp}{dr} = \frac{\rho v^2}{r}$ tells us that the pressure in the fluid varies with the radius of curvature of the streamline and increases with increase in the radius. Further, $v = \frac{A}{r}$ or $vr = A$, means that in this flow, the azimuthal component of velocity at the point under consideration is inversely proportional to the distance of the streamline through that point from the axis of rotation. Equation (3.3) suggests that the flow under consideration is the free vortex type. The vorticity $\vec{\Omega} = \vec{\nabla} \times \vec{v}$ is zero in this flow and therefore the flow is of irrotational type.

## CONCLUSIONS

Two-dimensional laminar free convective fluid flow through porous media voids is of free vortex type which has no requirement of adding energy from outside. Moreover the vorticity of the fluid flow comes out to be zero. The porosity of the medium affects the characteristics of this flow.

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