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ANALYTICAL FUNCTIONS IN ANALYSIS: AN OVERVIEW

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In the present paper we had discuss the concept of real analytical functions and complex analytical functions. And discuss some important results on it, we also discuss the condition of analytical functions for infinitely differentiability.

INTRODUCTION

As we all know that real analysis and complex analysis both are very important branch of Mathematics. A "Real Analysis" deals with problems which are closely connected with the notion of 'limit' and some other notions, such as the operations of 'differentiation' and 'integration' which are directly dependent on the concept of limit when all these operations are confined to the domain of real numbers.

The name "Complex Analysis" is deceiving, because the subject in fact analysis only those functions of complex numbers C that are differentiable at a point or for all $z \in C$ or some other open set $G \in C$. Complex analytical functions is a different from complex derivatives, it is defined as any complex differentiable functions in an open set is analytical.

PRELIMINARY FUNCTIONS:

Let *A* and *B* be two sets and let there be a rule which associate to each member *x* of *A*, a member *y* of *B*. Such a rule or a correspondence f under which to each element *x* of the set *A* there corresponds exactly one element *y* of the set *B* is called as a function (mapping). Symbolically we write $f: A \to B$, the set *A* is domain of the function and *B* is the co- domain of the function.

REAL VALUED FUNCTION:

Let X be a non empty set. *A* function $f: X \to R$ is called as a real valued function on *X*. For each $x \in X$, the f- image, denoted by $f(x)$ (which is also called the value of *f* at *x*), is a real number.

COMPLEX VALUED FUNCTION:

Let S be a subset of complex plane and a function f: $S \rightarrow C$ *i.e.* $W=f(z)$, $z \in S$ then f is said to be complex valued function.

Real-Valued functions of a complex variable:

Real-valued functions of a complex variable are functions $y = f(z)$ where z is a complex number and y is a real number. Then function $x=Re(z)$ and $r=|z|$ are both the examples of real- valued functions of a complex variable.

Complex-Valued functions of a real variable:

Complex-valued functions of a real variable are functions $w = f(t)$ where t is a real number and w is a complex number.

For example, $w(t) = 3t + i$ cost is a complex- valued functions of a real variable t.

Limit of Real functions:

Let $D \subseteq R$ and f: $D \rightarrow R$ be a function. Let c be a limit point of D. A real number l is said to be a limit of f at c if corresponding to a pre-assigned positive $\in$ there exist a positive δ such that $l - \in < f(x) < l + \in$, for all $x \in N'(c, \delta) \cap D$, where $N'(c, \delta) = \{x \in R: 0 < |x - c| < \delta\} = (c - \delta, c + \delta) \setminus \{c\}$.

Limit of Complex functions:

Let a function f be defined at all points z in some neighbourhood of z_o .The statement that the limit point of $f(z)$ as z approaches to z_o is a number w_o or $\lim_{z \to z_0} f(z) = w_0$,means that the point w, $f(z) = w_0$ can be made arbitrarily close to w_0 if we choose the point z close enough to z_o but distinct from it .That is for each positive number of ϵ , there is a positive δ such that $|f(z) - w| < \epsilon$ whenever $0 < |z - z_o| < \delta$.

Continuity of Real functions:

Let $D \subseteq R$. Consider a function $f : D \rightarrow R$ and a point $c \in D$. We say that f is continuous at c if for any given $\epsilon > 0$, $\delta > 0$ such that $x \in D$ and $|x - c| < \delta = |f(x) - f(c)| < \epsilon$.

Continuity of Complex functions:

A complex function f is continuous at a point z_o if $\lim_{z \to z_0} f(z) = z_0$.

Power Series of Real Numbers:

The series of the form $\sum_{n=0}^{\infty} a_0 X^n$ are called as power series of real numbers and a_0 are their coefficients.

Real Analytical functions:

A function *f*, with domain an open set $U \subseteq R$ and range either the real or complex numbers, is set to be real analytic at α if the function f may be represented by a convergent power series on some interval of positive radius centred at α; $f(x) = \sum_{i=0}^{\infty} a_i (x-a)^i$ is said to be real analytic on $V \subseteq U$ if it is real analytic at each α Є *V*.

SOME THEOREMS

Liouville Theorem: A bounded entire function is constant.

Morera Theorem:

If a function *f* (*z*) is continuous throughout a domain *D* and $\int_{\gamma} f(z)\,dz = 0$ for every closed contour *Y* in *D*, then *f* (*z*) is analytic throughout *D*.

Maximum Modulus Principle:

If *f* (*z*) is analytical in a domain *D*, then |*F*| does not attain its maximum value in *D* unless it is a constant.

Maximum Modulus Theorem:

Suppose *f* (*z*) is analytic in *D* and continuous on ∂D. Then the maximum of |f (z)|, which is always reached, occurs somewhere on the boundary of D and never in the interior.

Minimum Modulus Principle:

Let f (z) be analytic in *D* such that $f(z) \neq 0$, $z \in D$. Then |*f* (*z*)| does not attain its minimum unless it is a constant.

Minimum Modulus Theorem:

Let *f* (*z*) be an analytic function in *D* and continuous on ∂*D* such that $f(z) \neq 0, \forall z \in D$.

Then |*f* (*z*)| attains its minimum value somewhere on ∂ *D*.

Term by Term Integration:

A Power Series $\sum_{\kappa=0}^{\infty} a_k (z-z_0)^{\kappa}$ can be integrated term by term with in its circle of convergence $|z-z_0| = R$, for every contour *C* lying entirely within the circle of convergence.

Proof: Integrating a power series by term gives,

$$\int_c \sum_{k=0}^{\infty} a_0 (z-z_0)^{\kappa} dz = \sum_{k=0}^{\infty} \int_c a_k (z-z_0)^{\kappa} dz$$

Where C lies in the interior of $|z - z_o| = R$. Indefinite integration can also be carried out term by term $\int_c \sum_{k=0}^{\infty} a_k (z - z_0)^{\kappa} dz = \sum_{k=0}^{\infty} \int_c a_k (z - z_0)^{\kappa} dz = \sum_{k=0}^{\infty} \left\{ \frac{a_k}{k+1} \right\} (z - z_0)^{k+1} dz$ constant. The (2.25) can be used to be prove that both $\sum_{k=0}^{\infty} a_k (z - z_0)^{\kappa}$ and $\sum_{k=0}^{\infty} a_k \frac{1}{k+1} (z - z_0)^{\kappa+1}$ have the same radius of convergence $|z - z_0| = R$.

CONCLUSION

A function *f* is analytic if *f* is continuously differentiable on domain. Every analytic function is infinitely differentiable and has power series expansion about each point of its domain.

A real function possesses derivatives of all orders and also agrees with its Taylor's series in a neighborhood of every point, then such function is analytic.

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