OPTIMIZE FUZZY INVENTORY MODEL WITH RELIABILITY INFLUENCE DEMAND BY SIGNED DISTANCE METHOD

[1]ANUBHAV PRATAP SINGH, [2]RITU ARORA, [3]RENU SHARMA AND [4]ANAND CHAUHAN

[1, 2]Department of Mathematics and Statistics, Gurukul Kangri Vishwavidyalaya, Haridwar, India

[3]Department of Mathematics S.G.R.R. (PG) College, Dehradun, India

[4]Department of Mathematics Graphic Era Deemed to University, Dehradun, India

RECEIVED : 6 August, 2020

The research study provides a model for the quantity of economic order (EOQ) with reliability influenced demand rate and constant rate of deterioration. This model is starting with scarcity of inventory which is fully backlogged. Model is suitable for the advance booking of items such as mobiles booking, automobiles booking and gas cylinder booking, etc. The aim of this research is to bring reliable products of best-quality and adjustment of the stock to regulate the inventory management of the company for uncertainty of the cost functions. However, by optimising the procurement time point the average total cost is minimized using uncertain costs. Model is validate by numerically and sensitivity explore the various aspects of different parameters on total average cost TAC(t_ε) in crisp and fuzzy environment.

Keywords: Reliability dependent demand rate; Deteriorating items; Procurement time; Signed distance method; Trapezoidal fuzzy number.

[1]Email:drapsingh78@gmail.com

[2]Email: ritu.arora29@gmail.com

[3]Email: km.renuvishwkarma@gmail.com

[4]Email: Dranandchauhan83@gmail.com

INTRODUCTION

Classical models of inventories are built with no consideration of the effect of deterioration, but in reality we cannot ignore the effect of deterioration. Deterioration is spoilages or damage of the item by some other factors. Many items such that food items, automobiles, mobiles, electronic items and chemicals etc losses their value as time increases it means they deteriorated. Firstly, the effect on fashion goods of the deterioration is presented by Whitin [19] in their research study. After that, an inventory model with a constant rate of deterioration was explored by Ghare and Scharder [5].

The demand rate is also one of the important element in modelling of deteriorative inventory model. Generally, the demand fluctuate with time. In classical model of inventory, demand is considered to be constant. Therefore, the researchers started considering time-dependent demand, fluctuating demand, variable demand, time and price dependent demand etc. In modern time there is competition between companies to provide high reliable products for their customers, and earn good reputation in the market. Reliability deals with reducing failure over a period of time. Therefore, considering demand as reliability and time-dependent would be more realistic. For the present market circumstances the reliability and time depended demand is suitable. Companies have started to increase the reliability of the product. So many researchers started to develop inventory model by taking deterioration factor and reliability influenced demand in their research work. In 1999, the study on time dependent demand is done by Chang [2]. Then, Wu[20] explained an EOQ model in which deterioration of the Weibull distribution and demand for the ramp form is used. Shortages are partially backlogged.

Teng *et al.* [18] clarified the unfulfilled demand is partially backlogged with the waiting time at a negative exponential rate in their study. Chiang *et al.* [3] discussed an inventory model with backorder and fuzzifying the different variable such that the storing cost, demand,shortage quantity order placing cost, shortage quantity and backorder cost. Variables considered as a triangular fuzzy number and then defuzzified by a signed distance process in their research work. Skouri *et al.* [17] investigates the model of inventory with time reliant Weibull deterioration rate, ramp type demand rate, and partial backlog of unjustified demand for the product is treated. The proposed model is plotted under the various approaches: (a) with shortages and (b) without shortages. The optimal replacement technique is carried out with these two approaches. Purnomo *et al.* [9] described the median rule to find the minimization of the fuzzy variable. Dutta and Kumar [4] proposed a fuzzy inventory model with considering the set up and carrying cost as trapezoidal fuzzy number and defuzzification is done with signed distance approach. Their fuzzy inventory model is for without backorder. Sahoo et al. [13] investigate a fuzzy model of inventory with deterioration, salvage, and time-varying demand as trapezoidal fuzzy numbers. Numerically comparison with crisp to fuzzy is done and founded that the result in fuzzy sense is relatively better optimal solution than in crisp sense.

Singh and Rathore [15] analyze about reliable manufacturing method along with stock dependent manufacturing. In this research study demand is used as exponential time function and non-instantaneous deterioration rate is taken. Sen and Malakar [14] suggested a fuzzy inventory model without shortages. The related costs were taken as various fuzzy numbers such as parabolic fuzzy number, trapezoidal fuzzy number and for defuzzification graded mean integration method is considered. Under fuzzy approach Chandrasiri [1] described an inventory model without shortages. Khara *et al.* [6] described a model with defective production procedure. Here, demand is based on product’s reliabilityand selling price. In the manufacturing system a function of profit has been developed to discover the best possible values of reliability factor, productand duration of production such that a producer gains maximum benefits. Mahapatra *et al.* [8] presents the influence of deterioration with reliability and time dependent demand rate and partial backorder. Ray [12] studied about the fuzziness in inventory management. Rajalakshmi and Rosario [10] studied a fuzzy inventory model that is entirely backlogged and shortage is permitted. Signed distance approach is taken to perform the defuzzification of optimisation function of proposed research study. Khurana *et al.* [7] anticipated an EPQ model for decaying items. To accomplish the expectations and demand of the market, in this study the rate of production is a function of the rate of demand. Singh *et al.* [16] analysed an EOQ model for decaying items with ramp-type demand rate and time relative deterioration rate. Rajut et al. [11] investigated an EOQ model for three cases demand about pharmaceutical inventory using fuzzy and crisp parameters.

Therefore, by optimizing the timing of the purchase, the overall cost is minimized by using the trapezoidal fuzzy figure.. After this procedure, signed distance method is used to defuzzify the overall cost. To express this planned model the sensitivity analysis is presented in fuzzy environment. The function principle is used for fuzzy operations.The remainder of the paper is structured as: Preliminaries are given in Section 3. Notation and assumptions are provided in Section 4. An EOQ model for uncertain cost function in fuzzy environment is developed for reliability and time depended demand in Section 5. To test the model a numerical example is present in Section 6. In Section 7 analyse sensitivity is demonstrate using fuzzy approach and finally, conclusion of this study is discussed in last section.

3. Preliminaries

Definition 3.1: [1] Fuzzy Point: $\tilde{F}$ be a fuzzy set. It is said to be a fuzzy point on $R = (-\infty, \infty)$ if its membership function is

$$\mu_{\tilde{F}(\lambda)} = \begin{cases} 1 \, if \;\; \lambda = \upsilon \\ 0 \, if \;\lambda \neq \upsilon \end{cases}$$

Definition 3.2: [4] Fuzzy number: Consider the fuzzy set $\tilde{F}$ on R of real number.The membership grade of these functions has the form $\mu_{\tilde{F}} : R \to [0, 1]$. Then $\tilde{F}$ act as a fuzzy number if:

(i) $\tilde{F}$ must be normal fuzzy *set i.e.* $\mu_{\tilde{F}}(\lambda)$ =1 for at least one $\lambda \in \tilde{F}$

(ii) $\alpha_{\tilde{F}}$ must be closed for every $\alpha \in [0, 1]$, i.e. $\alpha \neq 0$

(iii) The support of $\tilde{F}$ must be bounded, *i.e.* a fuzzy number must be 1.

(iv) $\tilde{F}$ is convex fuzzy set.

Definition 3.3:[1] L-R Version of Fuzzy Numbers: A fuzzy set $\tilde{F} \subseteq R$ is a L–R type fuzzy if its membership function is

$$\mu_{\tilde{F}}(\lambda) = \begin{cases} L\left(\dfrac{\omega-\lambda}{a}\right), & \text{for } \lambda \le \omega; \\ 1 & ,\text{for } \omega \le \lambda \le \gamma; \\ R\left(\dfrac{\lambda-\gamma}{b}\right), & \text{for } \lambda \ge \gamma; \end{cases}$$

Where, ω is the mean numeric value of $\tilde{F}$, the left and right spreads are a and b with $a > 0$, $b > 0$.

Definition 3.4: [4] Trapezoidal Fuzzy Number: Let $\tilde{F} = (l, m, n, o)$, $l < m < n < o$,be a fuzzy set on $R = (-\infty, \infty)$. It is a trapezoidal fuzzy number if the membership functions of $\tilde{F}$ is defined as

$$\tilde{F}(\lambda) = \begin{cases} \dfrac{\lambda-1}{m-1}, & 1 \le \lambda \le m \\ 1, & m \le \lambda \le n \\ \dfrac{o-\lambda}{o-n}, & n \le \lambda \le o \\ 0, & \text{otherwise} \end{cases}$$

Definition 3.5:[3] Signed Distance Method for Defuzzification: For $\tilde{F} \in F$, the signed distance from $\tilde{F}$ to 0 is defined as: $d(\tilde{F}, 0) = \frac{1}{2}\left[\int_0^1 F_L(\alpha) + F_R(\alpha)\right] d\alpha$. Where $\alpha \in [0, 1]$; $F_L(\alpha) =$ left distance value from $\tilde{F}$ to 0 $F_R(\alpha) =$ right distance value from $\tilde{F}$ to 0. For trapezoidal fuzzy number $F_L(\alpha) = l + \alpha(m - l)$; $F_R(\alpha) = o - \alpha(o - n)$

Then $$d(\tilde{F}, 0) = \frac{1}{4}(l + m + n + o)$$

Definition 3.6: [4] Fuzzy Arithmetical Operations: Let $\tilde{\tau} = (\tau_1, \tau_2, \tau_3, \tau_4)$ and $\tilde{\omega} = (\omega_1, \omega_2, \omega_3, \omega_4)$ are two trapezoidal fuzzy numbers where $\tau_1, \tau_2, \tau_3, \tau_4, \omega_1, \omega_2, \omega_3, \omega_4$ are positive real number then fuzzy arithmetical operations are defined as:

(i) $\tilde{\tau} \oplus \tilde{\omega} = \tau_1 + \omega_1, \tau_2 + \omega_2, \tau_3 + \omega_3, \tau_4 + \omega_4$

(ii) $\tilde{\tau} \otimes \tilde{\omega} = (\tau_1\omega_1, \tau_2\omega_2, \tau_3\omega_3, \tau_4\omega_4)$

(iii) $\tilde{\tau} - \tilde{\omega} = (\tau_1 - \omega_1, \tau_2 - \omega_2, \tau_3 - \omega_3, \tau_4 - \omega_4)$

(iv) $\tilde{\tau} \varnothing \tilde{\omega} = \left(\dfrac{\tau_1}{\omega_4}, \dfrac{\tau_2}{\omega_3}, \dfrac{\tau_3}{\omega_2}, \dfrac{\tau_4}{\omega_1}\right)$

(v) $\alpha\tilde{\tau} = (\alpha\tau_1, \alpha\tau_2, \alpha\tau_3, \alpha\tau_4)$, where α is any real number.

4. Notation and Assumption

4.1: Notations

- θ: Deteriorating Rate.
- a : Demand rate.
- $I(t)$: The inventory level at any time t in the interval [0, T].
- $I_{\max}$ **:** Maximum level of inventory.
- T: Set time of the ordering cycle.
- t_ε **:** Procurement time and t_ε^* is optimum procurement time.
- I_0 **:** Ordering quantity and I_0^* is optimum ordering quantity of the inventory.
- o : Ordering cost for an order in crisp environment and $\tilde{o}$ is ordering cost for an order in fuzzy environment.
- d : Deterioration cost for a unit in crisp environment and $\tilde{d}$ is deteriorating cost for a unit in fuzzy environment.
- h : Holding cost for a unit per unit time in crisp environment and $\tilde{h}$ is holding cost for a unit per unit time in fuzzy environment.
- s : Shortage cost for a unit per unit of time in crisp environment and $\tilde{s}$: is shortage cost for a unit per unit time in fuzzy environment.
- ω : Reliability of item of this inventory system, *i.e.* ω% of perfect item present in the inventory.
- TAC $((t_\varepsilon^*)$ Optimal total average cost for a unit per unit time in crisp environment and $\widetilde{TAC}(t_\varepsilon^*)$ is optimal total average cost for a unit per unit time in fuzzy environment.

4.2: Assumptions

- Model deals with single type of product.
- The constant deterioration rate is used and deteriorated objects are not replaced.
- Demand is dependent on reliability as well as time factor.
- This model is for infinite point horizon.
- Shortages are legally recognized and they are completely back logged.
- Replenishment of items is instantaneous.
- The lead time (delivery time) is negligible.

5. Mathematical Model Development

5.1 Crisp Model

There is no inventory at t = 0. Due to advance booking of the item shortages occurs in interval $[0, t_\varepsilon]$. At $t = t_\varepsilon$ shortages are fully backlogged. Rest of the inventory $I_{\max}$ depletes due to demand and deterioration rate in $[t_\varepsilon, T]$. Finally, the inventory point reaches to zero at T.

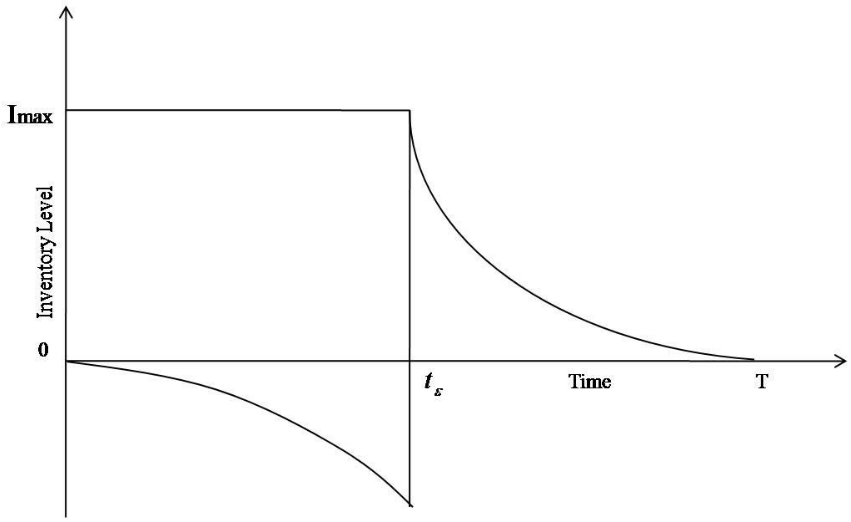

Fig.1. Representaion of Inventory Level with respect to time

$$\frac{dI(t)}{dt} = -at\omega', \; 0 \le t \le t_\varepsilon \qquad ...(1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -at\omega', \quad t_\varepsilon \le t \le T \qquad ...(2)$$

with $I(0) = 0, \; I(T) = 0$

Solution of the differential equation (1) and (2) are:

$$I(t) = \frac{-a}{\log[\omega]^2}[\omega'(-1 + t\log[\omega]) + 1] \qquad ... (3)$$

$$I(t) = \frac{a((\omega^t + \omega^T) - (t\omega^t + T\omega^T)(\theta + \log[\omega]))}{(\theta + \log[\omega])^2} \qquad ...(4)$$

at $t = t_\varepsilon$ we get the maximum inventory level (I_{max}) for each ordering rotation *i.e.*

$$I_{max} = \frac{a((\omega^{t_\varepsilon} + \omega^T) - (t_\varepsilon \omega^{t_\varepsilon} + T\omega^T)(\theta - \log[\omega]))}{(\theta + \log[\omega])^2} \qquad ... (5)$$

Backlogged demand $(b) = \int_0^{t_\varepsilon} at\omega' dt = \frac{a}{\log[\omega]^2}[\omega^{t_\varepsilon}(-1 + t_\varepsilon \log[\omega]) + 1]$ ... (6)

Initial inventory $I_0 = I_{max} + b$

$$= \frac{a((\omega^{t_\varepsilon} + \omega^T) - (t_\varepsilon \omega^{t_\varepsilon} + T\omega^T)(\theta + \log[\omega]))}{(\omega + \log[\omega])^2}$$

$$+ \frac{a}{\log[\omega]^2}[\omega^{t_\varepsilon}(-1 + t_\varepsilon \log[\omega]) + 1] \qquad ... (7)$$

The different costs are defined as:

(1) The cost of ordering (OC) = o

(2) The Deteriorating cost (DC) in

$$[t_\varepsilon, T] = d\left[\frac{a((\omega^{t_\varepsilon} + \omega^T) - (t_\varepsilon\omega^{t_\varepsilon} + T\omega^T)(\theta + \log[\omega]))}{(\theta + \log[\omega])^2} - \int_{t_\varepsilon}^{T} at\omega^t dt\right]$$

$$= d\left[\frac{a((\omega^{t_\varepsilon} + \omega^T) - (t_\varepsilon\omega^{t_\varepsilon} + T\omega^T)(\theta + \log[\omega]))}{(\theta + \log[\omega])^2} + \frac{a((\omega^{t_\varepsilon} + \omega^T) - a\log[\omega](t_\varepsilon\omega^{t_\varepsilon} + T\omega^T)}{\log[\omega]^2}\right]$$

...(9)

(3) The cost of holding (HC) in

$$[t_\varepsilon, T] = h\int_{t_\varepsilon}^{T} I(t)dt = h\int_{t_\varepsilon}^{T} \frac{a((\omega^t + \omega^T) - (t\omega^t + T\omega^T)(\theta + \log[\omega]))}{(\theta + \log[\omega])^2} dt$$

$$= \frac{ha}{(\theta + \log[\omega])^2}\left[T\omega^T + \frac{\omega^T - \omega^{t_\varepsilon}}{\log[\omega]} - \omega^{t_\varepsilon} t_\varepsilon\right.$$

$$\left. + \frac{(\theta + \log[\omega])(\omega^{t_\varepsilon}(-1 + \log[\omega_\varepsilon]t_\varepsilon + \omega^2(1 - T\log[\omega](1 + T\log[\omega]) + T\log[\omega]^2 t_\varepsilon))}{\log[\omega]^2}\right] \quad ...(10)$$

(4) The shortage cost (SC) in [0, t_ε]

$$= s\int_0^{t_\varepsilon} \frac{a}{\log[\omega]^2}[\omega'(-1 + t\log[\omega]) + 1]dt$$

$$= sa\left[\frac{2 - 2\omega^{t_\varepsilon} + (1 + \omega^{t_\varepsilon})\log[\omega]t_\varepsilon}{\log[\omega]^3}\right] \quad ...(11)$$

Total average cost $TAC(t_\varepsilon) = \dfrac{(OC + DC + HC + SC)}{T}$

$$= \left[\frac{o}{T} + \frac{da}{T}\left(\frac{(\omega^T + \omega^{t_\varepsilon}) - (T\omega^T + t_\varepsilon\omega^{t_\varepsilon})(\theta + \log[\omega])}{(\theta + \log[\omega])^2} + \frac{(\omega^T + \omega^{t_\varepsilon}) - a\log[\omega](T\omega^T + t_\varepsilon\omega^{t_\varepsilon})}{\log[\omega]^2}\right)\right.$$

$$+ \frac{ha}{T(\theta + \log[\omega])^2}\left(T\omega^T + \frac{\omega^T - \omega^{t_\varepsilon}}{\log[\omega]} - \omega^{t_\varepsilon} t_\varepsilon + \frac{1}{\log[\omega]^2}(\theta + \log[\omega])(\omega^{t_\varepsilon}(-1 + \log[\omega]t_\varepsilon)\right.$$

$$\left.\left. + \omega^T(1 - T\log[\omega](1 + T\log[\omega]) + T\log[\omega]^2 t_\varepsilon))\right) + \frac{s}{T}\left(-\frac{a(-1 + \omega^{t_\varepsilon})(2 - 2\omega^{t_\varepsilon} + (1 + \omega^{t_\varepsilon})\log[\omega]t_\varepsilon)}{\log[\omega]^3}\right)\right]$$

...(12)

5.2 Fuzzy Model

The different costs are defined as:

(1) The cost of ordering $\widetilde{OC} = \tilde{o}$ ...(13)

(2) The Deteriorating cost ($\widetilde{DC}$) in [t_ε , T]

$$= \tilde{d} \otimes \left[\frac{a((\omega^{t_\varepsilon} + \omega^T) - (t_\varepsilon \omega^{t_\varepsilon} + T\omega^T)(\theta + \log[\omega]))}{(\theta + \log[\omega])^2} - \int_{t_\varepsilon}^{T} at\omega^t dt \right]$$

$$= \tilde{d} \otimes \left[\frac{a((\omega^{t_\varepsilon} + \omega^T) - (t_\varepsilon \omega^{t_\varepsilon} + T\omega^T)(\theta + \log[\omega]))}{(\theta + \log[\omega])^2} + \frac{a(\omega^{t_\varepsilon} + \omega^T) - a\log[\omega](t_\varepsilon \omega^{t_\varepsilon} + T\omega^T)}{\log[\omega]^2} \right]$$

... (14)

(3) The inventory holding cost (($\widetilde{HC}$))in $[t_\varepsilon, T] = \tilde{h}\int_{t_\varepsilon}^{T} I(t)dt$

$$= \tilde{h} \otimes \int_{t_\varepsilon}^{T} \frac{a((\omega^{t_\varepsilon} + \omega^T) - (t_\varepsilon \omega^{t_\varepsilon} + T\omega^T)(\theta + \log[\omega]))}{(\theta + \log[\omega])^2} dt$$

$$= \frac{h \otimes a \left(T\omega^T + \frac{\omega^T - \omega^{t_\varepsilon}}{\log[\omega]} - \omega^{t_e} t_\varepsilon + \frac{(\theta + \log[\omega])(\omega^{t_\varepsilon}(-1 + \log[\omega]t_\varepsilon) + \omega^T(1 - T\log[\omega](1 + T\log[\omega]) + T\log[\omega]^2 t_\varepsilon))}{\log[\omega]^2} \right)}{(\theta + \log[\omega])^2}$$

...(15)

(4) The shortage cost ($\widetilde{SC}$) in $[0,\, t_\varepsilon] = \tilde{s} \otimes \int_0^{t_\varepsilon} \frac{a}{\log[\omega]^2}[\omega^t(-1 + t\log[\omega]) + 1]dt$

$$= s \otimes a \left[\frac{2 - 2\omega^{t_\varepsilon} + (1 + \omega^{t_\varepsilon})\log[\omega]t_\varepsilon}{\log[\omega]^3} \right] \quad ...(16)$$

Total average cost $\widetilde{TAC}(t_\varepsilon) = \frac{[\widetilde{OC} \oplus \widetilde{DC} \oplus \widetilde{HC} \oplus \widetilde{SC}]}{T}$

$$\left[\frac{\tilde{o}}{T} \oplus \frac{\tilde{d} \otimes a}{T} \left(\frac{(\omega^T + \omega^{t_\varepsilon}) - (T\omega^T + t_\varepsilon \omega^{t_\varepsilon})(0 + \log[\omega])}{(\theta + \log[\omega])^2} \right. \right.$$

$$\oplus \frac{(\omega^T + \omega^{t_\varepsilon}) - a\log[\omega](T\omega^T + t_\varepsilon \omega^{t_\varepsilon})}{\log[\omega]^2} \oplus \frac{\tilde{h} \otimes a}{T(\theta + \log[\omega])^2}$$

$$\left(T\omega^T + \frac{\omega^T - \omega^{t_\varepsilon}}{\log[\omega]} - \omega^{t_\varepsilon} t_\varepsilon + \frac{1}{\log[\omega]^2}(\theta + \log[\omega])(\omega^{t_\varepsilon}(-1 + \log[\omega] t_\varepsilon)\right.$$

$$\oplus\ \omega^T(1 - T\log[\omega](1 + T\log[\omega]) + T\log[\omega]^2 t_\varepsilon))) \oplus \frac{\tilde{s}}{T}$$

$$\otimes\left[-\frac{a(-1+\omega^{t_\varepsilon})(2 - 2\omega^{t_\varepsilon} + (1+\omega^{t_\varepsilon})\log[\omega] t_\varepsilon)}{\log[\omega]^2}\right] \quad ...(17)$$

5.3. Defuzzification by using signed distance method

Using signed distance method for trapezoidal fuzzy number in equation (17) we get

Total average cost $(\widetilde{TAC}(t_\varepsilon)) =$

$$\left[\frac{(o_1+o_2+o_3+o_4)}{4T} \oplus \frac{(d_1+d_2+d_3+d_4)\otimes a}{4T}\left(\frac{(\omega^T + \omega^{t_\varepsilon}) - (T\omega^T + t_\varepsilon\omega^{t_\varepsilon})(\theta + \log[\omega])}{(\theta+\log[\omega])^2}\right.\right.$$

$$\left.\oplus \frac{(\omega^T + \omega^{t_\varepsilon}) - a\log[\omega](T\omega^T + t_\varepsilon\omega^{t_\varepsilon})}{\log[\omega]^2}\right) \oplus \frac{(h_1+h_2+h_3+h_4)\otimes a}{4T(\theta+\log[\omega])^2}$$

$$\left(\frac{(\omega^T + \omega^{t_\varepsilon}) - (T\omega^T + t_\varepsilon\omega^{t_\varepsilon})(\theta + \log[\omega])}{(\theta+\log[\omega])^2} \oplus \frac{(\omega^T + \omega^{t_\varepsilon}) - a\log[\omega](T\omega^T + t_\varepsilon\omega^{t_\varepsilon})}{\log[\omega]^2}\right)$$

$$\oplus \frac{(h_1+h_2+h_3+h_4)\otimes a}{4T(\theta+\log[\omega])^2}\left(T\omega^T + \frac{\omega^T - \omega^{t_\varepsilon}}{\log[\omega]}\omega^{t_\varepsilon} t_\varepsilon + \frac{1}{\log[\omega]^2}(\theta+\log[\omega])(\omega^{t_\varepsilon}(-1+\log[\omega] t_\varepsilon\right.$$

$$\left.\oplus\, \omega^T(1 - T\log[\omega](1+T\log[\omega]) + T\log[\omega]^2 t_\varepsilon))\right) \oplus \frac{(s_1+s_2+s_3+s_4)}{4T}$$

$$\left.\otimes(-\frac{a(-1+\omega^{t_\varepsilon})(2-2\omega^{t_\varepsilon} + (1+\omega^{t_\varepsilon})\log[\omega] t_\varepsilon)}{\log[\omega]^3}\right] \quad \textbf{....(18)}$$

6. Computational Algorithm

To get the maximum benefit for the estimated overall cost $TAC(t_\varepsilon)$ with respect to t_ε the following steps are used to minimise $TAC(t_\varepsilon)$.

Step 1: Start with the first derivative of $TAC(t_\varepsilon)$ of equation (12) and equate it to zero i.e. $\frac{\partial TAC(t_\varepsilon)}{\partial(t_\varepsilon)} = 0$, we obtain the critical point t_ε.

Step 2: Take second derivative of $TAC(t_\varepsilon)$ i.e. $\frac{\partial^2 TAC(t_\varepsilon)}{\partial t_\varepsilon^2}$

Step 3: Use critical point t_ε in step 2 we get the second derivative of total average cost will be positive at this critical point i.e. $\frac{\partial^2 TAC(t_\varepsilon)}{\partial t_\varepsilon^2} > 0$. Therefore $TAC(t_\varepsilon)$ is minimum at this critical point t_ε.

Step 4: Then find the minimum value of $TAC(t_\varepsilon)$ by putting the value of t_ε in equation (12).

7. Numerical Example

Example: In this section, we gave an example of mathematical disclosure of the inventory model presented. The demand rate is not only depends on time but also on reliability of the product quality therefore the demand rate is at ω^t where let a = 0.01. Thus, let a new item be launched in the market of which 80% are of acceptable quality. The ordering cost is $20 per unit per year. To hold the item let it costs the retailer by $2 per unit per year. The item undergoes deterioration with time at the rate of 1% which cost $15 per unit per year. During shortages, Customers are eager to see the item on the market with cost $10 per unit. The total inventory system is considered for 2 years.

Thus input parameters for crisp model are: o = 20, d = 15, h = 2, s =10, a=0.01, T = 2, ω= 0.8, θ= 0.05. We get the optimal procurement time is t_ε^* = 6.73655 year. Putting t_ε^* = 6.73655 in equation (12) the optimum order quantity I_0^* = 0.537156 units and the optimumtotal average cost TAC (t_ε^*) = $14.465.

Input parameters for fuzzy model are:

$o_1 = 17$; $o_2 = 17$; $o_3 = 20$; $o_4 = 21$; $d_1 = 14.25$; $d_2 = 14.5$; $d_3 = 15.5$; $d_4 = 15.75$; $h_1 = 1.25$; $h_2 = 1.5$; $h_3 = 2.5$; $h_4 = 2.75$; $s_1 = 9.25$; $s_2 = 9.5$; $s_3 = 10.5$; $s_4 = 10.75$; $a = 0.01$; $T = 2$; $\omega = 0.8$; $\theta = 0.05$.

Then the procurement time remain same *i.e.* $t_\varepsilon^* = 6.7365543$ year. Putting $t_\varepsilon^* = 6.73655$ in equation (17) the optimal fuzzy total average cost $\widetilde{TAC}(t_\varepsilon^*) = \$13.9650136409$.

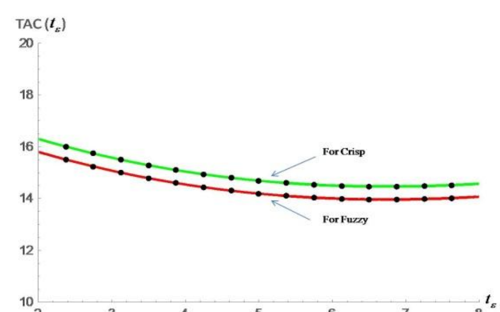

Fig 2: Comparison Graph between crisp and fuzzy model

8. Sensitivity Analysis

The changing effect of different parameters *o, d, h, s, a, T* and θ on optimal total average cost and optimal order quantity have presented in Table 1. The analysis is conducted using example 1. The analysis for crisp model is shown in Table 1 and the analysis for fuzzy model is shown in Table 2. The observed points from table 1 are:

1. t_ε, I_0 are not change by changing the ordering cost factor *o* and $TAC(t_\varepsilon)$ enhance by increasing in ordering cost factor *o* and reduce with decreasing in ordering cost factor o.

2. I_0 remain constant by changing the deterioration cost factor *d*. t_ε and $TAC(t_\varepsilon)$ enhance with increasing deterioration cost factor *d* and reduce with reducing in deterioration cost factor *d*.

3. I_0 remain constant by changing theholding costfactor *h*. t_ε Enhance with enhancingholding cost factor *h* and reduce with decrease in holding cost. $TAC(t_\varepsilon)$ reduce with increase inholding cost and increase with decrease in holding cost factor h.

4. I_0 remain constant by changing the shortage cost factor *s*. t_ε reduces with increase in shortage cost factor *s* and increase with reduce in shortage cost factor s. $TAC(t_\varepsilon)$ enhance with enhancing the shortage cost factor *s* and diminish with reduce the shortage cost factor *s*.

5. I_0, t_ε and ($TAC(t_\varepsilon)$ enhance with improving in parameter *a* and reduce with reducing in parameter *a*.

6. t_ε enhance with enhancing in parameter *T* and reduce with reducing in parameter T. I_0 and $TAC(t_\varepsilon)$ reduce with increase in parameter *T* and rise with decrease in parameter *T*.

7. I_0, t_ε and $TAC(t_\varepsilon)$ enhance with enhancing the parameter θ and reduce with reducing the parameter θ.

Table 1

Parameter	Percent change in Parameter	t_ε	Percent change in t_ε	$TAC(t_\varepsilon)$	Percent change in $TAC(t_\varepsilon)$	I_0	Percent change in I_0
o	50%	6.73655	0.00000%	19.465	34.56619%	0.537156	0.00000%
	10%	6.73655	0.00000%	15.465	6.91324%	0.537156	0.00000%
	-10%	6.73655	0.00000%	13.465	-6.91324%	0.537156	0.00000%
	-50%	6.73655	0.00000%	9.46501	-34.56613%	0.537156	0.00000%
d	50%	7.56581	12.30986%	16.7492	15.79122%	0.537156	0.00000%
	10%	6.91139	2.59539%	14.9298	3.21327%	0.537156	0.00000%

	-10%	6.5568	-2.66828%	13.9958	-3.24369%	0.537156	0.00000%
	-50%	5.78383	-14.14255%	12.0702	-16.55582%	0.537156	0.00000%
h	50%	7.39923	9.83708%	13.6953	-5.32112%	0.537156	0.00000%
	10%	6.87577	2.06664%	14.3161	-1.02938%	0.537156	0.00000%
	-10%	6.59331	-2.12631%	14.6111	1.01002%	0.537156	0.00000%
	-50%	5.97015	-11.37674%	15.1643	4.83443%	0.537156	0.00000%
s	50%	5.55413	-17.55231%	14.9663	3.46561%	0.537156	0.00000%
	10%	6.43932	-4.41220%	14.5854	0.83235%	0.537156	0.00000%
	-10%	7.07989	5.09667%	14.3304	-0.93052%	0.537156	0.00000%
	-50%	9.31569	38.28577%	13.5598	-6.25786%	0.537156	0.00000%
a	50%	6.7379	0.02004%	16.7045	15.48220%	0.805734	50.00000%
	10%	6.73682	0.00401%	14.9125	3.09367%	0.590872	10.00007%
	-10%	6.73629	-0.00386%	14.0177	-3.09229%	0.483441	-9.99989%
	-50%	6.73521	-0.01989%	12.2302	-15.44971%	0.268578	-50.00000%
T	50%	6.84298	1.57989%	9.62705	-33.44590%	0.509245	-5.19607%
	10%	6.76409	0.40881%	13.1441	-9.13170%	0.531681	-1.01926%
	-10%	6.70545	-0.46166%	16.0807	11.16972%	0.542544	1.00306%
	-50%	6.54044	-2.91113%	29.037	100.73972%	0.562805	4.77496%
θ	50%	7.35588	9.19358%	15.0426	3.99309%	0.669541	24.64554%
	10%	6.84884	1.66688%	14.5649	0.69063%	0.559291	4.12078%
	-10%	6.62934	-1.59147%	14.3714	-0.64708%	0.516731	-3.80243%
	-50%	6.24548	-7.28964%	14.0501	-2.86830%	0.448897	-16.43079%

The observed points from Table 2 are:

(1) t_ε and $TAC(t_\varepsilon)$ are increase with enhancing the parameter (*a*) and decrease with reducing the parameter (*a*).

(2) t_ε increases with enhancing the parameter (*T*) and decrease with reducing the parameter (*T*). But $TAC(t_\varepsilon)$ decrease with enhancing the parameter (*T*) and increase with reduce the parameter (*T*).

(3) t_ε and $TAC(t_\varepsilon)$ are increase with enhancing the parameter (θ) and decrease with reducing the parameter (θ) .

Table 2

Parameter	**Percent change in Parameter**	t_ε	**Percent Change in** t_ε	$\widetilde{TAC}(t_\varepsilon)$	**Percent change in** $\widetilde{TAC}(t_\varepsilon)$
a	50%	6.7379	0.02004%	16.2045	16.03641%
	10%	6.73682	0.00401%	14.4125	3.20434%
	-10%	6.73629	-0.00386%	13.5177	-3.20310%
	-50%	6.73521	-0.01989%	11.7302	-16.00294%
T	50%	6.84298	1.57989%	9.29372	-33.44997%

	10%	6.76409	0.40881%	12.6896	-9.13292%
	-10%	6.70545	-0.46166%	15.5251	11.17140%
	-50%	6.54044	-2.91113%	28.037	100.76601%
θ	50%	7.35588	9.19358%	14.5426	4.13596%
	10%	6.84884	1.66688%	14.0649	0.71527%
	-10%	6.62934	-1.59147%	13.8714	-0.67034%
	-50%	6.24548	-7.28964%	13.5501	-2.97109%

9. Conclusions and Future Scope

In the research study, an inventory model is described with reliability influenced demand in which shortages are permitted with the complete backlog, and the deterioration rate is constant. The proposed inventory model has a new strategy, which helps in making the decision to achieve the optimal profit. By analyzing the model we observed that the total worth of the system is highly sensitive to change in demand rate. In the proposed model all types of costs have taken as a trapezoidal fuzzy number then to defuzzifying the total average cost, the signed distance method has used. The optimal solution is a better approximation to the Fuzzy approach than the Crisp one. The model is appropriate for those retailers who have been done the advance booking of items such as automobiles, electronics, petroleum products, etc. This research helps in decision making for optimum level of stock sustaining with minimum total cost.

The model can be able to expand for different deterministic demands like stock dependent demand, stochastic demand, etc. It can also be expanded for time -dependant deterioration rate, for Weibull distribution deterioration rate of two-parameter and for three-parameter Weibull deterioration rate.

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