VERTEX PRIME LABELING FOR SOME DUPLICATE GRAPH

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RECEIVED : 30 August, 2019

A graph G(V, E) is said to have a vertex prime labeling if its edges can be labeled with distinct integers from {1, 2, 3,…,|E|} such that for each vertex of degree atleast 2 then the greatest common divisor of the labels on its incident edges is 1. A graph that admits a vertex prime labeling is called a vertex prime graph. In this paper, we prove that the duplicate graph of the wheel W_n the duplicate graph of the bistar graph $B_{n, n}$ the duplicate graph of the double star graph $D_{n, n}$ the duplicate graph of the crown graph C_n the duplicate graph of the lilly graph I_n and the duplicate graph of the H-graph H_n are vertex prime graphs.

Keywords: Graph labeling, wheel graph, bistar graph, double star graph, crown graph, lilly graph, H-graph, duplicate graph, vertex prime labeling.

INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967 [6]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain condition(s). In the intervening years various labeling of graphs have been investigated in over 2500 papers. A detailed survey on graph labeling has been done by Gallian [3]. In [2] T. Deretsky, S.M. Lee and J. Mitchem has introduced vertex prime labeling. A graph G(V, E) has vertex prime labeling if its edges can be labeled with distinct integers 1, 2, 3,…,|E| (*i.e.*) a function $f : E \to \{1, 2, ..., |E|\}$ defined such that for each vertex with degree atleast 2 then the greatest common divisor of the labels on its incident edges is 1. The edge labels are the actual images under function $f : E(G) \to \{1, 2, ..., |E|\}$.

They have shown that all forests, connected graphs, $C_{2k} \cup C_n, 5C_{2m}$ etc. are vertex prime. They have further proved that a graph with exactly 2 components, of which is not an odd cycle has a vertex prime labeling and a 2 – regular graph with atleast two odd cycles does not have a vertex prime labeling. Mukund V. Bapat have proved Some vertex prime graphs and a new type of graph labeling in [5].P. Kavitha and S. Meena proved Vertex prime labeling for some helm related graphs in [4].

The concept of duplicate graph was introduced by E. Sampath kumar and he proved many result on it [7]. K. Thirusangu, P.P. Ulaganathan and B. Selvam have proved that the duplicate graph of a path graph P_m is cordial [8]. K. Thirusangu, P.P Ulaganathan and P. Vijaya kumar have proved that the duplicate graph of ladder graph $L_m, m \geq 2$ is cordial, total cordial and prime cordial [9]. In this paper we consider only simple finite, Undirected and nontrivial graph G. For notation and terminology we refer to Bondy and Murthy [1]. Some important definitions which related to this research are as follows:

Definition 1.1: A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling called a vertex labeling (or an edge labeling).

Definition 1.2: Let $G(V,E)$ be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V^1$ and $V \cap V^1 = \phi$ and $V \cap V^1 = \phi$ is bijective (for $v \in V$, we write $f(v) = v'$) and the edge set E_1 of DG is defined as: The edge uv is in E if and only if both uv^1 and u^1v are edges in E_1.

Definition 1.3: The wheel graph W_n is join of the graphs C_n and K_1. *i.e.* $W_n = C_n + K_1$. Here vertices corresponding to C_n are called rim vertices and C_n is called rim of W_n while the vertex corresponds to K_1 is called apex vertex.

Definition 1.4: A bistar $B_{n,n}$ is a graph obtained from K_2 by joining n pendant edges to each end of K_2. The edge K_2 is called the central edge of $B_{n,n}$ and the vertices of K_2 are called the central vertices of $B_{n,n}$.

Definition 1.5 : Double star $DS_{n,n}$ is a tree $K_{1,n,n}$ obtained from the star $K_{1,n}$ by adding the new pendant edge of the existing n pendant vertices. It has $2n+1$ vertices and $2n$ edges.

Definition 1.6: The Crown graph C_n^* is obtained from a cycle C_n by attaching a pendent edge at each vertex of the n-cycle.

Definition 1.7: The Lilly graph I_n, $n \geq 2$ can be constructed by two star graphs $2K_{1,n}, n \geq 2$ joining two path graphs $2P_n, n \geq 2$ with sharing a common vertex.

i.e. $$I_n = 2K_{1,n} + 2P_n.$$

Definition 1.8: The H graph of a path P_n is the graph obtained from two copies of P_n with vertices $u_1, u_2, \ldots u_n$ and $v_1, v_2, \ldots v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$ if n is even.

MAIN RESULTS

Theorem 2.1: The duplicate graph of wheel W_n is vertex prime for all integers $n \geq 3$.

Proof : Let W_n be the wheel graph with vertex set $V(W_n) = \{v_i / 0 \leq \text{i} \leq n\}$ and the edge set is $E(W_n) = \{v_0 v_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_1 v_n\}$.

Let G be the duplicate graph of wheel W_n. Let $v_0, v_1, v_2, ..., v_n, v_0^{'}, v_1^{'}, v_2^{'}, ..., v_n^{'}$ be the new vertices and the new edges are $e_1, e_2, ..., e_{2n}, e_1^{'}, e_2^{'}, ..., e_{2n}^{'}$ respectively.

Then $$V(G) = \{v_i, v_i^{'} / 0 \leq i \leq n\}$$

$$E(G) = \{v_0 v_i^{'} / 1 \leq i \leq n\} \cup \{v_i v_{i+1}^{'} / 1 \leq i \leq n-1\} \cup \{v_0^{'} v_i / 1 \leq i \leq n\}$$

$$\cup \{v_i^{'} v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_1 v_n^{'}\} \cup \{v_1^{'} v_n\}$$

Now $$|V(G)| = 2(n+1), |E(G)| = 4n.$$

Define a labeling $f : E\ (G) \to \{1, 2, 3, ..., 4n\}$ as follows

$$f(v_i v_{i+1}^{'}) = 2 + i \qquad \text{if } i = \begin{cases} 1, 3, 5, ..., n-2 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 1, 3, 5, ..., n-1 \end{cases}$$

$$f(v_i v_{i+1}^{'}) = 2 + n + i \quad \text{if } i = \begin{cases} 2, 4, 6, ..., n-1 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 2, 4, 6, ..., n-2 \end{cases}$$

$$f(v_0 v_1^{'}) = 2$$

$$f(v_0^{'} v_1) = 1$$

$$f(v_i^{'} v_{i+1}) = n + i + 2 \quad \text{if } i = \begin{cases} 1, 3, 5, ..., n-2 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 1, 3, 5, ..., n-1 \end{cases}$$

$$f(v_i^{'} v_{i+1}) = 2 + i \qquad \text{if } i = \begin{cases} 2, 4, 6, ..., n-1 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 2, 4, 6, ..., n-2 \end{cases}$$

$$f(v_0 v_i^{'}) = 2n + 2i - 1 \quad \text{for } 2 \leq i \leq n, \ \textit{for all } n$$

$$f(v_0^{'} v_1) = 2n + 2i \qquad \text{for } 2 \leq i \leq n, \ \textit{for all } n$$

If 'n' is odd $f(v_1 v_n^{'}) = 2n + 2$ and $f(v_1^{'} v_n) = n + 2$.

If 'n' is even $f(v_1 v_n^{'}) = n + 2$ and $f(v_1^{'} v_n) = 2n + 2$.

Clearly all the edge labels are distinct.

Now consider

$$f^*(v_0) = \text{gcd of labels of all edges incident at } v_0$$

$$= \gcd(f(v_0 v_1'), f(v_0 v_i')) \qquad \text{for } 2 \le i \le n$$

$$= \gcd(2, \text{odd})$$

$$= 1$$

$$f^*(v_0') = \text{gcd of labels of all edges incident at } v_0'$$

$$= \gcd(f(v_0' v_1) f(v_0' v_i)) \qquad \text{for } 2 \le i \le n$$

$$= \gcd(1, f(v_0' v_i)) \qquad \text{for } 2 \le i \le n$$

$$= 1$$

$$f^*(\mathrm{v}_i) = \gcd(f(v_i v_{i-1}'), f(v_i v_{i+1}'), f(v_i v_0'))$$

$$\text{if } i = \begin{cases} 3, 5, 7 \ldots, n-2 \text{ for n is odd} \\ \text{(or)} \\ 3, 5, 7 \ldots, n-1 \end{cases}$$

$$= \gcd(i+1, i+2, 2n+2i)$$

$$\text{if } i = \begin{cases} 3, 5, 7 \ldots, n-2 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 3, 5, 7 \ldots, n-1 \end{cases}$$

$$f^*(\mathrm{v}_i) = 1$$

Since each vertex v_i has degree $3,$ two of the edges have consecutive integers.

$$f^*(\mathrm{v}_i) = \gcd(f(v_i v_{i-1}'), f(v_i v_{i+1}'), f(v_i v_0'))$$

$$\text{if } i = \begin{cases} 2, 4, 6, \ldots, n-1 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 2, 4, 6, \ldots, n-2 \end{cases}$$

$$= \gcd(n+i+1, n+i+2, 2n+2i)$$

$$\text{if } i = \begin{cases} 2, 4, 6, \ldots, n-1 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 2, 4, 6, \ldots, n-2 \end{cases}$$

$$= 1$$

Since each vertex v_i has degree 3, two of the edges have consecutive integers.

$$f^*(\mathrm{v}_1) = \gcd(f(v_1 v_0'), f(v_1 v_2'), f(v_1 v_n'))$$

$$= \gcd(1, 3, n+2) \qquad \text{for } n \text{ is even}$$

$$= 1$$

$$f^*(\mathrm{v}_1) = \gcd(f(v_1 v_0'), f(v_1 v_2'), f(v_1 v_n'))$$

$$= \gcd(1, 3, 2n+2) \qquad \text{for } n \text{ is odd}$$

$$= 1$$

$$f^*(v_1') = \gcd(f(v_1' v_0), f(v_1' v_2), f(v_1' v_n))$$

$$= \gcd(2, n+3, n+2) \qquad \text{for } n \text{ is odd}$$

$$= 1$$

Since each vertex v_i has degree 3, two of the edges have consecutive integers.

$$f^*(v_1') = \gcd(f(v_1' v_0), f(v_1' v_2), f(v_1' v_n))$$

$$= \gcd(2, n+3, 2n+2) \qquad \text{for } n \text{ is even}$$

$$= 1$$

Since each vertex v_i has degree 3, one of the edge having odd label.

$$f^*(v_i') = \gcd(f(v_i' v_{i-1}), f(v_i' v_{i+1}), f(v_i' v_0))$$

$$\text{if } i = \begin{cases} 3, 5, 7, \ldots, n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 3, 5, 7, \ldots, n-1 \end{cases}$$

$$= \gcd(\mathrm{n+i+1}, n+i+2, 2(n+i)-1)$$

$$\text{if } i = \begin{cases} 3, 5, 7, \ldots, n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 3, 5, 7, \ldots, n-1 \end{cases}$$

$$= 1$$

Since each vertex v_i has degree 3 and two of them are Consecutive Integers.

$$f^*(v_i') = \gcd(f(v_i' v_{i-1}), f(v_i' v_{i+1}), f(v_i' v_0))$$

$$\text{if } i = \begin{cases} 2,4,6,\ldots,n-1 \text{ for } n \text{ is odd} \\ \text{or} \\ 2,4,6,\ldots,n-2 \end{cases}$$

$$= \gcd(\text{i}+1, i+2, 2(n+i)-1)$$

$$\text{if } i = \begin{cases} 2,4,6,\ldots,n-1 \text{ for } n \text{ is odd} \\ \text{or} \\ 2,4,6,\ldots,n-2 \end{cases}$$

$$= 1$$

Since each vertex v_i has degree 3 and two of them are Consecutive Integers.

Thus $f^*(\text{v}_i) = f^*(v_i^{'}) = 1$ for $0 \le i \le n$.

Hence G is a vertex prime graph.

Illustration 2.1

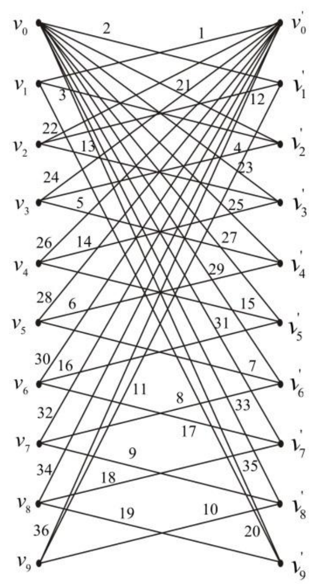

Figure 1. Vertex Prime labeling of duplicate graph of wheel W_9

Theorem 2.2: The duplicate graph of the bistar graph $B_{n,n}, n \ge 2$ admits vertex prime labeling.

Proof : Let $B_{n,n}$ be the bistar with vertex set $V(B_{n,n}) = \{v_0, u_0, v_i / 1 \le i \le 2n\}$ and the edge set $E(B_{n,n}) = \{v_0 v_i / 1 \le i \le n\} \cup \{u_0 v_i / n+1 \le i \le 2n\} \cup \{v_0 u_0\}$.

Let G be the duplicate graph of bistar $B_{n,n}, n \geq 2$. Now the new set of vertices and edges are $\{v_0, u_0, v_1, v_2, ..., v_{2n}, v_0^{'}, u_0^{'}, v_1^{'}, v_2^{'}, ..., v_{2n}^{'}\}$ and $\{e_1, e_2, ..., e_{2n+1}, e_1^{'}, e_2^{'}, ..., e_{2n+1}^{'}\}$ respectively.

Then $V(G) = \{u_0, u_0^{'}, v_i, v_i^{'} / 0 \leq i \leq 2n\}$

and $E(G) = \{v_0 v_i^{'} / 1 \leq i \leq n\} \cup \{v_0^{'} v_i / 1 \leq i \leq n\} \cup \{u_0 v_i^{'} / n+1 \leq i \leq 2n\}$

$\cup \{u_0^{'} v_i / n+1 \leq i \leq 2n\} \cup \{v_0 u_0^{'}\} \cup \{v_0^{'} u_0\}$.

Then $|V(G)| = 4n+4$ and $|E(G)| = 4n+2$.

Define a labeling $f : \mathrm{E}(G) \to \{1, 2, 3, ..., 4n+2\}$ as follows

$$f(v_0 v_1^{'}) = 1$$

$$f(v_0^{'} v_1) = 2$$

$$f(v_0 v_i^{'}) = 2i \qquad \text{for } 2 \leq i \leq n$$

$$f(v_0 u_0^{'}) = 2(n+1)$$

$$f(v_0^{'} v_i) = 2i-1 \qquad \text{for } 2 \leq i \leq n$$

$$f(v_0^{'} u_0) = 2n+1$$

$$f(u_0 v_i^{'}) = 2(n+1)+i \qquad \text{for } n+1 \leq i \leq 2n$$

$$f(u_0^{'} v_i) = i+n+2 \qquad \text{for } n+1 \leq i \leq 2n$$

Clearly all the labels are distinct.

$$f^*(v_0) = \gcd(f(v_0 v_1^{'}), f(v_0 v_i^{'}), f(v_0 u_0^{'})) \qquad \text{for } 2 \leq i \leq n$$

$$= \gcd(1, f(v_0 v_i^{'}), f(v_0 u_0^{'})) \qquad \text{for } 2 \leq i \leq n$$

$$=1$$

$$f^*(v_0^{'}) = \gcd(f(v_0^{'} v_1), f(v_0^{'} v_i), f(v_0^{'} u_0)) \qquad \text{for } 2 \leq \mathrm{i} \leq n$$

$$= \gcd(2, 2\mathrm{i}-1, 2\mathrm{n}+1) \qquad \text{for } 2 \leq \mathrm{i} \leq n$$

$$= \gcd(2, \mathrm{odd}) = 1$$

$$f^*(u_0) = \gcd(f(u_0 v_i^{'}), f(u_0 v_0^{'})) \qquad \text{for } n+1 \leq i \leq 2n$$

$$= \gcd(2(n+1)+i, 2n+1) \qquad \text{for } n+1 \leq i \leq 2n$$

$$=1$$

Since u_0 has a degree $n+1$, there n edges has consecutive integers.

$$f^*(u_0^{'}) = \gcd(f(u_0^{'}v_i), f(u_0^{'}v_0)) \qquad \text{for } n+1 \le i \le 2n$$

$$= \gcd(n+i+2, 2n+2) \qquad \text{for } n+1 \le i \le 2n$$

$$= 1$$

Since u_0^1 has a degree $n+1$, there n edges has consecutive integers.

Thus $f^*(u_0) = f^*(u_0^{'}) = f^*(v_i) = f^*(v_i^{'}) = 1$ for all i.

Hence G is a vertex prime graph.

Illustration 2.2

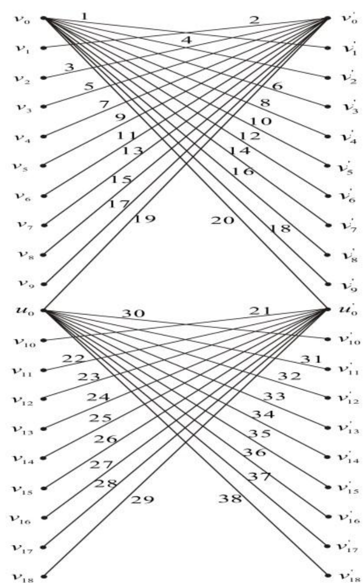

Figure 2. Vertex Prime labeling of duplicate graph of bistar graph $B_{9,9}$

Theorem 2.3: The duplicate graph of double star $DS_{n,n}$ where $n \ge 2$ is a vertex prime graph.

Proof: Let $DS_{n,n}$ be the double star with vertex set $V(DS_{n,n}) = \{v_i / 0 \le i \le 2n\}$ and edge set $E(DS_{n,n}) = \{v_0 v_i \ /1 \le i \le n\} \cup \{v_i \, v_{i+n} /1 \le i \le n\}$.

Let G be the duplicate graph of double star $DS_{n,n}$ where $n \ge 2$. So we get $v_0, v_1, v_2, ..., v_n v_{n+1}, ..., v_{2n}, v_0^{'}, v_1^{'}, v_2^{'}, ..., v_{2n}^{'}$ and $e_1, e_2, ..., e_{2n}, e_1^{'}, e_2^{'}, ..., e_{2n}^{'}$ respectively be the new set of vertices and edges of the duplicate graph of double star graph $DS_{n,n}$.

Now $V(G) = \{v_i, v_i^{'} / 0 \le i \le 2n\}$

and $E(G) = \{v_0 v_i' / 1 \le i \le n\} \cup \{v_0' v_i / 1 \le i \le n\}$

$\cup \{v_i v_{i+n}' / 1 \le i \le n\} \cup \{v_i' v_{n+i} / 1 \le i \le n\}$

Then $|V(G)| = 4n + 2$ and $|E(G)| = 4n$.

Define a labeling $f : \mathrm{E}(G) \to \{1, 2, 3, ..., 4n\}$ as follows

$f(v_0 v_i') = 2i - 1$ for $1 \le i \le n$

$f(v_0' v_i) = 2(n+i) - 1$ for $1 \le i \le n$

$f(v_i' v_{n+i}) = 2i$ for $1 \le i \le n$

$f(v_i v_{n+i}') = 2(n+i)$ for $1 \le i \le n$

Clearly all the labels are distinct.

Now $f^*(v_0) = \text{gcd of label of all edges incident at } v_0$

$= \gcd(f(v_0 v_1'))$ for $1 \le i \le n$

$=1$

Since all the edge labels are consecutive odd integers.

$f^*(v_0') = \text{gcd of label of all edges incident at } v_0'$

$= \gcd(f(v_0' v_i))$ for $1 \le i \le n$

$= 1$

Since all the edge labels are consecutive odd integers.

$f^*(v_i) = \gcd(f(v_0' v_i), f(v_i v_{n+i}'))$ for $1 \le i \le n$

$= \gcd(2(n+i) - 1, 2(n+i))$ for $1 \le i \le n$

$=1$

Since all the edge labels are consecutive integers.

$f^*(v_i') = \gcd(f(v_0 v_i'), f(v_i' v_{n+i}))$ for $1 \le i \le n$

$= \gcd(2i - 1, 2i))$ for $1 \le i \le n$

$= 1$

Since all the edge labels are consecutive integers.

Thus $f^*(v_i) = f^*(v_i') = 1$ for all i.

Hence G is a vertex prime graph

Illustration 2.3

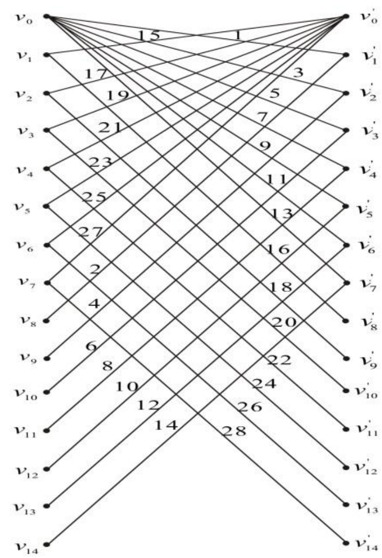

Figure 3. Vertex Prime labeling of duplicate graph of double star graph $DS_{7,7}$

Theorem 2.4: The duplicate graph of Crown graph C_n where $n \geq 3$ is a vertex prime graph.

Proof: Let C_n be the Crown graph with vertex set $V(C_n) = \{u_i, v_i / 1 \leq i \leq n\}$ and the edge set $E(C_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_n u_1\} \cup \{u_i v_i / 1 \leq i \leq n\}$.

Let G be the duplicate graph of crown graph C_n where $n \geq 3$.

Here $u_1, u_2, u_3, ..., u_n, v_1, v_2, ..., v_n, u_1', u_2', ..., u_n', v_1', v_2', ..., v_n'$ and $e_1, e_2, ..., e_{2n}, e_1', e_2', ..., e_{2n}'$ be the set of vertices and edges of the duplicate graph of a crown graph C_n. Then the vertex set and edge sets are $V(G) = \{u_i, u_i', v_i, v_i' / 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1}', u_i' u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_1 u_n', u_1' u_n\} \cup \{u_i v_i', u_i' v_i / 1 \leq i \leq n\}$.

Then $|V(G)| = 4n$ and $|E(G)| = 4n$.

Define a labeling $f : \mathrm{E}(G) \rightarrow \{1, 2, 3, ..., 4n\}$ as follows

Case(i): *For n is even*

$$f(u_1 u_2') = 1$$

$$f(u_1 u_n') = 3$$

$$f(u_1' u_n) = 4n - 1$$

$$f(u_1' v_1) = 4n$$

$$f(u_1 v_1') = 2$$

$f(u_iu'_{i-1}) = 2(n-1+i)-1$ for $i = 2,4,6,...,n$

$f(u_iu'_{i-1}) = 2(n+1-i)+3$ for $i = 3,5,7,...,n-1$

$f(u_iu'_{i+1}) = 2(n+i)-1$ for $i = 2,4,6,...,n-2$

$f(u_iu'_{i+1}) = 2(n-i+1)+1$ for $i = 3,5,7,...,n-1$

$f(u'_iv_i) = 2(n+2-i)$ for $i = 2,4,6,...,n$

$f(u'_iv_i) = 2(n+i-1)$ for $i = 3,5,7,...,n-1$

$f(u_iv'_i) = 2(n+i-1)$ for $i = 2,4,6,...,n$

$f(u_iv'_i) = 2(n-i+2)$ for $i = 3,5,7,...,n-1$

Case(ii): *For n is odd*

$f(u_1u'_2) = 1$

$f(u_1v'_1) = 2$

$f(u_1u'_n) = 3$

$f(u_iu'_{i-1}) = 2(n+1-i)+3$ for $i = 2,4,6,...,n-1$

$f(u_iu'_{i-1}) = 2(2n-i+1)+3$ for $i = 3,5,7,...,n$

$f(u_iu'_{i+1}) = 2(n-i+1)+1$ for $i = 2,4,6,...,n-1$

$f(u_iu'_{i+1}) = 2(2n-i+1)+1$ for $i = 3,5,7,...,n-2$

$f(u'_1u_n) = 2n+3$

$f(u_iv'_i) = 2(n+2-i)$ for $i = 2,4,6,...,n-1$

$f(u_iv'_i) = 2(2n+2-i)$ for $i = 3,5,7,...,n$

$f(u'_iv_i) = 2(n+2-i)$ for $i = 1,3,5,...,n$

$f(u'_iv_i) = 2(2n+2-i)$ for $i = 2,4,6,...,n-1$

Clearly all the labels are distinct.

For n is odd:

$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1}), f(u_iv'_i))$ for $i = 2,4,6,....n-1$

$= \gcd(2(n+1-i)+3, 2(n+1-i)+1, 2(n+1-i)+2)$ for $i = 2,4,6,....n-1$

$=1$ Since all the edge labels are consecutive integers.

$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1}), f(u_iv'_i))$ for $i = 3, 5, 7, \ldots n-2$

$= \gcd(2(2n+1-i)+3, 2(2n+1-i)+1, 2(2n+1-i)+2)$ for $i = 3, 5, 7, \ldots n-2$

$= 1$ Since all the edge labels are consecutive integers.

$f^*(u_n) = \gcd(f(u_nu'_{n-1}), f(u_nu'_1) f(u_nv'_n))$

$= \gcd(2n+5, 2n+3, 2n+4)$

$= 1$ Since all the edge labels are consecutive integers.

Since all the edge labels are consecutive positive integers.

$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1}), f(u'_iv_i))$ for $i = 4, 6, 8, \ldots n-1$

$= \gcd(2(2n+1-i)+3, 2(2n+1-i)+1, 2(2n+1-i)+2)$ for $i = 4, 6, 8, \ldots n-1$

$= 1$ Since all the edge labels are consecutive integers.

$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1}), f(u'_iv_i))$ for $i = 3, 5, 7, \ldots n-2$

$= \gcd(2(n+1-i)+3, 2(n+1-i))+1, 2(n+1-i)+2))$ for $i = 3, 5, 7, \ldots n-2$

$= 1$ Since all the edge labels are consecutive integers.

$f^*(u'_1) = \gcd(f(u'_1u_2), f(u'_1u_n), f(u'_1v_1))$

$= \gcd(2n+1, 2n+3, 2n+2)$

$= 1$ Since all the edge labels are consecutive integers.

For n is even:

$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1}), f(u_iv'_i))$ for $i = 2, 4, 6, \ldots n-2$

$= \gcd(2(n-1+i)-1, 2(n-1+i)+1, 2(n-1+i))$ for $i = 2, 4, 6, \ldots n-2$

$= 1$ Since all the edge labels are consecutive integers.

$f^*(u_n) = \gcd(f(u_nu'_1), f(u_nu'_{n-1}), f(u_nv'_n))$

$= \gcd(4n-1, 4n-3, 4n-2)$

$= 1$ Since all the edge labels are consecutive integers.

$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1}), f(u_iv'_i))$ for $i = 3, 5, 7, \ldots n-1$

$= \gcd(2(n+1-i)+3, 2(n+1-i)+1, 2(n+1-i)+2)$ for $i = 3, 5, 7, \ldots n-1$

$= 1$ Since all the edge labels are consecutive integers.

$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1}), f(u'_iv_i))$ for $i = 4, 6, 8, \ldots n-2$

$= \gcd(2(n+1-i)+3, 2(n+1-i)+1, 2(n+1-i)+2)$ for $i = 4, 6, 8, \ldots n-2$

$= 1$ Since all the edge labels are consecutive integers.

$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1}), f(u'_iv_i))$ for $i = 3, 5, 7, \ldots n-1$

$$=\gcd(2(n-1+i)-1, 2(n-1+i)+1, 2(n-1+i)) \quad \text{for } i=3,5,7,...n-1$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u_1^{'}) = \gcd(f(u_1^{'}u_n), f(u_1^{'}u_2), f(u_1^{'}v_1))$$

$$=\gcd(4n-1, 2n+1, 4n)$$

$$=1$$

Since each vertex v_i has degree 3, two of the edges have consecutive integers.

In both the cases,

$$f^*(u_1) = \gcd(f(u_1u_2^{'}), f(u_1u_n^{'}), f(u_1v_1^{'}))$$

$$= \gcd(1,2,3) \quad \text{for all } n$$

$$=1$$

$$f^*(u_2^{'}) = \gcd(f(u_2^{'}u_1), f(u_2^{'}v_2), f(u_2^{'}u_3))$$

$$= \gcd(1, f(u_2^{'}v_2), f(u_2^{'}u_3)) \quad \text{for all } n$$

$$=1$$

$$f^*(u_n^{'}) = \gcd(f(u_n^{'}u_1), f(u_n^{'}u_{n-1}), f(u_n^{'}v_n))$$

$$=\gcd(3,5,4) \quad \text{for all } n$$

$$=1$$

Thus $f^*(u_i) = f^*(u_i^{'}) = 1$ for all i.

Hence G is a vertex prime graph.

Illustration 2.4

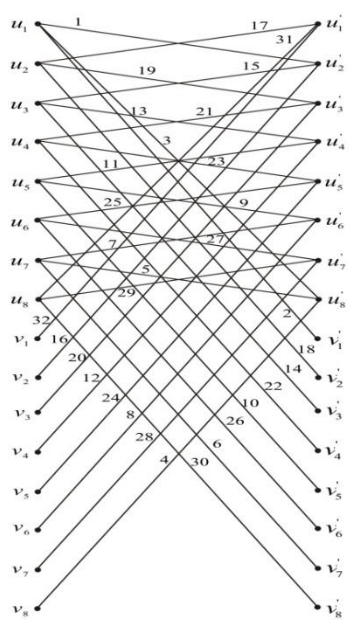

Figure 4. Vertex Prime labeling of duplicate graph of crown graph C_8

Theorem 2.5: The duplicate graph of Lilly graph I_n where $n \geq 2$ is a vertex prime graph.

Proof: Let I_n be the Lilly graph with vertex set $V(I_n) = \{u_i / 1 \leq i \leq 4n-1\}$ and the edge set

$$E(I_n) = \{u_1u_i / 2 \leq i \leq 2n+1\} \cup \{u_1u_{3n}\} \cup \{u_1u_{3n+1}\}$$
$$\cup \{u_iu_{i+1} / 2n+2 \leq i \leq 3n-1, 3n+1 \leq i \leq 4n-2\}$$

Let G be the duplicate graph of lilly graph for all integer $n \geq 2$. Now we get $u_1, u_2, u_3, ..., u_{4n-1}, u_1', u_2', ..., v_{4n-1}'$ and $e_1, e_2, ..., e_{4n-2}, e_1', e_2', ..., e_{4n-2}'$ respectively be the new set of vertices and edges of the duplicate graph of lilly graph I_n.

Then $V(G) = \{u_i, u_i' / 1 \leq i \leq 4n-1\}$ and

$$E(G) = \{u_1u_i' / 2 \leq \text{i} \leq 2n+1\} \cup \{u_1'u_i / 2 \leq i \leq 2n+1\}$$
$$\cup \{u_1'u_{3n}\} \cup \{u_1u_{3n}'\} \cup \{u_1'u_{3n+1}\} \cup \{u_1u_{3n+1}'\}$$
$$\cup \{u_iu_{i+1}' / 2\text{n}+2 \leq i \leq 3n-1, 3n+1 \leq i \leq 4n-2\}$$
$$\cup \{u_iu_{i-1}' / 2\text{n}+3 \leq i \leq 3n, 3n+2 \leq i \leq 4n-1\}$$

Then $|V(G)| = 8n-2$ and $|E(G)| = 8n-4$.

Define a labeling $f : \text{E}(G) \to \{1, 2, 3, ..., 8n-4\}$ as follows

$$f(u_1u_2') = 2$$

$$f(u_1'u_2) = 1$$

$$f(u_1u_i') = 2(i+2n-3)-1 \qquad \text{for } 3 \leq i \leq 2n+1$$

$$f(u_1'u_i) = 2(i+2n-3) \qquad \text{for } 3 \leq i \leq 2n+1$$

$$f(u_iu_{i+1}') = 2(2n+1)-i \quad \text{if } i = \begin{cases} 2n+2, 2n+4, 2n+6, ..., 3n-1 \text{for n is odd} \\ \text{(or)} \\ 2n+2, 2n+4, 2n+6, ..., 3n-2 \end{cases}$$

$$f(u_iu_{i+1}') = i-1 \quad \text{if } i = \begin{cases} 2n+3, 2n+5, 2n+7, ...3n-2 \text{ for } n \text{ is odd} \\ \text{(or)} \\ 2n+3, 2n+5, 2n+7, ...3n-1 \end{cases}$$

$$f(u_iu_{i-1}') = i-2 \quad \text{if } i = \begin{cases} 2n+3, 2n+5, 2n+7, ...3n \text{ for } n \text{ is odd} \\ \text{(or)} \\ 2n+3, 2n+5, 2n+7, ...3n-1 \end{cases}$$

$$f(u_iu_{i-1}') = 2(2n+1)-i+1 \quad = \begin{cases} 2\text{n}+4, 2n+6, 2n+8, ..., 3n-1 \text{for n is odd} \\ \text{(or)} \\ 2\text{n}+4, 2n+6, 2n+8, ..., 3n \end{cases}$$

For n is odd:

$$f(u_1u'_{3n}) = n+2$$

$$f(u_1u'_{3n+1}) = 3n$$

$$f(u'_1u_{3n}) = 3n-1$$

$$f(u'_1u_{3n+1}) = n+1$$

$$f(u_iu'_{i+1}) = 4n+1-i \qquad \text{for } i = 3n+1, 3n+3, 3n+5, ..., 4n-2$$

$$f(u_iu'_{i+1}) = i \qquad \text{for } i = 3n+2, 3n+4, 3n+6, ..., 4n-3$$

$$f(u_iu'_{i-1}) = i-1 \qquad \text{for } i = 3n+2, 3n+4, 3n+6, ..., 4n-1$$

$$f(u_iu'_{i-1}) = 4n-i+2 \qquad \text{for } i = 3n+3, 3n+5, 3n+7, ..., 4n-2$$

For n is even:

$$f(u_1u'_{3n}) = 3n-1$$

$$f(u_1u'_{3n+1}) = n+1$$

$$f(u'_1u_{3n}) = n+2$$

$$f(u'_1u_{3n+1}) = 3n$$

$$f(u_iu'_{i+1}) = i \qquad \text{for } i = 3n+1, 3n+3, 3n+5, ..., 4n-3$$

$$f(u_iu'_{i+1}) = 4n+1-i \qquad \text{for } i = 3n+2, 3n+4, 3n+6, ..., 4n-2$$

$$f(u_iu'_{i-1}) = 4n-i+2 \qquad \text{for } i = 3n+2, 3n+4, 3n+6, ..., 4n-2$$

$$f(u_iu'_{i-1}) = i-1 \qquad \text{for } i = 3n+3, 3n+5, 3n+7, ..., 4n-1$$

Clearly all the labels are distinct.

Now

$$f^*(u_1) = \gcd(f(u_1u'_2), f(u_1u'_i), f(u_1u'_{3n}), f(u_1u'_{3n+1})) \qquad \text{for } 3 \le i \le 2n+1$$

$$= \gcd(2, f(u_1u'_i), f(u_1u'_{3n}), f(u_1u'_{3n+1})) \qquad \text{for } 3 \le i \le 2n+1$$

$=1$ Since all the edges except $u_1u'_2$ are having odd labels.

$$f^*(u'_1) = \gcd(f(u'_1u_2), f(u'_1u_i), f(u'_1u_{3n}), f(u'_1u_{3n+1})) \qquad \text{for } 3 \le i \le 2n+1$$

$$= \gcd(1, f(u'_1u_i), f(u'_1u_{3n}), f(u'_1u_{3n+1})) \qquad \text{for } 3 \le i \le 2n+1$$

$$= 1$$

$$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1}))$$

$$\text{if } i = \begin{cases} 2n+3, 2n+5, 2n+7, ..., 3n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 2n+3, 2n+5, 2n+7, ..., 3n-1 \end{cases}$$

$= \gcd(\text{i-2,i-1})$

$$\text{if } i = \begin{cases} 2n+3, 2n+5, 2n+7, ..., 3n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 2n+3, 2n+5, 2n+7, ..., 3n-1 \end{cases}$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u_i) = \gcd(f(u_i u'_{i-1}), f(u_i u'_{i+1}))$$

$$\text{if } i = \begin{cases} 2\text{n}+4, 2n+6, 2n+8, ..., 3n-1 \text{ for } n \text{ is odd} \\ \text{or} \\ 2\text{n}+4, 2n+6, 2n+8, ..., 3n-2 \end{cases}$$

$= \gcd(2(2n+1)-i+1, 2(2n+1)-i)$

$$\text{if } i = \begin{cases} 2\text{n}+4, 2n+6, 2n+8, ..., 3n-1 \text{ for } n \text{ is odd} \\ \text{or} \\ 2\text{n}+4, 2n+6, 2n+8, ..., 3n-2 \end{cases}$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_i u_{i-1}), f(u'_i u_{i+1}))$$

$$\text{if } i = \begin{cases} 2n+3, 2n+5, 2n+7, ..., 3n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 2n+3, 2n+5, 2n+7, ..., 3n-1 \end{cases}$$

$= \gcd(2(2n+1)-i+1, 2(2n+1)-i)$

$$\text{if } i = \begin{cases} 2n+3, 2n+5, 2n+7, ..., 3n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 2n+3, 2n+5, 2n+7, ..., 3n-1 \end{cases}$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_i u_{i-1}), f(u'_i u_{i+1}))$$

$$\text{if } i = \begin{cases} 2\text{n}+4, 2n+6, 2n+8, ..., 3n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 2\text{n}+4, 2n+6, 2n+8, ..., 3n-1 \end{cases}$$

$$= \gcd(i-2, i-1)$$

$$\text{if } i = \begin{cases} 2\text{n}+4, 2n+6, 2n+8, ..., 3n-2 \text{ for } n \text{ is odd} \\ \text{or} \\ 2\text{n}+4, 2n+6, 2n+8, ..., 3n-1 \end{cases}$$

$=1$ Since all the edge labels are consecutive integers.

For n is odd:

$$f^*(u_i) = \gcd(f(u_i u'_{i-1}), f(u_i u'_{i+1})) \quad \text{for } i = 3n+2, 3n+4, 3n+6, ...4n-3$$

$$= \gcd(i-1, i) \quad \text{for } i = 3n+2, 3n+4, 3n+6, ...4n-3$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u_i) = \gcd(f(u_i u'_{i-1}), f(u_i u'_{i+1})) \quad \text{for } i = 3n+3, 3n+5, 3n+7, ...4n-2$$

$$= \gcd(4n+2-i, 4n+1-i) \quad \text{for } i = 3n+3, 3n+5, 3n+7, ...4n-2$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_i u_{i-1}), f(u'_i u_{i+1})) \quad \text{for } i = 3n+2, 3n+4, 3n+6, ...4n-3$$

$$= \gcd(4n+2-i, 4n+1-i) \quad \textit{for } i = 3n+2, 3n+4, 3n+6, ...4n-3$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_i u_{i-1}), f(u'_i u_{i+1})) \quad \text{for } i = 3n+3, 3n+5, 3n+7, ...4n-2$$

$$= \gcd(i-1, i) \quad \text{for } i = 3n+3, 3n+5, 3n+7, ...4n-2$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u_{3n}) = \gcd(f(u_{3n} u'_1), f(u_{3n} u'_{3n-1}))$$

$$= \gcd(3n-1, 3n-2)$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u_{3n+1}) = \gcd(f(u_{3n+1} u'_1), f(u_{3n+1} u'_{3n+2}))$$

$$= \gcd(n+1, n)$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u'_{3n}) = \gcd(f(u'_{3n} u_1), f(u'_{3n} u_{3n-1}))$$

$$= \gcd(n+2, n+3)$$

$=1$ Since all the edge labels are consecutive integers.

$$f^*(u'_{3n+1}) = \gcd(f(u'_{3n+1}u_1), f(u'_{3n+1}u_{3n+2}))$$
$$= \gcd(3n, 3n+1)$$
$= 1$ Since all the edge labels are consecutive integers.

For n is even:

$$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1})) \quad \text{for } i = 3n+2, 3n+4, 3n+6, \ldots 4n-2$$
$$= \gcd(4n-i+2, 4n-i+1) \quad \text{for } i = 3n+2, 3n+4, 3n+6, \ldots 4n-2$$
$= 1$ Since all the edge labels are consecutive integers.

$$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1})) \quad \text{for } i = 3n+3, 3n+5, 3n+7, \ldots 4n-3$$
$$= \gcd(i-1, i) \quad \text{for } i = 3n+3, 3n+5, 3n+7, \ldots 4n-3$$
$= 1$ Since all the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1})) \quad \text{for } i = 3n+2, 3n+4, 3n+6, \ldots 4n-2$$
$$= \gcd(i-1, i) \quad \text{for } i = 3n+2, 3n+4, 3n+6, \ldots 4n-2$$
$= 1$ Since all the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1})) \quad \text{for } i = 3n+3, 3n+5, 3n+7, \ldots 4n-3$$
$$= \gcd(4n+2-i, 4n+1-i) \quad \text{for } i = 3n+3, 3n+5, 3n+7, \ldots 4n-3$$
$= 1$ Since all the edge labels are consecutive integers.

$$f^*(u_{3n}) = \gcd(f(u_{3n}u'_1), f(u_{3n}u'_{3n-1}))$$
$$= \gcd(n+2, n+3)$$
$=1$ Since all the edge labels are consecutive integers.

$$f^*(u_{3n+1}) = \gcd(f(u_{3n+1}u'_1), f(u_{3n+1}u'_{3n+2}))$$
$$= \gcd(3n, 3n+1)$$
$=1$ Since all the edge labels are consecutive integers.

$$f^*(u'_{3n}) = \gcd(f(u'_{3n}u_1), f(u'_{3n}u_{3n-1}))$$
$$= \gcd(3n-1, 3n-2)$$
$=1$ Since the edge labels are consecutive integers.

$$f^*(u'_{3n+1}) = \gcd(f(u'_{3n+1}u_1), f(u'_{3n+1}u_{3n+2}))$$

$$= \gcd(n+1, n)$$

$$= 1 \quad \text{Since the edge labels are consecutive integers.}$$

Thus $f^*(u_i) = f^*(u_i') = 1$ for all i

Hence G is a vertex prime graph.

Illustration 2.5

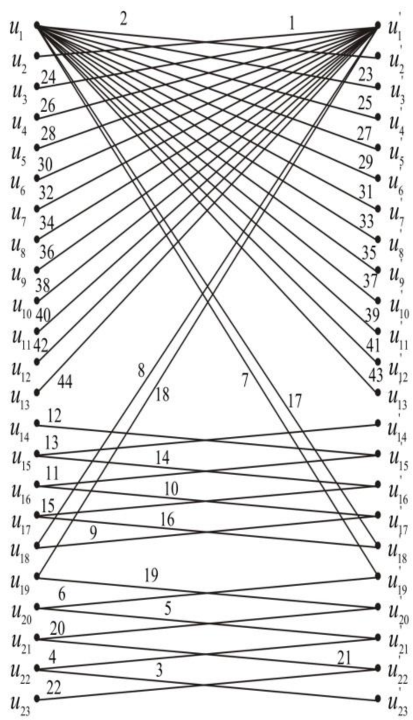

Figure 5. Vertex Prime labeling of duplicate graph of Lilly graph I_6

***Theorem 2.6*:** The duplicate graph of H-graph H_n is a vertex prime for all integer $n \geq 3$.

Proof: Let H_n be the *H*-graph with vertex set $V(H_n) = \{u_1, u_2, u_3, ..., u_n, v_1, v_2, ..., v_n\}$ and the edge set $E(H_n) = \{u_i u_{i+1}, v_i v_{i+1} / 1 \leq \text{i} \leq n-1\} \cup \left\{ u_{\frac{n+1}{2}} v_{\frac{n+1}{2}} \text{ if } n \text{ is odd} \right\}$ (or) $\left\{ u_{\frac{n}{2}+1} v_{\frac{n}{2}} \text{ if } n \text{ is even} \right\}$

Now let G be the duplicate graph of H_n then the new set of vertices and new set of edges are explained below:

$$V(G) = \{u_i, v_i, u_i', v_i' / 1 \leq i \leq n\}$$

$$E(G) = \{v_i v_{i+1}', u_i u_{i+1}' / 1 \leq \text{i} \leq n-1\} \cup \{v_i v_{i-1}', u_i u_{i-1}' / 2 \leq i \leq n\}$$

$$\cup \left\{ u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}', u_{\frac{n+1}{2}}' v_{\frac{n+1}{2}} \text{ if } n \text{ is odd} \right\} \text{ (or) } \left\{ u_{\frac{n}{2}+1} v_{\frac{n}{2}}', v_{\frac{n}{2}} u_{\frac{n}{2}+1}' \right\}$$

$$|V(G)| = 4n \text{ and } |E(G)| = 4n-2 .$$

Define the labeling $f : E(G) \to \{1,2,3,...,4n-2\}$ as

$$f(u_i u'_{i+1}) = i \qquad \text{if } i = \begin{cases} 1,3,5,...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 1,3,5,...,n-2 \end{cases}$$

$$f(v_i v'_{i+1}) = n+i-1 \qquad \text{if } i = \begin{cases} 1,3,5,...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 1,3,5,...,n-2 \end{cases}$$

$$f(u_i u'_{i+1}) = 2n+i-2 \qquad \text{if } i = \begin{cases} 2,4,6,...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6,...,n-1 \end{cases}$$

$$f(v_i v'_{i+1}) = 3(n-1)+i \qquad \text{if } i = \begin{cases} 2,4,6,...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6,...,n-1 \end{cases}$$

$$f(u'_i u_{i+1}) = 2n+i-2 \qquad \text{if } i = \begin{cases} 1,3,5,...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 1,3,5,...,n-2 \end{cases}$$

$$f(v'_i v_{i+1}) = 3(n-1)+i \qquad \text{if } i = \begin{cases} 1,3,5,...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 1,3,5,...,n-2 \end{cases}$$

$$f(u'_i u_{i+1}) = i \qquad \text{if } i = \begin{cases} 2,4,6,...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6,...,n-1 \end{cases}$$

$$f(v'_i v_{i+1}) = n+i-1 \qquad \text{if } i = \begin{cases} 2,4,6,...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6,...,n-1 \end{cases}$$

If n is odd then $f\left(u_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}\right) = 4n-3$ *and* $f\left(v_{\frac{n+1}{2}} u'_{\frac{n+1}{2}}\right) = 4n-2$

If n is even then $f\left(u_{\frac{n}{2}+1} v'_{\frac{n}{2}}\right) = 4n-3$ *and* $f\left(v_{\frac{n}{2}} u'_{\frac{n}{2}+1}\right) = 4n-2$

Clearly all the edge labels are distinct.

$$f^*(u_i) = \gcd(f(u_iu'_{i+1}), f(u_iu'_{i-1})) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$$= \gcd(i,\ i\text{-}1) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

$$f^*(u_i) = \gcd(f(u_iu'_{i-1}), f(u_iu'_{i+1})) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$$= \gcd(2n+i-3, 2n+i-2) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1})) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$$= \gcd(2n+i-3, 2n+i-2) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

$$f^*(u'_i) = \gcd(f(u'_iu_{i-1}), f(u'_iu_{i+1})) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$$= \gcd(i-1, i) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

$$f^*(v'_i) = \gcd(f(v'_iv_{i-1}), f(v'_iv_{i+1})) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$$= \gcd(n+i-2, n+i-1) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

$$f^*(v_i^{'}) = \gcd(f(v_i^{'}v_{i-1}), f(v_i^{'}v_{i+1})) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$$= \gcd(3(n-1)+i-1, 3(n-1)+i) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

$$f^*(v_i) = \gcd(f(v_i v_{i-1}^{'}), f(v_i v_{i+1}^{'})) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$$= \gcd(3(n-1)+i-1, 3(n-1)+i) \quad \text{if } i = \begin{cases} 2,4,6...,n-2 \text{ for } n \text{ is even} \\ \text{or} \\ 2,4,6...,n-1 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

$$f^*(v_i) = \gcd(f(v_i v_{i-1}^{'}), f(v_i v_{i+1}^{'})) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$$= \gcd(n+i-2, n+i-1) \quad \text{if } i = \begin{cases} 3,5,7...,n-1 \text{ for } n \text{ is even} \\ \text{or} \\ 3,5,7...,n-2 \end{cases}$$

$=1$ Since the edge labels are consecutive integers.

For n is odd:

$$f^*\left(u_{\frac{n+1}{2}}\right) = \gcd\left(f\left(u_{\frac{n+1}{2}}u^{'}_{\frac{n-1}{2}}\right), f\left(u_{\frac{n+1}{2}}u^{'}_{\frac{n+3}{2}}\right), f\left(u_{\frac{n+1}{2}}v^{'}_{\frac{n+1}{2}}\right)\right)$$

$$= \gcd\left(f\left(u_{\frac{n+1}{2}}u^{'}_{\frac{n-1}{2}}\right), f\left(u_{\frac{n+1}{2}}u^{'}_{\frac{n+3}{2}}\right), 4n-3\right)$$

$$= 1$$

Since the degree of $u_{\frac{n+1}{2}}$ is 3 and two of the edge labels $\left(u_{\frac{n+1}{2}} u'_{\frac{n-1}{2}} \textit{ and } u_{\frac{n+1}{2}} u'_{\frac{n+3}{2}} \right)$ are consecutive integer.

$$f^*\left(v_{\frac{n+1}{2}}\right) = \gcd\left(f\left(v_{\frac{n+1}{2}} v'_{\frac{n-1}{2}}\right), f\left(v_{\frac{n+1}{2}} v'_{\frac{n+3}{2}}\right), f\left(v_{\frac{n+1}{2}} u'_{\frac{n+1}{2}}\right)\right)$$

$$= \gcd\left(f\left(v_{\frac{n+1}{2}} v'_{\frac{n-1}{2}}\right), f\left(v_{\frac{n+1}{2}} v'_{\frac{n+3}{2}}\right), 4n-2\right)$$

$$= 1$$

Since the degree of $v_{\frac{n+1}{2}}$ is 3 and two of the edge labels $\left(v_{\frac{n+1}{2}} v'_{\frac{n-1}{2}} \textit{ and } v_{\frac{n+1}{2}} v'_{\frac{n+3}{2}} \right)$ are consecutive integer.

$$f^*\left(v'_{\frac{n+1}{2}}\right) = \gcd\left(f\left(v'_{\frac{n+1}{2}} v_{\frac{n-1}{2}}\right), f\left(v'_{\frac{n+1}{2}} v_{\frac{n+3}{2}}\right), f\left(v'_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right)\right)$$

$$= \gcd\left(f\left(v'_{\frac{n+1}{2}} v_{\frac{n-1}{2}}\right), f\left(v'_{\frac{n+1}{2}} v_{\frac{n+3}{2}}\right), 4n-3\right)$$

$$= 1$$

Since the degree of $v'_{\frac{n+1}{2}}$ is 3 and two of the edge labels $\left(v'_{\frac{n+1}{2}} v_{\frac{n-1}{2}} \textit{ and } v'_{\frac{n+1}{2}} v_{\frac{n+3}{2}} \right)$ are consecutive integer.

$$f^*\left(u'_{\frac{n+1}{2}}\right) = \gcd\left(f\left(u'_{\frac{n+1}{2}} u_{\frac{n-1}{2}}\right), f\left(u'_{\frac{n+1}{2}} u_{\frac{n+3}{2}}\right), f\left(u'_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right)\right)$$

$$= \gcd\left(f\left(u'_{\frac{n+1}{2}} u_{\frac{n-1}{2}}\right), f\left(u'_{\frac{n+1}{2}} u_{\frac{n+3}{2}}\right), 4n-2\right) = 1$$

Since the degree of $u'_{\frac{n+1}{2}}$ is 3 and two of the edge labels $\left(u'_{\frac{n+1}{2}} u_{\frac{n-1}{2}} \textit{ and } u'_{\frac{n+1}{2}} u_{\frac{n+3}{2}} \right)$ are consecutive integer.

For n is even:

$$f^*\left(u_{\frac{n}{2}+1}\right)=\gcd\left(f\left(u_{\frac{n}{2}+1}u'_{\frac{n}{2}}\right),f\left(u_{\frac{n}{2}+1}u'_{\frac{n}{2}+2}\right),f\left(u_{\frac{n}{2}+1}v'_{\frac{n}{2}}\right)\right)$$

$$=\gcd\left(f\left(u_{\frac{n}{2}+1}u'_{\frac{n}{2}}\right),f\left(u_{\frac{n}{2}+1}u'_{\frac{n}{2}+2}\right),4n-3\right)$$

$$=1$$

Since the degree of $u_{\frac{n}{2}+1}$ is 3 and two of the edge labels $\left(u_{\frac{n}{2}+1}u'_{\frac{n}{2}} \textit{ and } u_{\frac{n}{2}+1}u'_{\frac{n}{2}+2}\right)$ are consecutive integer.

$$f^*\left(u'_{\frac{n}{2}+1}\right)=\gcd\left(f\left(u'_{\frac{n}{2}+1}u_{\frac{n}{2}}\right),f\left(u'_{\frac{n}{2}+1}u_{\frac{n}{2}+2}\right),f\left(u'_{\frac{n}{2}+1}v_{\frac{n}{2}}\right)\right)$$

$$=\gcd\left(f\left(u'_{\frac{n}{2}+1}u_{\frac{n}{2}}\right),f\left(u'_{\frac{n}{2}+1}u_{\frac{n}{2}+2}\right),4n-2\right)$$

$$=1$$

Since the degree of $u'_{\frac{n}{2}+1}$ is 3 and two of the edge labels $\left(u'_{\frac{n}{2}+1}u_{\frac{n}{2}} \textit{ and } u'_{\frac{n}{2}+1}u_{\frac{n}{2}+2}\right)$ are consecutive integer.

$$f^*\left(v_{\frac{n}{2}}\right)=\gcd\left(f\left(v_{\frac{n}{2}}v'_{\frac{n}{2}-1}\right),f\left(v_{\frac{n}{2}}v'_{\frac{n}{2}+1}\right),f\left(v_{\frac{n}{2}}u'_{\frac{n}{2}+1}\right)\right)$$

$$=\gcd\left(f\left(v_{\frac{n}{2}}v'_{\frac{n}{2}-1}\right),f\left(v_{\frac{n}{2}}v'_{\frac{n}{2}+1}\right),4n-2\right)$$

$$=1$$

Since the degree of $v_{\frac{n}{2}}$ is 3 and two of the edge labels $\left(v_{\frac{n}{2}}v'_{\frac{n}{2}-1} \textit{ and } v_{\frac{n}{2}}v'_{\frac{n}{2}+1}\right)$ are consecutive integer.

$$f^*\left(v'_{\frac{n}{2}}\right)=\gcd\left(f\left(v'_{\frac{n}{2}}v_{\frac{n}{2}-1}\right),f\left(v'_{\frac{n}{2}}v_{\frac{n}{2}+1}\right),f\left(v'_{\frac{n}{2}}u_{\frac{n}{2}+1}\right)\right)$$

$$=\gcd\left(f\left(v'_{\frac{n}{2}}v_{\frac{n}{2}-1}\right),f\left(v'_{\frac{n}{2}}v_{\frac{n}{2}+1}\right),4n-3\right)=1$$

Since the degree of $v'_{\frac{n}{2}}$ is 3 and two of the edge labels $\left(v'_{\frac{n}{2}} v_{\frac{n}{2}-1} \text{ and } v'_{\frac{n}{2}} v_{\frac{n}{2}+1}\right)$ are consecutive integer.

Thus $f^*(u_i) = f^*(u_i') = f^*(v_i) = f^*(v_i') = 1$ for all i.

Hence G is a vertex prime graph.

Illustration 2.6

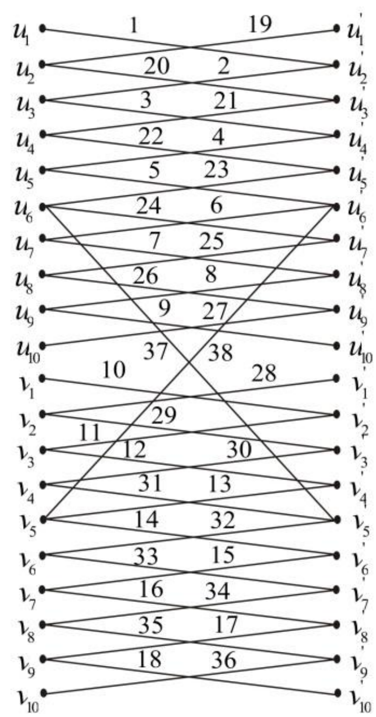

Figure 6. Vertex Prime labeling of duplicate graph of H-graph H_{10}

*C*ONCLUSION:

Here we investigate six corresponding results on vertex prime labeling analogues work can be carried out for other families also.

*A*CKNOWLEDGEMENT:

The authors are highly thankful to the anonymous referees for their kind suggestions and comments.

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