Acta Ciencia Indica, Vol. XXXIX P, No. 2, 63 (2013)

LEPTON FLAVOUR MIXING AND CP VIOLATION IN AN EXTENDED $SU(2)_H$ GAUGE MODEL

KRISHNANATH BANDYOPADHYAY

Department of Physics, R.S.P. College, Jharia, Dhanbad – 828111, Jharkhand (Vinoba Bhave University, Hazaribag)

NIRAJ KUMAR

Department of Physics, T.M.B. University, Bhagalpur – 812007, Bihar (Research Scholar)

AND

ABHAY KUMAR

Department of Physics, R.D. & D.J.College, Munger – 811201, Bihar (T.M.B. University, Bhagalpur)

RECEIVED : 18 April, 2013

Neutrino oscillations have been observed and successfully established by the data from solar, atmospheric and long-base line neutrino experiments. However, for non-zero neutrino masses and lepton flavour mixing we should look for the gauge models beyond the standard model. In the present work, we consider left-right- symmetric gauge model with $SU(2)_H$ as horizontal gauge group for three generations of leptons. With the proper choice of minimal Higgs fields, Fritzsch type mass matrices can be achieved in the lepton sector. We observe a normal hierarchy among the neutrino masses and could also relate the mass eigenvalues of charged leptons and of the neutrinos to the mixing angles. The suggested values of atmospheric mixing angle and the mixing element V_{3e} in the model are consistent with the experimental values. Leptonic CP violation is strongly favoured in the model due to exchange of horizontal gauge bosons at tree level.

Introduction

It is well known [W. Grimus and M.N. Rebelo 1997] that pure gauge theories do not violate CP. In gauge theory, fermionic as well as vector boson sectors are CP symmetric. In the electroweak standard model (SM), CP violation arises in the quark sector only for three generations of fermions through well-known Cabibbo-Kobayashi-Maskawa (CKM) mechanism [N. Cabibbo 1963; M. Kobayashi and K. Maskawa 1973]. However, in the framework of SM, no CP violation aries in the lepton sector because of the fact that leptonic mixing in the charged currents can be rotated away by redefining neutrino fields since neutrinos are massless in SM. Therefore, in any extension of the SM with non-zero neutrino masses and mixing, leptonic CP violation can take place. Leptonic CP violation is an important and indispensable phenomenon for understanding neutrino masses and lepton flavour mixing. On the other hand, recent experimental data on atmospheric and solar neutrinos and cosmological observations provide evidence for non-vanishing neutrino masses and possible non-trivial lepton mixing [G.L. Fogli, E. Lisi, A. Marrone and A. Palazzo 2004].

146/P013

In the SM, no Dirac mass term can arise due to the absence of right-handed neutrinos and no Majorana mass can be generated at the tree level due to simple Higgs structure of the SM and at higher orders also due to exact B-L conservation. Again, neutrino oscillation experiments [Y. Grossman 2004] provide evidence for non-vanishing neutrino masses and mixings. Hence, at this stage the phenomenological construction of lepton mass matrices might be an important step towards the understanding of phenomenology of extended gauge models beyond the standard model and investigation of the CP violation effects in the lepton sector have become more and more important.

Again, horizontal or inter-generation symmetry has often been proposed to understand different properties of fermions of different generations [K. Bandyopadhyay, A.K. Ray and U. Sarkar 1986]. This symmetry for the three generation case could be either $SU(2)_H$ or $SU(3)_H$ local symmetry. $SU(3)_H$ symmetry requires additional fermions to cancel anomalies and hence, we choose $SU(2)_H$ symmetry as horizontal symmetry in the present investigation. It has the property that if it operates on right-handed charged leptons, cancellation of global witten anomaly requires at least two right-handed neutrinos transforming as a doublet under $SU(2)_H$. Hence, masses of right handed neutrinos are connected to the breaking of $SU(2)_H$ symmetry.

In the present work, we consider an extended gauge model based on the gauge groups $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$ to study mass matrices of leptons, mixing angles and matrix element V_{3e} of the mixing matrix V which could be observed in the upcoming reactor neutrino experiments.The mass matrices obtained for leptons are of the same texture as obtained by Fritzsch and Xing [H. Fritzsch and Z. Xing 2006] with the choice of minimal Higgs fields. A normal hierarchy among neutrino masses has been observed in the model. The values of atmospheric mixing angle and mixing element V_{3e} have been suggested in the model which are consistent with the experimental values. It has already been shown [K. Bandyopadhyay and G. Rakshit 2006] that the model may give rise to leptonic CP violation because of non-vanishing CP-violating rephasing invariant quantity ($J'_{CP} \approx 10^{-8} G_F$) which is large compared to that obtained in the quark sector. It is to be noted that the horizontal gauge bosons are responsible for tree level flavour changing neutral current (FCNC), but the effective four fermion gauge coupling should be of superweak ($G_H \approx 10^{-8} G_F$) in strength.

The plan of the paper is as follows. Section 2 contains the phenomenology of the model under consideration. We discuss result of the work in section 3. Conclusions are given in section 4.

THE PHENOMENOLOGY OF THE MODEL

For three generations of fermions, we consider the left-right-symmetric gauge model with $SU(2)_H$ as horizontal symmetry based on the gauge groups $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$. The representation contents of the leptons (ψ) as well as Higgs fields (Φ, X, η, Δ_L, Δ_R) along with their VEV'S under the gauge groups are given as follows.

$$\psi_L(2, 1, -1, 3) = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\zeta \\ e^- & \mu^- & \zeta^- \end{pmatrix}_L \quad ...(1)$$

$$\psi_R(1, 2, -1, 3) = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\zeta \\ e^- & \mu^- & \zeta^- \end{pmatrix}_R \quad ...(2)$$

$$\langle\phi_{ij}\rangle\,(2, 2, 0, 5) = \begin{pmatrix} a_{ij}\exp(i\theta_{ij}) & 0 \\ 0 & a'_{ij}\exp(i\theta'_{ij}) \end{pmatrix} \quad ...(3)$$

$$\langle X_{ij}\rangle\,(2, 2, 0, 3) = \begin{pmatrix} b_{ij}\exp(i\alpha_{ij}) & 0 \\ 0 & b'_{ij}\exp(i\alpha'_{ij}) \end{pmatrix} \quad ...(4)$$

$$\langle\eta\rangle(1, 1, 0, 3) = h_k \exp(i\beta_k) \quad ...(5)$$

where $i,\ j, k = 1, 2, 3$

$$< \Delta_L > (3, 1, 2, 1) = \begin{pmatrix} 0 & 0 \\ V_L & 0 \end{pmatrix} \quad ...(6)$$

$$< \Delta_R > (1, 3, 2, 1) = \begin{pmatrix} 0 & 0 \\ V_R & 0 \end{pmatrix} \quad ...(7)$$

The Higgs fields Φ and X are responsible for generating masses of the leptons and left-handed, right-handed as well as horizontal gauge bosons. η gives masses to the horizontal gauge bosons only while Δ_L and Δ_R have been introduced to break the left-right-symmetry.

The most general renormalizable Higgs potential is

$$V = V_\phi + V_X + V_\eta + V_\Delta + V_{\phi\Delta} + V_{X\Delta} + V_{\eta\Delta} + V_{X\eta} + V_{\phi X} + V_{\phi\eta} \quad ...(8)$$

The minimization of the Higgs potential gives rise to the following non-vanishing parameters and phase conditions which are listed below as

$$a_{11} = 0,\quad a_{12} \neq 0,\quad a_{13} = 0,\quad a_{22} = 0,\quad a_{23} \neq 0,\quad a_{33} = 0,$$

$$a'_{11} = 0,\quad a'_{12} \neq 0,\quad a'_{13} = 0,\quad a'_{22} = 0,\quad a'_{23} \neq 0,\quad a'_{33} = 0,$$

$$b_{11} = 0,\quad b_{22} \neq 0,\quad b_{33} \neq 0,\quad b'_{11} = 0,\quad b'_{22} \neq 0,\quad b'_{33} \neq 0,$$

and $\theta_{ij} = \theta'_{ij}, \qquad \alpha_{ij} = \alpha'_{ij}$ ... (9)

The most general lepton – Higgs interaction is

$$L = g_1 \overline{\psi^i}_{aL}\psi'_{\beta R}\,\Phi^{ij}{}_{\alpha\beta} + g_2 \overline{\psi}^i{}_{aL}\ \psi^i{}_{\beta R}\ X^{ij}{}_{\alpha\beta} + h.C. \quad ...(10)$$

where, $i,j = 1, 2, 3$ and $\alpha, \beta = 1, 2$

It is to be noted that g_1 and g_2 are independent of $SU(2)_L$, $SU(2)_R$, $SU(2)_H$ indices and hence, g_1 and g_2 are real numbers. Furthermore, $\tilde{\Phi}$ and $\tilde{X}$ terms are prohibited using a discrete symmetry ($\tilde{\Phi} \rightarrow -\tilde{\Phi}, \tilde{X} \rightarrow -\tilde{X}$, but $\Phi \rightarrow \Phi$ and $X \rightarrow X$) for achieving natural flavour conservation in lepton-Higgs coupling. The lepton mass matrices take the forms [H. Fritzsch and Z. Xing 2006] as

$$M_{\text{neutrino}} = \begin{pmatrix} 0 & A_1 & 0 \\ A_1 & B_1 & C_1 \\ 0 & C_1 & D_1 \end{pmatrix}, \quad M_{\text{charged lepton}} = \begin{pmatrix} 0 & A_2 & 0 \\ A_2 & B_2 & C_2 \\ 0 & C_2 & D_2 \end{pmatrix} \qquad \text{...(11)}$$

where

$$A_1 = a_{12} \exp(i\theta_{12}), \quad A_2 = a'_{12} \exp(i\theta_{12}),$$
$$B_1 = b_{12} \exp(i\alpha_{22}), \quad B_2 = b'_{22} \exp(i\alpha_{22}),$$
$$C_1 = a_{23} \exp(i\theta_{23}), \quad C_2 = a'_{23} \exp(i\theta_{23}),$$
$$D_1 = b_{33} \exp(i\alpha_{33}), \quad D_2 = b'_{33} \exp(i\alpha_{33}), \qquad \text{...(12)}$$

The parameters A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 can be expressed in terms of lepton mass eigen values and one free parameter ε' (identical in both the charged lepton and neutrinos mass matrices) whose value could be determined from neutrino experimental results [Y. Ashie *et al*. 2004; and G.C. Branco, and M.N. Rebelo 2005] and found to lie between 1.3×10^{-2} eV and 2.4×10^{-2} eV. The parameters can be written as

$$A_1 = (2m_1m_2)^{1/2}, B_1 = (m_3/3)[1+2.(m_1-m_2/m_3)], C_1 = (m_3/2)[1-(m_2-m_1/m_3)],$$
$$D_1 = m_3/2, \; A_2(m_e m_\mu)^{1/2}, B_2 = m_\mu, \; C_2 = (\varepsilon' m_\zeta)^{1/2}, \; D_2 = m_\zeta \qquad \text{... (13)}$$

In equation (13), charged lepton mass hierarchy ($m_e \ll m_\mu \ll m_\zeta$) is considered and m_1, m_2 and m_3 represent neutrino masses considering the possibility that the neutrino masses are nearly degenerate although only mass squared differences can be measured in neutrino oscillation experiments.

Now, we can write the mixing matrix as (in case of $SU(2)_H$ gauge model we can have majorana neutrinos with negligibly small mixing and, hence do not consider in the present work)

$$V = \begin{pmatrix} C_1 & S_1 & 0 \\ -S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \exp(-i\phi) & 0 & 0 \\ 0 & C & S \\ 0 & -S & C \end{pmatrix} \begin{pmatrix} C_\nu & -S_\nu & 0 \\ S_\nu & C_\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{...(14)}$$

In the above expression, S_ν represent θ_ν (θ_ν : Solar mixing angle), S stands for sin θ (θ : atmospheric mixing angle) and S_1 denotes $\sin\theta_1$ (θ_1 : reactor mixing angle) and similarly we can write cosine for C_ν, C and C_1. The reactor mixing angle has not been measured precisely in all experiments. However, from CHOOZ experiment [M. Apollonio et al.1999] an upper limit of 13° has been given. The experimental results for the solar and atmospheric mixing angles [Y. Ashie *et al*.2004; and G.C. Branco, and M.N. Rebelo 2005] are

$$30° \le \theta_\nu \le 39° \quad \text{and} \quad 37° \le \theta_\nu \le 53° \qquad (15)$$

Thus, neutrino oscillation can be thought of in this model since the neutrinos are mixtures of mass eigen states and it will be discussed elsewhere. In general, non-vanishing phase term, exp(–iΦ), exists in mixing matrix V and the phase can never be roated away. Thus, CP violation in lepton sector is strongly favoured in the model. This is mainly because of exchange of horizontal gauge bosons at tree level (K. Bandyopadhyay, A. K. Ray, and A. Raychaudhuri 1988) since CKM matrices in both left-and right-handed sectors are real. The relations between mixing angles and mass eigen values can be obtained as

$$\tan\theta_1 = (m_e / m_\mu)^{1/2} \approx 0.07$$

and $$\tan\theta_{\nu} = (m_1 / m_2)^{1/2} \qquad ...(16)$$

We get the value of the ratio m_1/m_2 as 0.42 if we take solar mixing angle as 33°. Again, oscillation experiments (H. Fritzsch 2009) provide the mass squared differences of the three neutrino mass eigen states:

$$\Delta m^2{}_{21} \approx 8\times10^{-5}\,\text{eV}^2, \quad \Delta m^2{}_{32} \approx 2.5\times10^{-3}\,\text{eV}^2 \qquad ...(17)$$

We also can get the neutrino mass hierarchy (in eV) as

$$3.6\times10^{-3} \le m_1 \le 5.9\times10^{-3}, \quad 8.5\times10^{-3} \le m_2 \le 1.4\times10^{-2}$$

and $$4.4\times10^{-2} \le m_3 \le 5.8\times10^{-2} \qquad ...\ (18)$$

In the present model, the atmospheric mixing angle (θ) could be estimated and it is expressed as

$$\theta = \theta_1 + \theta_2 \approx 38°,$$

where $$\tan\theta_1 = (m_\mu / m_\zeta)^{1/2},\ \tan\theta_2 = (m_2 / m_3)^{1/2} \text{ and } \theta_1 \approx 14°,\ \theta_2 \approx 24° \qquad ...\ (19)$$

This Value (38°) of atmospheric mixing angle is consistent with the experiment. Furthermore, the value of the matrix element V_{3e} of the mixing matrix *V* can be suggested as

$V_{3e} = \sin\theta \sin\theta_1 \approx 0.707\,(m_e / m_\mu)^{1/2} \approx 0.05$ and this could be observed in the upcoming reactor neutrino experiments.

RESULT AND DISCUSSION

We have studied neutrino masses and mixing in the three generation $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$ gauge model. The mass matrices obtained for leptons are of the same texture as obtained by Fritzsch and Xing and S. Kang with the choice of minimal Higgs fields. We could relate the mass eigenvalues of the charged leptons and of the neutrinos to the mixing angles and could also suggest the mass limits of the neutrinos. A normal hierarchy among the neutrino masses has been observed in the model. The values of atmospheric mixing angle and the mixing element V_{3e} have been suggested in the model which are consistent with the experimental values. Furthermore, neutrino oscillation phenomenon and leptonic CP violation are found to be strongly favoured in this extended gauge model since the neutrinos are the mixtures of mass eigen states and the CP violating phase in mixing matrix can not be removed even by redefining the fermion fields. CP violation in the lepton sector arises exclusively due to the exchange of horizontal gauge bosons at tree level (~T eV scale). The value of CP violating rephasing invariant quantity is $J'_{CP} \approx (5.6 - 8.0) \times 10^{-3}$ where $J'_{CP} = (1/m_1 + m_2)$ $(m_1 \, m_2 \, m_e / 8m_\mu)^{1/2}$. This value of J'_{CP} is quite large compared to that in the quark sector (K. Bandyopadhyay, N. Kumar and A. Kumar 2013). Flavour changing neutral current (FCNC) is suppressed in the model due to super weak gauge coupling $G_H \approx 10^{-8} G_P$.

Conclusions

We thus conclude that phenomena of neutrino oscillation and leptonic *CP* violation can arise in the left-right-symmetric gauge model with $SU(2)_H$ as horizontal symmetry. The model can explain smallness of neutrino masses and suggest the experimentally consistent values for neutrino mass hierarchy and atmospheric mixing angle. The mixing element, connecting the first neutrino with the electron, is also suggested to be 0.05 in the model and this could be verified in the upcoming reactor neutrino experiments. *CP* violating rephasing invariant quantity is quite large compared to that in the quark sector and horizontal gauge bosons play dominant role for leptonic *CP* violation.

References

1. Grimus, W., and Rebelo, M. N., *Phys. Rep.,* **281**, 239 (1997).
2. Cabibbo, N. , *Phys. Rev. Lett.* **10**, 531(1963).
3. Kobayashi, M., and Maskawa,T., *Prog. Theor. Physics*, **49**, 652 (1973).
4. Fogli, G.L., Lisi, E., Marrone, A., and Palazzo, *A., Phys. Lett.* **B583**, 149 (2004).
5. Grossman, Y., *Pramana*, **62**, 269 (2004).
6. Bandyopadhyay, K., Ray, K. A., and Sarkar, U., *Phys. Rev*. D33, 3293 (1986) and references therein.
7. Bandyopadhyay, K., and Rakshit, G., *Ind. J. Phys.* 80(1), **67** (2006).
8. Fritzsch, H., and Xing, Z., *Phys. Lett.* **B634**, 514 (2006).
9. Ashie, Y., et al. [Super-Kamiokande Collaboration], *Phys. Rev. Lett.,* **93**,101801 (2004).
10. Branco, C.G., and Rebelo, N.M., *New J. Phys.* 7, 86 (2005) and references therein.
11. Apollonio, M., *et al*. [CHOOZ Collaboration], *Phys. Lett.* **B466**, 415(1999) .
12. Bandyopadhyay, K., Ray, A. K., and Raychaudhuri, A., *Mod. Phys. Lett.* **A3**, 607 (1998).
13. Fritzsch, H., arxiv 0906. 1066 v1[hep-ph] (2009).
14. Bandyopadhyay, K., Kumar, N., and Kumar, A., *Proc. XX* DAE-BRNS HEP Symposium, p.61-62, 13-18 January, Visva-Bharti (2013).

❑