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UNSTEADY FREE CONVECTION FLOW OF A NON-NEWTONIAN FLUID PAST AN IMPULSIVELY STARTED POROUS WALL WITH HEAT AND MASS TRASNFER

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Heat and mass transfer in unsteady free convection flow of a visco-elastic fluid past an impulsively started porous wall have been studied. Rivlin-Ericksen model of the fluid has been chosen. The constitutive equations of the problem have been formulated and solved by perturbation technique applying the boundary conditions Expressions for velocity and temperature skin-friction and rate of heat transfer have been obtained. Graphs are plotted to present the velocity, temperature and concentration profiles, whereas the values of skin-friction, Nusselt number and concentration gradient are entered in tables. It is observed that the decrease in the value of permeability factor reduces the mean velocity as well as the transient velocity of the fluid. Concentration is affected by chemical reaction whereas temperature is influenced by source/sink parameter.

KEYWORDS : Non-Newtonian flow, heat and mass transfer, porous medium.

INTRODUCTION

The unsteady free convectional flow problems are very important from technology point of view as it has got wide applications in the field of chemical engineering, aeronautics, electronics etc. Soundalgekar and Pop [1], have discussed the unsteady flow past an infinite vertical porous plate with constant or variable suction. The free convection flow of Walters B' liquid past an infinite porous plate with constant suction when the wall temperature is a function of time is studied by Soundalgekar [2]. Mishra [3] *et al*. have solved the energy equation in the case of a particular class of non-Newtonian fluid in which the co-effecients of viscosity, besides being functions of physical properties, is also function of invarants of the rate of strain tensor. Agarwal and Upmanyu [4] have analysed the heat transfer in the presence of temperature dependent heat sources in a second order fluid flowing over a flat plate with uniform suction. The problem of unsteady free convective flow from vertical plate in the presence of a magnetic field with the wall temperature as a function of time has been presented by Mishra and Mohapatra [5]. However, the analysis of temperature field as modified by the generation or absorption of heat in moving fluids is important in view of several physical problems such as (i) problems concerned with chemical reaction [6] and (ii) problems concerned with dissociating fluids [7, 8]. Teipe [9] studied the problem of the impulsive motion of a flat plate in a visco-elastic fluid. Choubey [10] has analysed the hydromagnetic flow of an electrically conducting liquid (Rivlin-Ericksen) [11] near an infinite horizontal flat plate started impulsively from rest in its own plane with constant velocity subjected to an externally applied uniform transverse magnetic field. Yadav and Singh [12] have studied the impulsive motion of a porous flat plate in an elastico-viscous liquid (Rivlin-Ericksen) under the influence of uniform transverse magnetic field. They have observed that the velocity of the liquid is maximum near the porous plate *i.e.* $y = 0$ and decreases with increase of y, when the magnetic field is being considered as constant.

Singh and Naveen Kumar [13] have studied the free convection flow of an incompressible viscous fluid past an exponentially accelerated infinite vertical plate. Free convection effect on the flow of an elasto-viscous fluid past an exponentially accelerated vertical plate has been studied by Dash and Biswal [14]. Dash and Ojha [15] have studied the MHD unsteady free convection effect on the flow past an exponentially accelerated vertical plate. Biswal and Mahalik [16] have analysed the unsteady free convection flow and heat transfer of a viscoelastic fluid past an impulsively started porous flat plate with heat sources/sinks. Same researchers have investigated heat transfer in the free convection flow of a viscoelastic fluid inside a porous vertical channel with constant suction and heat sources [17]. Biswal [18] alone has studied heat and mass transfer effects of oscillatory hydromagnetic free convective flow of a viscoelastic fluid past an infinite vertical porous flat plate in the presence of Hall current. Further, Biswal [19] has analysed the unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous wall.

The objective of the present problem is to study the flow, heat and mass transfer of a visco-elastic fluid (Rivlin-Ericksen) past an impulsively started porous wall in the presence of pores.

Basic equations

The second order approximation of the general constitutive equation given by Rivlin-Ericksen [11] can be written as

$$T = -PI + \mu A_1 + \alpha A_1^2 + \beta A_2 \qquad \dots (2.1)$$

where T is the stress tensor
P is the pressure
I is the unit tensor
A_1 and A_2 are the first two Rivlin-Ericksen tensors.
μ, α and β are three material constants.
A_1 and A_2 are given by the symmetric matrices defined by

$$A_1 = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \qquad \dots (2.2)$$

$$A_2 = \frac{\partial}{\partial x_j}\left(\frac{DV_i}{Dt}\right) + \frac{\partial}{\partial X_j}\left(\frac{DV_j}{Dt}\right) + 2\frac{\partial V_m}{\partial X_i}.\frac{\partial V_m}{\partial X_j} \qquad \dots (2.3)$$

where, ($i. j$, $m = 1, 2, 3$)

Here the wall is porous and horizontal. We take X' axis along the flat wall and Y' axis normal to it. u' is the velocity of the fluid along X' axis and v' is the velocity along Y' axis. Consequently u' is a function of Y' and but v' is independent of Y'.

Let a constant impulsive velocity u be given to the plate in its own plane. For the boundary condition it is assumed that there is no slip at the wall. Thus the flow is governed by the following equations.

Equation of Continuity :

$$\frac{\partial V'}{\partial Y'} = 0 \Rightarrow V' = \text{constant} = -V_0 \tag{2.4}$$

We take V_0 as the suction velocity and negative sign indicates that suction is towards the plate.

Equation of Motion :

$$\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial y'^2} + \frac{K_0}{\rho}\frac{\partial^3 u'}{\partial t' \partial y^2} - \frac{v}{K'}u' + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \quad \ldots (2.5)$$

Equation of Energy :

$$V'\frac{\partial T'}{\partial Y'} + \frac{\partial T'}{\partial t'} = \frac{K}{\rho C_P}\frac{\partial^2 T'}{\partial Y'^2} + S'\left(T' - T'_\infty\right) \quad \ldots (2.6)$$

Equation of Concentration :

$$\frac{\partial C'}{\partial t'} + V'\frac{\partial C'}{\partial y'} = D\frac{\partial C'}{\partial y'^2} + \lambda', \quad \ldots (2.7)$$

where $\lambda' = -k''\left(C' - C'_\infty\right)^n$

ρ, density of the fluid

$v\left(= \frac{\eta_0}{\rho}\right)$, the co-efficient of kinematic viscosity

k'', the reaction rate constant and n is the order of the reaction followed from Aris[20]

K_0, the volume coefficient of elasticity of the fluid

K, thermal conductivity of the fluid

C, the specific heat of the fluid at constant pressure,

β, the co-efficient of thermal expansion

β^*, the co-efficient of mass expansion

g, acceleration due to gravity, S, the source/sink term.

K', the dimensional permeability parameter.

S', the source/sink parameter,

The boundary conditions imposed are

$$t' \le 0, \quad u' = 0, \ T' = T'_\infty, \ C' = C'_\infty, \text{ for } Y' \ge 0$$

$$t' > 0,\ u' = U,\ T' = T'_\infty + \in \left(T'_w - T'_\infty\right) e^{iw't'}, C' = C'_\infty + \in \left(T'_w - T'_\infty\right) e^{iwt''}\ for\ Y' = 0$$

$$u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty\ \ for\ Y' \to \infty \qquad \ldots (2.7)$$

We introduce the following non-dimensional quantities

$$\left.\begin{aligned} & Y = \frac{Y'}{\sqrt{vT}}, u = \frac{u'}{U}, t = \frac{t'}{T}, R_c = \frac{K_0}{\eta_0 T} \\ & V = \frac{v'}{U}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ P_r = \frac{\nu\rho C_p}{K} \\ & G_c = \frac{\nu g\beta^*\left(C'_w - C'_\infty\right)}{U^3},\ G_r = \frac{vg\beta\left(T'_w - T'_\omega\right)}{U^3},\ S = \frac{4S'\nu_1}{U^2} \\ \text{and} \quad & K^* = \frac{K^1U^2}{v^2},\ K_1 = \frac{\nu K''}{U^2} \end{aligned}\right\} \ldots (2.8)$$

With $\omega = \dfrac{vw'}{U^2}$ and $\nu = \dfrac{n}{\rho}$ in order to transform the equation of the motion and energy into their corresponding non-dimensional form as

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + R_c\frac{\partial^3 u}{\partial t \partial y^2} - \frac{1}{K^*}u + Gr\theta + G_c C \qquad (2.9)$$

$$\frac{\partial \theta}{\partial t} + V\frac{\partial \theta}{\partial y} = \frac{1}{P}\frac{\partial^2 \theta}{\partial y^2} + S\theta \qquad \ldots (2.10)$$

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_1 C \qquad \ldots (2.10a)$$

where,

R_c, the elastic parameter

T', temperature of the fluid

T'_w, temperature of the plate

T'_∞, temperature of the fluid at infinite

G_r, the Grashof Number

G_c, modified Grashof number,

K^*, the dimensionless permeability parameter,

$P_r\left(= \dfrac{\eta_0 C_p}{K}\right)$, the Prandtl number,

K_1, non-dimensional chemical reaction

S, source/sink parameter,

η_0, the coefficient of viscosity of the fluid

Now the modified boundary conditions are

$$T > 0 \begin{cases} u = 1,\ \theta = 1 + \varepsilon e^{i\omega t}\ C = 1 + \varepsilon e^{i\omega t} \ \text{ at } y = 0 \\ u = 0,\ \theta = 0\ C = 0, \text{ at } y \to \infty \end{cases} \quad \dots (2.11)$$

SOLUTIONS OF THE EQUATIONS

Equation (2.9) is a third order differential equation because of the presence of elastic parameter R_c. It requires three boundary conditions to be solved, while the present problem provides only two. Therefore, we apply small parameter perturbation technique following Beard and Walters [21] and assume that

$$u = u_0 + \varepsilon e^{i\omega t} u_1$$

$$\theta = \theta_0 + \varepsilon e^{i\omega t} \theta_1,\ C = C_0 + \varepsilon e^{i\omega t} C_1 \text{ where } \varepsilon << 1 \quad \dots (3.1)$$

Substituting equation (3.1) in (2.9) and (2.10) and then equating the coefficients of θ_0, θ, u_0 and u_1, we obtain the zeroth and first order equation for velocity and temperature as :

Zeroth order equations:

$$u_0'' - Vu_0' - \frac{1}{K^*} u_0 = -G_r \theta_0 - G_c C_0 \quad \dots (3.2)$$

$$\frac{1}{P_r} \theta_0'' - V\theta_0' + S\theta_0 = 0\,, \quad \dots (3.3)$$

$$\frac{1}{S_c} C_0'' - VC_0' - K_1 C_0 = 0 \quad \dots (3.3a)$$

First order equations:

$$\left(1 + i\omega R_c\right) u_1'' - Vu_1' - \left(i\omega + \frac{1}{K^*}\right) u_1 = -G_r \theta_1 - G_c \theta_1 \quad \dots (3.4)$$

$$\frac{1}{P_r} \theta_1'' - V\theta_1' - i\omega\theta_1 = 0, \quad \dots (3.5)$$

$$\frac{1}{S_c} C_1'' - VC_1' - \left(i\omega + K_1\right) C_1 = 0, \quad \dots (3.5a)$$

where the Prime $(')$ denotes differentiation with respect to Y.

The boundary conditions given in equation (2.11) are further modified as

$$t > 0 \begin{cases} u_0 = 1,\ u_1 = 0,\ \theta_0 = 1,\ C_0 = 1,\ \theta_1 = C_1 = 0 \text{ at } y = 0 \\ u_0 = 0,\ u_1 = 0,\ C_0 = 0,\ \ \theta_0 = 0,\ \theta_1 = C_1 = 0 \text{ at } y \to \infty \end{cases} \quad \dots (3.6)$$

Equation (3.3) and (3.5) are second order homogeneous differential equations which are solved separately with the help of boundary conditions given in (3.6) to give.

$$\theta_0 = e^{-P_r v_0 Y} \quad \dots (3.7)$$

$$C_0 = e^{-S_c V_0 y} \quad \dots (3.7a)$$

and $$\theta_1 = e^{-\frac{1}{2}\left[P_r V_0 + \sqrt{\left(P_r^2 V_0^2 + 4i\omega\left(P_r + S\right)\right)}\right]y} = e^{-a_1 y} \quad \ldots (3.8)$$

$$C_1 = e^{\frac{1}{2}\left[S_c V_0 + \sqrt{S_c^2 V_0^2 + Aiw\left(S_c + K_1\right)}\right]} = e^{-a_2 y} \quad \ldots (3.8a)$$

where $$a_1 = \frac{1}{2}\left[P_r V_0 + \sqrt{\left(P_r^2 V_0^2 + 4i\omega\left(P_r + S\right)\right)}\right],$$

$$a_2 = \frac{1}{2}\left[S_c V_0 + \sqrt{S_c^2 V_0^2 + 4iw\left(S_c + K_1\right)}\right],$$

and $V = -V_0$, obtained from the solution of equation of continuity (2.4) Hence, the temperature field is given by

$$\theta = \theta_0 + \in e^{i\omega t}\theta_1 = e^{-P_r V_0 Y} + \in e^{i\omega t} e^{-a_1 Y}, \quad \ldots (3.9)$$

$$C = C_0 + \varepsilon e^{i\omega t} C_1 = e^{-S_c V_0 y} + \varepsilon e^{iwt} e^{-a_2 y}, \quad \ldots (3.9a)$$

where $i = \sqrt{-1}$ the imaginary term.

Next, the values of θ_0 and C_0 from equation (3.7) and (3.7a) respectively are put in equation (3.2) which is a second order inhomogeneous differential equation. The complimentary function (C.F) and Particular integral (P.I.) of this equation are found out to be

$$\text{C.F.} = C_1 e^{-a_3 y} + C_2^{a_3 y}$$

and $$\text{P.I.} = -\frac{G_r e^{-P_r V_0 Y}}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}} - \frac{G_c e^{-S_c V_0 Y}}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}$$

Hence, the complete solution of equation (3.2) is u_0 = C.F. + P.I.

Using the boundary conditions (3.6) the constants C_1 and C_2 are evaluated to be

$$C_1 = 1 + \frac{G_r + G_c}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}} \text{ and } C_2 = 0$$

Thus $$u_0 = \left[1 + \frac{G_r + G_c}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}\right] e^{-a_3 Y} - \left[\frac{G_r e^{-P_r V_0 Y}}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}\right] - \left[\frac{G_c e^{-S_c V_0 y}}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}\right] \quad \ldots (3.10)$$

where $$a_3 = \frac{1}{2}\left[V_0 - \sqrt{\left(V_0^2 + 4\left(\frac{1}{K^*}\right)\right)}\right], \; a_4 = \frac{1}{2}\left[V_0 - \sqrt{V_0^2 + 4\left(\frac{4}{K^*}\right)}\right]$$

In the similar fashion, equation (3.4) is solved with the help of equation (3.8) and (3.8a) to give u_1 as

$$u_1 = \left[\frac{(G_r + G_c)e^{-a_5 y}}{(1+i\omega R_c)a_1^2 - V_0 a_1 - \left(i\omega + \frac{1}{K^*}\right)}\right] - \left[\frac{G_r e^{-a_1 y}}{(1+i\omega R_c)a_1^2 - V_0 a_1 - \left(i\omega + \frac{1}{K^*}\right)}\right]$$
$$- \left[\frac{G_c e^{-a_2 y}}{(1+i\omega R_c)a_1^2 - V_0 a_1 - \left(i\omega + \frac{1}{K^*}\right)}\right] \quad \dots (3.11)$$

where $$a_5 = \frac{1}{2}\left[\frac{V_0}{1+i\omega R_c} + \sqrt{\left\{\left(\frac{V_0}{1+i\omega R_c}\right)^2 + 4\left(\frac{i\omega + \frac{1}{K^*}}{1+i\omega R_c}\right)\right\}}\right]$$

Finally, the velocity field is given by

$$u = u_0 + \varepsilon\, e^{i\omega t} u_1$$
$$= \left[1 + \frac{G_r + G_c}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}\right] e^{-a_3 Y} - \left[\frac{G_r e^{-P_r V_0 Y}}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}\right] - \left[\frac{G_c e^{-S_c V_0 y}}{P_r^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}\right]$$
$$+ \in e^{i\omega t}\left[\frac{(G_r + G_c)e^{-a_5 Y}}{(1+i\omega R_c)a_1^2 - V_0 a_1 - \left(i\omega + \frac{1}{K^*}\right)} - \frac{G_r e^{-a_1 Y}}{(1+i\omega R_c)a_1^2 - V_0 a_1 - \left(i\omega + \frac{1}{K^*}\right)}\right.$$
$$\left. - \frac{G_c e^{-a_2 Y}}{(1+i\omega R_c)a_1^2 - V_0 a_1 - \left(i\omega + \frac{1}{K^*}\right)}\right] \quad \dots (3.12)$$

It is observed that the final expression for velocity (3.12) and temperature (3.9) contain imaginary terms along with real parts. In order to separate the real and imaginary parts, equations (3.12) and (3.9) can be written as

$$u = P_5 e^{a_5 y} - P_4\left[e^{-P_r V_0 Y} + e^{-S_c V_0 y}\right] + \in\left[(M_r \cos\omega t - M_i \sin\omega t)\right.$$
$$\left. + i\,(M_r \sin\omega t + M_i \cos\omega t)\right] \quad \dots (3.13)$$

and $$\theta = \theta_0 + \varepsilon\,[(\theta_r \cos\omega t) - \theta_i \sin\omega t) + i\,(\theta_r \sin\omega t + \theta_i \cos\omega t)] \quad (3.14)$$

$$C = C_\infty + \varepsilon\,(C_r \cos\omega t) - C_2 \sin\omega t) + i\,(C_r \sin\omega t + C_i \cos\omega t)] \quad (3.14a)$$

Where the constants are

$$P_1 = \frac{1}{2}P_r V_0 + \frac{1}{2}\sqrt{\left(\frac{\gamma_1 + x_1}{2}\right)}$$

$$Q_1 = \frac{1}{2}\sqrt{\left(\frac{\gamma_1 - x_1}{2}\right)}$$

$$x_1 = P_r{}^2 V_0{}^2$$

$$\gamma_1 = \sqrt{\left\{\left(P_r^2 V_0^2\right)^2 + \left(4\omega P_r\right)^2\right.}$$

$$A_1 = \frac{V_0}{2\left(1+\omega^2 R_c^2\right)}$$

$$A_2 = \frac{\omega V_0 R_c}{2\left(1+\omega^2 R_c^2\right)}$$

$$A_3 = \frac{4\left(\frac{1}{K^*} + \omega^2 R_c\right)}{1+\omega^2 R_c^2}$$

$$A_4 = \frac{4\omega\left(1 - \frac{1}{K^*} R_c\right)}{1+\omega^2 R_c^2}$$

$$A_5 = 4\,(A_1{}^2 - A_2{}^2)\,A_3, \quad A_6 = A_4 - 8A_1A_2$$

$$P_2 = A \;+ \frac{1}{2}\sqrt{\left(\frac{\gamma_2 + x_2}{2}\right)}$$

$$Q_2 = \frac{1}{2}\sqrt{\left(\frac{\gamma_2 - x_2}{2}\right)}$$

$$\gamma_2 = \sqrt{\left(A_5^2 + A_6^2\right)}$$

$$x_2 = A_5$$

$$Q_2' = Q_2 - A_2$$

$$A_7 = \left(P_1^2 - Q_1^2 - 2\omega R_c P_1 Q_1 - V_0 P_1 - \frac{1}{K^*}\right)$$

$$A_8 = \left(2P_1 Q_t + \omega R_c P_1^2 - \omega R_c Q_1^2 - V_0 Q_1 - \omega\right)$$

$$P_3 = \frac{A_7\left(G_r + G_c\right)}{A_7^2 + A_8^2}$$

$$Q_3 = \frac{A_8\left(G_r + G_c\right)}{A_7^2 + A_8^2}$$

$$P_4 = \frac{G_r + G_c}{P^2 V_0^2 - P_r V_0^2 - \frac{1}{K^*}}$$

$$P_5 = 1 + P_4$$

$$P_6 = e^{P_2 Y} \cos Q_2' Y$$

$$P_7 = e^{-P_2 Y} \sin Q_2' Y$$

$$P_8 = e^{-P_1 Y} \cos Q_1' Y \ , P_9 = e^{-P_1 Y} \sin Q_1' Y$$

$$M_r = P_3 P_6 - Q_3 P_7 - P_3 (P_8 + P_{10}) + Q_3 (P_9 + P_{11}]$$

$$M_i = Q_3 [P_8 + P_{10}] + P_3 [P_9 + P_{11}] - Q_3 P_6 - P_3 P_7$$

$$\theta_r = P_8, \theta_I = P_9$$

$$C_r = P_{10}, C_i = P_{11}$$

Thus the velocity and temperature field in terms of the fluctuating parts (real) are

$$u(y, t) = u_0(y) + \varepsilon (M_r \cos \omega t - M_i \sin \omega t) \quad \ldots (3.15)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon (\theta_r \cos \omega t - \theta_1 \sin \omega t) \quad \ldots (3.16)$$

$$C(y, t) = C_0(y) + \varepsilon (C_r \cos \omega t - C_i \sin \omega t), \quad \ldots (3.17a)$$

Now, the transient velocity and transient temperature for $\omega t = \frac{\pi}{2}$ are given by

$$u = u_0 - \varepsilon M_i$$

$$\theta = \theta_0 - \varepsilon \theta_i, \ C = C_0 - \varepsilon C_1 \quad \ldots (3.17)$$

Mean Skin Friction :

The value of the mean skin friction at the wall is given by

$$\tau_w^m = \left.\frac{du_0}{dy}\right|_{y=0} + R_c \left.\frac{d^2 u_0}{dy^2}\right|_{y=0} \quad \ldots (3.18)$$

and putting $u_0 = P_5 e^{-a_3 y} - P_4 \left[e^{-P_r v_0 y} + e^{-S_c V_0 y} \right]$,

mean Skin friction is found out to be

$$\tau_w^m = P_5 (R_c a_3 - 1) a_3 - P_4 (R_c P V_0 - 1) P V_0 \quad \ldots (3.19)$$

Skin Friction :

The value of the skin friction at the wall is given by

$$\tau_w^m = \left.\frac{du}{dy}\right|_{y=0} + R_c \left.\frac{d^2 u}{dy^2}\right|_{y=0} \quad \ldots (3.20)$$

and is found out to be

$$\tau_w = T_w^m + \in [(B_r \cos \omega t - B_i \sin \omega t) + i (B_i \cos \omega t + B_r \sin \omega t) \quad \ldots (3.21)$$

where
$$Br = (P_1 P_3 - P_2 P_3 + Q_1 Q_3 - Q_2' Q_3) + R_c$$
$$(P_2^2 P_3 - P_3 Q_2'^2 - P_1^2 P_3 + P_3 Q_1^2 + 2 P_2 Q_3 Q_2' - 2 P_1 Q_1 Q_3)$$

$$B_i = (P_3 Q_1 - P_3 Q_2' - P_1 Q_3 + P_2 Q_3) + R_c$$
$$(2 P_3 P_3 Q_2' - 2 P_1 P_3 Q_1 - P_2^2 Q_3 + Q_2'^2 Q_3 + Q_3 P_1^2 - Q_1^2 Q_3)$$

Taking only the real part of equation (3.21) we get

$$\tau_w = T_w^m + \in [(B_r \cos \omega t - B_i \sin \omega t) = \tau_w^m + \varepsilon\, |B| \cos(\omega t + \alpha) \qquad \dots (3.22)$$

where $|B| = \sqrt{\left(B_r^2 + B_i^2\right)}$

$$\tan\,\alpha = \frac{B_i}{B_r}$$

Mean rate of heat transfer :

The mean rate of heat transfer is given by

$$Nu_0 = -\left.\frac{\partial \theta_0}{\partial y}\right|_{y=0} = PV_0 \qquad \dots (3.23)$$

Rate of heat transfer:

The rate of heat transfer at the wall is given by

$Nu = -\left.\dfrac{\partial \theta}{\partial y}\right|_{y=0}$, which contains the real and imaginary parts as well.

Taking the real part only, we have

$$\begin{aligned} Nu &= Nu_0 + \varepsilon\,(H_r \cos \omega t - H_i \sin \omega t) \\ &= Nu_0 + \varepsilon\,|\,H\,| \cos(\omega t - \beta) \qquad \dots (3.24) \end{aligned}$$

where

$$H_r = P_1$$

$$H_i = Q_1$$

$$|\,H\,| = \sqrt{\left(H_r^2 + H_i^2\right)}$$

$$\tan\beta = \frac{H_i}{H_r}$$

Concentration gradient :

$$CG_0 = -\left.\frac{\partial C_0}{\partial y}\right|_{y=0}, \qquad \dots (3.25)$$

and

$$CG = -\left.\frac{\partial C}{\partial y}\right|_{y=0}, \qquad \dots (3.26)$$

Putting the value of C_0 from (3.7a) in the equation (3.25), we obtain

$$\begin{aligned} CG_0 &= -\frac{\partial}{\partial y}\left.\left(e^{-S_c V_0 y}\right)\right|_{y=0} = S_c V_0\, e^{-S_c V_0 y}\Big|_{y=0} \\ &= S_c V_{0,} \qquad (3.27) \end{aligned}$$

Similarly, putting the value of C from equation (3.9a) in the equation (3.26), we have

$$CG = -\frac{\partial}{\partial y}\left(e^{-S_c V_0 y} + \in e^{iwt} e^{-a_2 y}\right)\Big|_{y=0}$$

$$= S_c V_0 + \varepsilon a_2\, e^{i\omega t} = S_c V_0 + \varepsilon a_2\,[\cos \omega t + \sin \omega t]$$

$$= CG_0 + \varepsilon\,(Q_r \cos \omega t - Qi \sin \omega t)$$

$$= CG_0 + \varepsilon\,|\,Q\,|\cos(\omega t - r), \qquad \ldots (3.28)$$

where $Q_r = P_2,\ Q_i = Q_2,\ |\,Q\,| = \sqrt{\left(Q_r^2 + Q_i^2\right)},\ \tan r = \dfrac{Q_i}{Q_r}$

RESULTS AND DISCUSSIONS

Numerical results for velocity, temperature, skin friction and the rate of heat transfer are obtained with the help of the software facilities available at the computer centre of Utkal University, Bhubaneswar. Velocity and temperature profiles are plotted and shown by the curves of Fig. 1-5. while the values of shear treases and the rate of heat transfer at the wall are entered in Tables 1- 8.

Fig. 1 exhibits the effects of G_c on the velocity *u*. It is observed that the increase in G_c increases the velocity *u*.

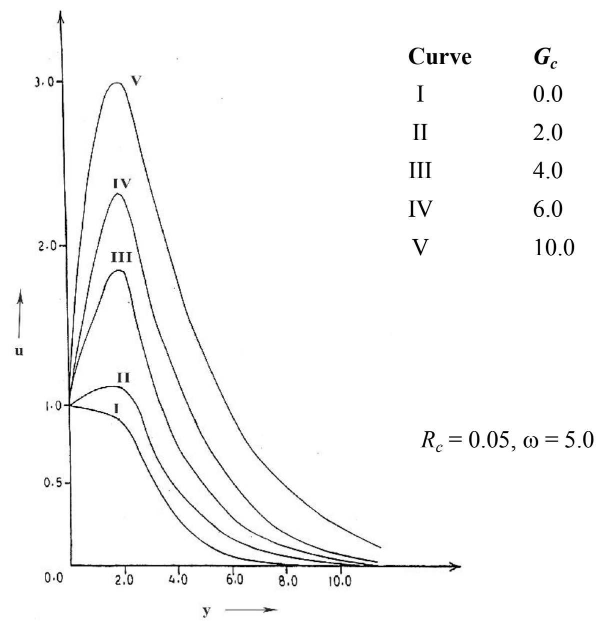

Fig 1. Effects of G_c on transient velocity *u*.

The effects of R_c, G_r and K^* on the transient velocity *u* are shown in Fig. 2. The behaviour of transient velocity with the elastic property of the fluid is evidently very peculiar (curves I and II). The transient velocity first rises and then fall switch the rise of R_c. The point of transition lies at a distance $2.0 < Y < 4.0$. Increase in Grashof number produces a sharp rise in the transient velocity while a decrease in the value of the permeability parameter reduces the transient velocity. Thus the permeability of the pores controls the flow of the fluid.

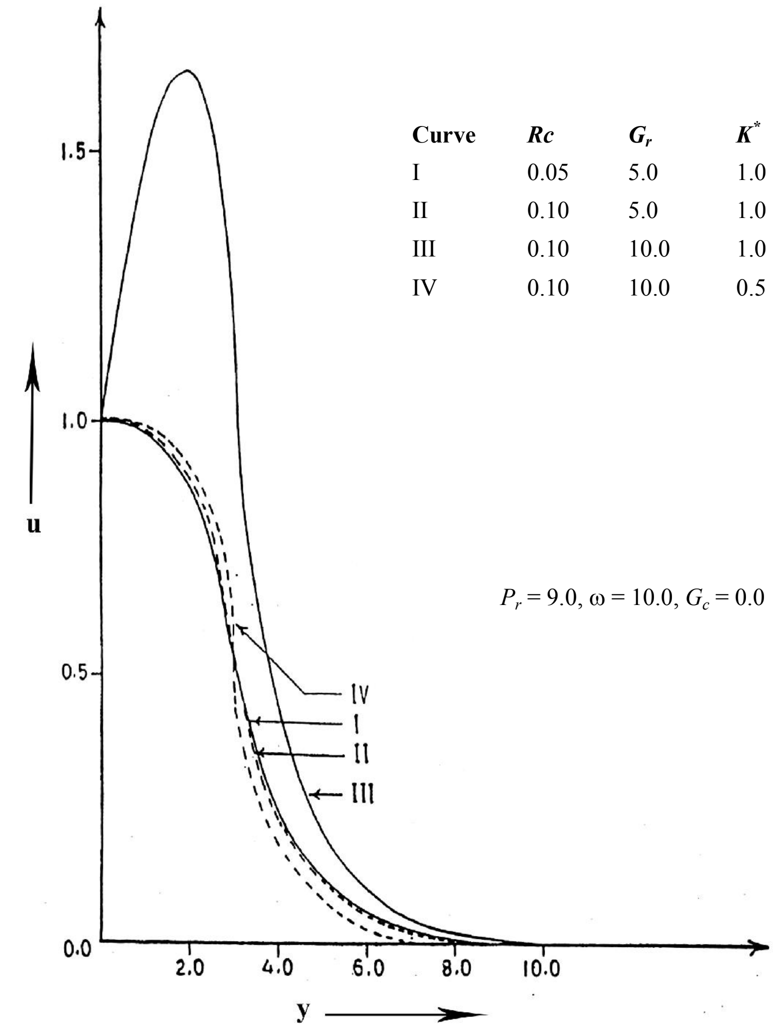

Fig 2 : Effects of R_c G_r and K^* on transient velocity (u).

Fig. 3 shows the influences of P_r, ω and K^* on transient velocity. As K^* decreases the transient velocity decreases from the same maximum $u = 1.0$. It is also noticed that the rise in the values of ω enhances the mean velocity slowly (curves II and III). Increase in Prandtl number decreases the transient velocity similar to that observed in case of permeability parameter K^*. However, the nature of the curves of transient velocity for all types of variations of P_r, ω and K^* is alike with the increase of distance (y) from the plate.

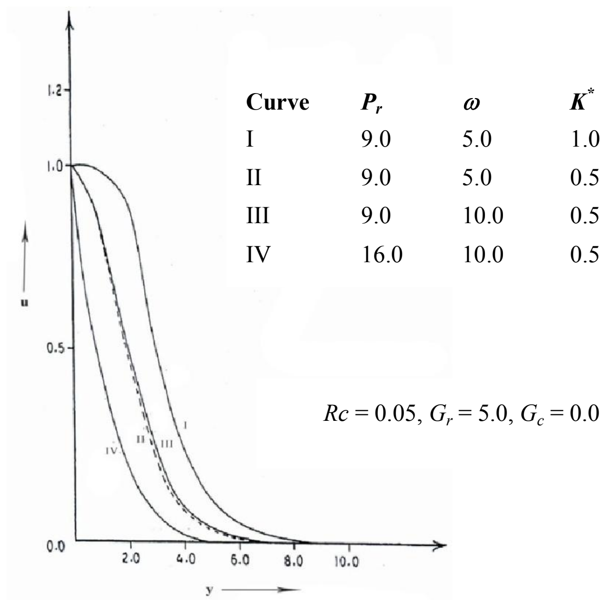

Fig 3 : Effects of P_r ω and K^* on transient velocity (u).

Mean temperature distributions in the fluid with Prandtl number are shown by the curves of Fig. 4. It is observed that the increases in Prandtl number decreases the mean temperature. Fig. 5 exhibits the effects of P_r and ω on transient temperature 0. As ω increases the transient temperature decreases and similar effect is marked in case of P_r.

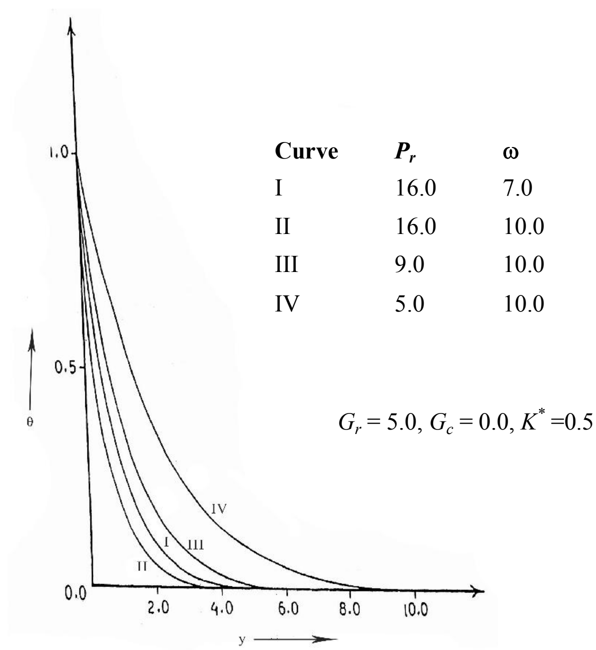

Fig. 4 : Effect of P_r and ω on transient temperature (θ)

Fig. 5 shows the effects of S_c on concentration while the other parameters are fixed. It is observed that the concentration decreases with the rise of S_c but the nature of the profiles with respect to the distance from the porous wall remains same irrespective of the values of Schmidt number.

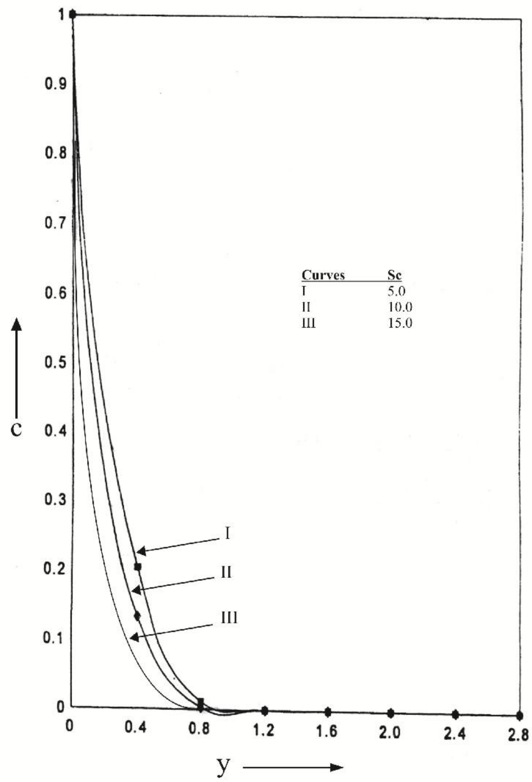

Fig. 5 : Effects of S_c on concentration C for $P_r = 2.0$, $K_1 = 4$

Table : 1. Effect of P_r, G_r, K^* and R_c on mean skin-friction (τ_W^m) for $\omega = 5.0$.

P_r	G_r	K^*/R_c	1.0	2.0	3.0
		100	1.6496	0.7423	0.1531
	5.0	20	1.44138	5.8860×10^{-1}	5.2366×10^{-2}
9.0		10	1.2331	0.4349	– 0.484
		100	4.3505	2.9497	2.0890
	10.0	20	3.8788	2.5350	1.7285910
		10	3.4070	2.1202	1.3681
		100	0.9086	0.2212	– 0.2597
	5.0	20	7.0403×10^{-1}	7.0082×10^{-2}	-3.5837×10^{-1}
		10	0.4995	– 0.0810	– 0.4571
		100	2.89684	1.9075	1.2634
	10.0	20	2.4041	1.4979	9.071153×10^{-1}
		10	1.9397	1.0884	0.5508

Table 1 has been prepared to show the effects of elastic parameter. Prandtl number. Grashof number and permeability parameter on the mean skin friction. It is noticed that the mean skin friction decreases with increase of R_c and P_4 but increases with rise of Grashof number, keeping all other parameters fixed. The mean skin friction decreases with the decrease of permeability parameter K^*.

Table – 2. Effects of P_r, G_r, ω, K^* and R_c on mean skin-friction (τ_w)

P_r	G_r	ω	K^*/R_c	1.0	2.0	3.0
			0.0	1.7302	8241	2358
		5.0	0.05	1.4558250	6.048250×10^{-1}	7.006954×10^{-2}
	5.0		0.10	1.2720	0.4718	– 0.0132
			0.0	1.7060310	0.7994	0.2107
		10.0	0.05	1.4533400	5.997936×10^{-1}	0.281819×10^{-2}
			0.10	1.2761710	0.4773	– 0.0066
9.0			0.0	4.5117	3.1134	2.2545
		5.0	0.05	3.9076	2.5674	1.7639980
	10.0		0.10	3.4848	2.1941	1.4385
			0.0	4.4635	3.0638	2.2041
		10.0	0.05	3.9027	2.5574	1.74495
			0.10	3.49307990	2.2050730	1.4517
			0.0	0.9727	0.2860	– 0.1493
		5.0	0.05	7.056734×10^{-1}	7.281967×10^{-1}	-3.547109×10^{-1}
	5.0		0.10	0.5473	– 0.0345	– 0.4116
			0.0	0.9536	0.2666	– 0.2140
		10.0	0.05	0.7221	8.762772×10^{-2}	-3.412881×10^{-1}
			0.10	0.5444	– 0.0365	– 0.4129
16.0			0.0	2.9966	2.0371	1.3942
		5.0	0.05	2.4073	1.5034	9.144366×10^{-1}
	10.0		0.10	2.0353	1.1815	0.6417
			0.0	2.9585	1.9983	1.35548
		10.0	0.05	2.4401	1.5330	0.9413
			0.10	2.0296	1.1775	0.6392

Table 2 enables us to study the dependence of skin friction on P, G, ω, K^* and R_c. It is observed that the increase in the value of Prandtl number (P) causes a lowering in the value of skin friction. A similar effect is also observed for R_c and $\frac{1}{K^*}$. But the increase in Grashof number enhances the skin friction. What is most interesting is that the skin friction falls with the rise in ω for viscous fluid ($R_c = 0$) while for visco-elastic fluid ($R_c > 0$) there is no remarkable change

Some numerical values of the amplitude $| B |$ and phase angle tan α of the skin friction are presented in tables 3 and 4 respectively. From table 3 it is evident that the amplitude $| B |$

decreases with the fall in the value of K^*. R_c and ω. However as G increases $|B|$ also increases.

Table 3. Effects of G, ω, K^* and R_c on $|B|$ for $P = 9.0$

P_r	ω	K^*/R_c	1.0	0.5	0.3
		0.0	0.5404	0.5377	0.5339
	5.0	0.05	3.731497×10^{-1}	3.667688×10^{-1}	3.596417×10^{-2}
5.0		0.10	0.3112	0.3011	0.2909
		0.0	3.862851×10^{-1}	0.3857	0.3849
	10.0	0.05	2.788343×10^{-1}	2.264832×10^{-1}	2.239676×10^{-1}
		0.10	2.178482×10^{-1}	0.2148	0.2118
		0.0	1.0807	1.0754	1.0644
	5.0	0.5	0.7463	0.7335	7.192835×10^{-1}
10.0		0.10	0.6225	0.6022	0.5818
		0.0	0.7726	0.7715	0.7699
	10.0	0.05	0.4577	0.4530	0.4479
		0.10	4.356963×10^{-1}	4.296915×10^{-1}	4.236445×10^{-1}

Table – 4. Effects of P_r, ω, K^* and R_c on phase then α of the skin friction

P_r	ω	K^*/R_c	1.0	0.5	0.3
		0.0	– 0.8932	– 0.8526	– 0.8162
	5.0	0.05	– 5.068940	– 4.4096190	– 3.9379840
9.0		0.10	1.2490	1.2879	1.3173
		0.0	-9.325015×10^{-1}	– 0.9102	– 0.8888
	10	0.05	3.6937500	3.9219060	4.1673190
		0.10	1.57683×10^{-1}	0.1610	0.1633
		0.00	– 0.8738	– 0.7424	– 0.8139
	5.0	0.05	– 33.4173300	– 19.6549200	– 14.3895200
16.0		0.10	0.6172	0.6200	0.6186
		0.0	– 0.9163	– 0.8989	– 0.8822
	10.0	0.05	1.6996	1.7360450	1.7711130
		0.10	– 0.1228	– 0.1230	– 0.1239

Table 4 envisages an enhancement of phase angle with the decrease in the value of permeability parameter. It is important to note here that the phase angle is negative for Newtonian fluid *i.e.* $R_c = 0$. For small value of ω = (5.0) it is noticed that tan α decreases as the value of R_c goes from 0 to 0.05 and then increases for $R_c = 0.1$. However, this effect is reversed for ω (= 10.0). As regards the effect of Prandtl number on the phase angle tan α it is concluded that the rise in the value of P_r causes a rise in the tan α value in case of Newtonian fluid and for non-Newtonian fluid it causes a fall in the value of tan α.

Further it is noticed that for Newtonian fluid (R_c = 0.0) and for non-Newtonian fluid having elastic-parameter R_c (= 0.1) the phase tan α decreases with the rise of ω but it increases for R_c = 0.05.

Table 5. Effect of P_r on mean rate of heat transfer (Nu_0) for various values of P_r when V_0 = 0.01

P_r	5.0	9.0	12.0	16.0
Nu_0	0.5	0.9	1.2	1.60

Table 6. Effect of P_r and ω on heat transfer (Nu) for V_0 = 0.1, ε = 0.2

ω/P	5.0	10.0
9.0	1.9408200	2.3331510
16.0	3.0299810	3.5524360

Table 5 and 6 show the effects of P and ω on mean rate of heat transfer and the rate of heat transfer respectively. It is worthy to note here that the increase in Prandtl number increases the value of mean rate of heat transfer and the rate of heat transfer. An increase in ω increases the rate of heat transfer while the mean rate of heat transfer remains unaffected as its mathematical expression does not include ω.

Table 7. Effects of P_r, ω and K^* on | H | for R_c = 0.05, G = 5.0

P_r	ω/K^*	1.0	0.5	0.3
9.0	5.0	3.731497×10^{-1}	3.667688×10^{-1}	3.596417×10^{-1}
	10.0	2.288343×10^{-1}	2.264832×10^{-1}	2.2367×10^{-1}
16.0	5.0	2.743506×10^{-1}	2.639454×10^{-1}	2.640074×10^{-1}
	10.0	1.777074×10^{-1}	1.757561×10^{-1}	1.737325×10^{-1}

Table 8. Effects of P_r and ω on tan β

P_r	ω	tan β
9.0	5.0	$9.0.4282 \times 10^{-1}$
	10.0	9.350950×10^{-1}
16.0	5.0	8.810339×10^{-1}
	10.0	9.143862×10^{-1}

The effect of P_r, ω and K^* on | H | are shown in Table-7, which leads to conclude that | H | decreases with the increase in any one of the above mentioned parameters. The increase in the value of ω increases the phase tan β of the rate of heat transfer as evident from the readings of table 8. It also reveals that the Prandtl number increase lowers the tan β value.

Conclusions

The systematic study of the above problems leads to the following conclusions :

- The velocity of the liquid decreases with the decrease of permeability

- The velocity is maximum near the porous wall and decrease with the increases of distance from the wall.
- The increase in Prandtl number decreases the temperature of the liquid, which is maximum near the plate and reduces with the increase of distance from the plate.

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