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VISCOUS FLUID UNIVERSE COUPLED WITH MASSIVE SCALAR FIELD

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The analytic solutions for viscous-fluid Universe coupled with massive scalar field in the spherically symmetric isotropic line element are investigated, and their physical and geometrical properties are studied from various angles.

INTRODUCTION

The meson particles with the charge of the electron and masses of the order of magnitude of 200 electron masses are found in cosmic rays. These particles have a good deal to do with the nuclear forces. The scalar meson field is a matter field and is associated with zero-spin chargeless particles such as π and k mesons. The study of such a field in general relativity has been initiated to provide an understanding of the nature of space-time and the gravitational field associated with neutral elementary particles of zero spin. Scalar fields, as they help in explaining the creation of matter in cosmological theories, represent matter fields with spinless quanta and can describe the gravitational fields. Yukawa's theory (Yukawa introduced the short-range meson field) is based on the assumption that all interactions must be transmitted through space from point to point by the mediation of a field, which is consistent with the principle of relativity; that is, the equations must be Lorentz-invariant.

The motivation for taking the scalar field in addition to the viscous fluid as energy-momentum tensor is with a view to obtaining solutions for the cosmological model and to study its physical properties. It is noted that all the normal stresses are equal due to the spherical symmetry assumed and the shear viscosity factor drops from the field equations. The bulk viscosity need not be zero for the viscous fluid distribution coupled with the scalar field. The coefficient of bulk viscosity ζ in the process of studying the solutions is found to be accompanied by a change in volume (that is, in density).

FIELD EQUATIONS AND THEIR SOLUTIONS

The line element considered is

$$ds^2 = e^{\gamma} dt^2 - e^{\beta}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right), \qquad \dots (1)$$

where β is a function of r and t; and γ is a function of t only.

The energy-momentum tensor for a viscous fluid is given by

$$T_{\mu\nu} = V_{\mu\nu} + S_{\mu\nu},$$

where $V_{\mu\nu}$ and $S_{\mu\nu}$ are the energy-momentum tensors for the viscous fluid and the massive scalar field. Here

$$V_{\mu\nu} = \rho u_\mu u_\nu + (p - \zeta\theta) H_{\mu\nu} - 2\eta\sigma_{\mu\nu}, \qquad \ldots (2)$$

where p is the isotropic pressure; ρ, the fluid density; ζ and η, the co-efficients of bulk and shear viscosity. $\theta = u^\mu_\mu$; is the expansion factor of the fluid lines, $H_{\mu\nu}$ is the projection tensor defined by $H_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$. $\sigma_{\mu\nu}$ is the shear tensor given by

$$\sigma_{\mu\nu} = \frac{1}{2}\left(u_{\mu;\tau} H^\tau_\nu + u_{\nu;\tau} H^\tau_\mu\right) - \frac{1}{3}\theta H_{\mu\nu}$$

and u_μ is the flow vector satisfying the relation ... (3)

$$g^{\mu\nu} u_\mu u_\nu = 1$$

Thus, in addition, $$S_{\mu\nu} = \phi_\mu \phi_\nu - \frac{1}{2} g_{\mu\nu}\left(\phi_\alpha \phi^\alpha - M^2\phi^2\right), \qquad \ldots (4)$$

where the scalar potential $\phi(r,t)$ satisfies the Klein-Gordon equation

$$g^{\mu\nu}\phi_{;\mu\nu} + M^2\phi = \in \qquad \ldots (5)$$

Here, $\in(r,t)$ is the source density of the scalar field and M is related to the mass of the zero-spin particle by $M = \dfrac{m}{\hbar}$ ($\hbar = \dfrac{h}{2\pi}$, where h is Planck's constant).

Considering the comoving coordinate system, we get

$$u^1 = u^2 = u^3 = 0, \quad u^4 = e^{-\gamma/2}. \qquad \ldots (6)$$

The orthogonality conditions for viscous fluid are satisfied identically, namely

$$H_{\mu\nu}u^\nu = 0, \ \ \sigma_{\mu\nu}u^\nu = 0, \quad \omega_{\mu\nu}u^\nu = 0 \quad \dot{u}_\nu u^\nu = 0 \qquad \ldots (7)$$

where $\omega_{\mu\nu} = \frac{1}{2}\left(u_{\mu;\tau}H^\tau_\nu - u_{\nu;\tau}H^\tau_\mu\right)$ are the rotation tensors, and $\dot{u}_\nu = u_{\nu;\mu}H^\mu$ are acceleration components.

The Einstein field equation $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu}$ gives

$$-e^{-\beta}\left(\frac{\beta'^2}{4} + \frac{\beta'}{r}\right) + e^{-\gamma}\left(\ddot{\beta} + \frac{3}{4}\dot{\beta}^2 - \frac{\dot{\beta}\dot{\gamma}}{2}\right) = -8\pi G(p - \zeta\theta) + 4\pi G(e^{-\beta}\phi'^2 + e^{-\gamma}\dot{\phi}^2 + M^2\phi^2) \quad \ldots (8)$$

$$-e^{-\beta}\left(\frac{\beta'^2}{4} + \frac{\beta'}{r}\right) + e^{-\gamma}\left(\ddot{\beta} + \frac{3}{4}\dot{\beta}^2 - \frac{\dot{\beta}\dot{\gamma}}{2}\right) = -8\pi G(p - \zeta\theta) - 4\pi G\left(e^{-\beta}\phi'^2 - e^{-\gamma}\dot{\phi}^2 - M^2\phi^2\right) \quad \ldots (9)$$

$$-e^{-\beta}\left(\beta'' + \frac{1}{4}\beta'^2 + \frac{2\beta'}{r}\right) + \frac{3}{4}e^{-\gamma}\dot{\beta} = 8\pi G\rho + 4\pi G\left(e^{-\beta}\phi'^2 + e^{-\gamma}\dot{\phi}^2 + M^2\phi^2\right) \qquad \ldots (10)$$

$$-\dot{\beta}' = 8\pi G\phi'\dot{\phi} \qquad \ldots (11)$$

The overhead dot and prime, respectively, denote differentiations w.r. to t and r; and a semi-colon followed by a subscript denotes covariant differentiation.

Subtracting (9) from (8), we have

$$\frac{\beta''}{2} - \frac{\beta'}{2r} - \frac{\beta'^2}{4} = -8\pi G\phi'^2 \qquad \ldots (12)$$

Case I : Here we assume $8\pi G\phi'^2 = \dfrac{\beta'}{2r}$. $\qquad \ldots (13)$

Then from (12) we get $\dfrac{\beta''}{2} - \dfrac{\beta'^2}{4} = 0$ which gives

$$\beta = m - 2\log\left(n - \frac{r}{2}\right), \qquad \ldots (14)$$

where n is an arbitrary constant and m is an arbitrary function of time. Then from (13) and (14) we get

$$\phi = \frac{2}{(8\pi G)^{1/2}} \sin^{-1}\left(\frac{r}{2n}\right)^{1/2} + a, \qquad \ldots (15)$$

where a is an arbitrary constant.

Now from (8) and (9) we have

$$p = \frac{1}{16\pi G}\left[24\pi G\zeta\dot{m}^{-\gamma/2} + 8\pi GM^2\left\{\frac{2}{\sqrt{8\pi G}}\sin^{-1}\left(\frac{r}{2n}\right)^{1/2} + a\right\}^2\right]$$
$$+\frac{3n}{2r}e^{-m} + 2e^{-\gamma}\left(\frac{1}{2}\dot{m}\dot{\gamma} - \ddot{m} - \frac{1}{4}\dot{m}^2\right) - \frac{1}{4}e^{-m} \qquad \ldots (16)$$

Again, from (10) we get

$$\rho = \frac{1}{8\pi G}\left[\frac{3}{4}e^{-\gamma}\dot{m}^2 + \frac{3}{8}e^{-m} - \frac{9n}{4}r^{-1}e^{-m} - 4\pi GM^2\left\{2(8\pi G)^{1/2}\sin^{-1}\left(\frac{r}{2n}\right)^{1/2} + a\right\}^2\right] \qquad \ldots (17)$$

Then from (5) we have

$$\varepsilon = M^2\left\{2(8\pi G)^{-1/2}\sin^{-1}\sqrt{\frac{r}{2c}} + a\right\} + \frac{3}{4}n(8\pi G)^{-1/2}e^{-m}r^{-3/2}\sqrt{2n - r}. \qquad \ldots (18)$$

Case II : We take here $\qquad 16\pi G\phi'^2 = \beta'^2 \qquad \ldots (19)$

Then from (12) we get $\dfrac{\beta''}{2} + \dfrac{\beta'^2}{4} - \dfrac{\beta}{2r} = 0$ which gives

$$\beta = 2\log\left(r^2 - 4g\right) - d, \qquad \ldots (20)$$

where d is an arbitrary constant and g is an arbitrary function of time. Then from (19) we get

$$\phi' = \frac{r}{\sqrt{\pi G}\left(r^2 - 4g\right)} \quad \ldots (21)$$

Again from (11) and (21) we have

$$\dot{\phi} = -\frac{2\dot{g}}{\sqrt{\pi G}\left(r^2 - 4g\right)} \quad \ldots (22)$$

Then (21) and (22) give

$$\phi = \frac{1}{2\sqrt{\pi G}} \log\left(r^2 - 4g\right) \quad \ldots (23)$$

Now from (8) and (9) we get

$$P = \frac{1}{2\pi G}\Big[16e^d\left(r^2 - 4g\right)^{-3} + e^{-\gamma}\left(r^2 - 4g\right)^{-2}\left(2r^2\ddot{g} + 4g\dot{g}\dot{\gamma} - 8g\ddot{g} - 20\dot{g}^2 - \dot{g}\dot{\gamma}r^2\right)$$
$$-24\pi G\zeta\dot{g}e^{-\gamma/2}\left(r^2 - 4g\right)^{-1} - 4e^{-\gamma}\dot{g}^2\left(r^2 - 4g\right)^{-2} + \frac{1}{4}M^2\left\{\log\left(r^2 - 4g\right)\right\}^2\Big] \quad \ldots (24)$$

Again, from (10) we get

$$\rho = \frac{1}{\pi G}\left[e^{-\gamma}\dot{g}^2\left(r^2 - 4g\right)^{-2} - 256e^d\left(r^2 - 4g\right)\left(r^2 - 6g\right) - \frac{M^2}{8}\left\{\log\left(r^2 - 4g\right)\right\}^2\right] \quad \ldots (25)$$

From (5) we have

$$\in = \frac{1}{\sqrt{\pi G}}\Big[48\left(r^2 - 4g\right)^{-3}\exp(\gamma + d) - 16\dot{g}^2\left(r^2 - 4g\right)^{-2}$$
$$+\left(2\ddot{g} - \dot{\gamma}\dot{g}\right)\left(r^2 - 4g\right)^{-1} + \frac{1}{2}M^2\log\left(r^2 - 4g\right)\Big]$$

PHYSICAL AND GEOMETRICAL PROPERTIES

Case I : The metric takes the form

$$ds^2 = e^{\gamma}dt^2 - \left(n - \frac{1}{2}r\right)^{-2} e^m\left(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right),$$

where γ and m are arbitrary functions of time. The fluid pressure and density both are found to be decreasing with the increase of radial distance. The "expansion factor" of the fluid lines is given by $\theta = \frac{3}{2}e^{-\gamma/2}\dot{m}$. Here the source density of the scalar field is a decreasing function of r. In this case, the spectral shift will be $\frac{(\lambda+\delta\lambda)}{\lambda}=be^{-\gamma/2}$, where b is an arbitrary constant.

Case II : In this case the metric takes the form

$$ds^2 = e^{\gamma}dt^2 - e^{-d}\left(r^2-4g\right)^2\left(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right),$$

where γ and g are arbitrary functions of time. Thus pressure and density of the fluid here, which are obtained respectively, both are found to be decreasing functions of r. The "expansion factor" of the fluid lines is given by $\theta = 12\dot{g}\left(4g-r^2\right)^{-1}e^{-\gamma/2}$. The source density of the scalar field is a decreasing function of r and the scalar potential ϕ is a decreasing function of time.

The rotation tensors $\omega_{\mu\nu}$ and the shear σ are come out to be zero. For this case, the spectral shift in wavelength, as measured at the origin, will be

$$\frac{(\lambda+\delta\lambda)}{\lambda}=ce^{-\gamma/2},$$

where c is an arbitrary constant.

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