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HEAT AND MASS TRANSFER EFFECTS OF FREE CONVECTION FLOW OF VISCO-ELASTIC FLUID INSIDE A POROUS VERTICAL CHANNEL WITH CONSTANT SUCTION AND HEAT SOURCES INCLUDING CHEMICAL REACTION

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This paper deals with heat and mass transfer effects of free convection flow of a visco-elastic fluid inside a porous vertical channel with constant suction and heat sources including chemical reaction. Mathematical formulation of the problem is developed with Walters' B' fluid model and the velocity equation is solve with the help of small parameter regular perturidation technique. It is deserved that the increase of chemical reaction parameter reduces the concentration.

KEYWORDS : Heat and mass transfer, Visco-elastic fluid, porous channel, constant suction, heat sources, chemical reaction.

Introduction

The phenomenon of mass transfer is try common in the theories of stellar structure, at least on the solar surface. Its origin is attributed to differences in temperature caused by the non-homogeneous production of heat which, in many cases can result not only in the formation of convective currents but also in violent explosions. Mass transfer certainly occurs within the mantles and cores of planets of the size of or larger than the Earth. Moreover, in many processes, mass transfer and heat transfer occur simultaneously. In free convection, these may either hinder or aid one another. Frequent occurrence of transport processes in nature are the atmospheric flows and the flows in water. The atmospheric flows at all scales are driven by both temperature and water concentration difference. The flows in water are caused by the effects of difference, in temperature, concentration or dissolved substances and suspended particles of matter. In technological field, mass transfer cooling methods suggested by Hartnett and Eckert [1] are used to maintain the rate of heat transfer from hot fluid layers to solid surfaces at a minimum level. In recent years, analytical solutions to such problems have been presented by Sparrow [2], Sparrow, Minkowyoz and Eckert [3], Eichhorn [4], Gebhart and Pera [5] and a number of others. In all these papers, only the effects of mass transfer on the free convection flow have been presented in the absence of external magnetic field.

In many engineering applications, combined effects of thermal diffusion and diffusion of chemical species are of importance. In some processes, the foreign gases are injected. This causes a reduction in wall shear stress, the mass transfer conductance or the rate of heat transfer. Usually, H, H_2, O, H_2O, CO_2 etc. are the foreign gases which arc injected ill the air flowing past bodies. Sometimes, the evaporating material is coated on the surface of the body and this evaporates due to heating of the body and mixes with the flow of air past bodies. Such processes are the cases of steady free-convective flow with mass transfer. There are many other transfer processes in both industry and environment where both heat and mass transfer occur simultaneously Such branches are ocean dynamics, chemical engineering, aero-space engineering and pollution studies. Mass transfer effects in the absence of external magnetic field have been analysed by many researchers in case of both viscous and visco-elastic fluids Gebhart [6] and also Gebhart and Mollendorf [7] have studied the mass transfer effects on the natural convective flow when the flow field is of extreme size or at extremely low temperature or in high gravity field. Soundalgekar [8] has studied the mass transfer effects in case of an infinite vertical porous plate with suction. Das and Biswal [9] have analysed the mass transfer effects of free convection flow of a Visco-elastic fluid inside a porous vertical channel with heat sources. Chandrasekhar and Radha Narayan [10] have studied heat and mass transfer by natural convection near a vertical surface embedded in a variable porosity medium.

The investigations of the flows of conducting non-Newtonian fluid in the presence of external transverse magnetic field are of necessity for the possibility of using new working substances in various degrees in every real material. These investigations have revealed a number of effects related to the presence of non-Newtonian properties in the fluid, for instance, in strong magnetic field, even in the presence of insignificant deviations from Newtonian properties, the character of the flow of the fluid can differ considerably from the flow of a Newtonian fluid [11]. Hence, the study of hydromagnetic flow and heat transfer without the variation of species concentration has attracted many researchers. Among them, the works of Mori [12], Yu [13], Soundalgekar and Haldavnekar [14], Gupta and Gupta [15], Sarpkaya [16] and Ray and Agrawal [17] have drawn our attention. To the best knowledge of ours, very few efforts have been taken so far to study the effects of mass transfer on the flow of non Newtonian fluids through porous medium with internal heat generating sources and constant suction. Hossain and Rashid [18] have theoretically analysed the Hall effects on hydromagnetic free convection flow along a porous flat plate with mass transfer. Dash and Ojha [19] have studied the free convection flow of an electrically conducting visco-elastic fluid inside a porous vertical channel with constant suction and heat sources. Datta, Biswal and Sahoo have analysed the magnetohydrodynamic unsteady free convection flow with heat sources and sinks [20]. Combined free and forced convection effects on the magnetohydrodynamic flow of a visco-inelastic fluid through a channel with mass transfer have been investigated by Biswal and Mishra [21]. Biswal and Pradhan [22] have studied the MHD unsteady free convection flow past an infinite plate with constant suction and heat sinks including dissipative heat. Biswal and Mahalik [23] have analysed the unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous flat plate with heat sources/sinks. Biswal, Ray and Mishra [24] have investigated the problem of mass transfer effects on oscillatory hydromagnetic free convective flow past an infinite vertical porous flat plate with Hall current. Rath, Dash and patra [25] have studied the effect of Hall current and chemical reaction on MHD flow along an exponentially accelerated porous flat plate with internal heat absorption/generation. Biswal, Dash and Nayak [26] have analysed the magnetohydrodynamic flow of a viscous conducting fluid past a stretched vertical permeable surface with heat sources/sink and chemical reaction. Sankar, Reddy and Rao [27] have studied the radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption. Barik, Pattnaik and Biswal [28] have investigated the problem of mass transfer effects on magnetohydrodynamic free convection flow of a visco-elastic fluid past an exponentially accelerated vertical porous plate. Very recently Mishra, Ray, Biswal and Jena [29] have studied the Problem of Hall effects on hydromagnetic non-Newtonian convective flow in a rotating channel with mass transfer. Though the literature is replete with copious such studies on MHD flow of visco-elastic fluids, no researcher, so far, has tried to investigate this particular problem with the source effects, chemical reaction and porosity of the medium. In this problem, our aim is to study the mass transfer effects of free convection flow of a visco-elastic fluid inside a porous vertical channel with constant suction and heat sources, taking permeability of the porous medium and chemical reaction into consideration.

Mathematical analysis

The channel under consideration consists of two porous vertical parallel plates P_1 and P_2 separated by a distance 'h'. The plates are infinitely stretched in both the directions of x'-axis, so that at large distance from the entry, the flow is fully developed. The x'-axis is taken in the vertical direction and the y'-axis is perpendicular to it. All the physical variables are therefore considered to be dependent of y'-axis only. The positive direction is taken from P_1 to P_2.

Let u' and v' be the components of the velocity at (x', y'). The derivations of the governing equations for the free convective flow with constant suction and heat sources and without the dissipative heat are presented following Walters' B' liquid model in the presence of porosity of the medium and mass transport.

In accordance with Walters' B' fluid model given by

$$P_{ik} = pq_{ik} + P'_{ik} \qquad \dots (2.1)$$

where P_{ik} = stress tensor,

P = an arbitrary isotropic pressure,

$q_{ik}(x)$ = metric tensor of a fixed co-ordinate system x^i.

We have the equations of continuity, momentum, energy and concentration as follows:

Equation of continuity:

$$\frac{dy'}{dy'} = 0 \qquad \dots (2.2)$$

Which on integration gives

$$v' = \text{Constant}, \qquad \dots (2.3)$$

Hence, we take a constant suction velocity $-v_0$ normal to the wall. Obviously the suction is towards the plate

Equation of motion :

$$\rho_0 v' \frac{du'}{dy'} = \eta_0 \frac{d^2u'}{dy'^2} - k_0^* v' \frac{d^3u'}{dy'^3} + \rho_0 g\beta\left(T' - T_2'\right)\rho_0 g\beta^*\left(C' - C_2'\right) - \frac{n_0}{K'}u' \qquad \dots (2.4)$$

Equation of energy :

$$v'\frac{dT'}{dy'} = \frac{k_0}{\rho_0 C_p}\frac{d^2T'}{dy'^2} + S'\left(T' - T_2'\right) \qquad \ldots (2.5)$$

Equation of Concentration :

$$v'\frac{dc'}{dy'} = D\frac{d^2C'}{dy'^2} + \lambda' \qquad \ldots (2.6)$$

where ρ_0 is the density of the fluid,

G is the acceleration due to gravity

β is the co-efficient of thermal expansion of the fluid,

η_0 is the co-efficient of viscosity of the fluid.

K_0^* is the dimensional elastic parameter

C_p is the specific heat at constant parameter

K_0 is the thermal conductivity of the fluid,

S' is the source/sink term,

D is the chemical molecular diffusivity,

β' is the volumetric co-efficient of expansion with concentration

K' is the dimensional permeability parameter.

Here $\lambda' = -K'(C' - C_2')^n$

where K' is the reaction rate constant and n is the order of the reaction as hold by Aris [30]

The boundary conditions of the physical problem are

$$Y' = 0 \ : \ u' = 0\ ,\ T' = T_1',\ \ C' = C_1'$$

$$Y' = h \ : \ u' = 0\ ,\ T' = T_2',\ \ C' = C_2' \qquad \ldots (2.7)$$

On introducing the following non-dimensional parameters,

$$Y = \frac{Y'}{h},\ \ u = \frac{u'h}{v},\ \ v = \frac{\eta_0}{\rho_0},\ T = \frac{T' - T_2'}{T_1' - T_2'},\ \ C = \frac{c' - c_2'}{c_1' - c_2'}$$

$K = \dfrac{k_0}{\rho_0 C_p}$, the thermal diffusivity,

$K^* = \dfrac{K'}{h^2}$, non-dimensional permeability parameter,

$R_c = \dfrac{k_0^*}{\rho_0 h^2}$, the Elastic number,

$R = \dfrac{v_0 h}{v}$, the Reynolds number,

$G_T = g\beta h^3 \dfrac{(T_1' - T')}{v^2}$, the Grashof number

$G_c = g\beta^* h^3 \dfrac{(C_1' - C_2')}{v^2}$, the Modified Grashof number

$P = \eta_0\, C_p / k_0$ the Prandtl number,

$S = 4\, S'\ v/V_0^2$, the Source parameter,

$S_c = v/D$, the Schmidt number,

$K_1 = \dfrac{\nu k'}{v^2}$, chemical reaction parameter

in equation (2.4) to (2.6), we get

$$RR_c \frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} + R\frac{du}{dy} = -G_T T - G_c C + \frac{1}{K^*} u \qquad \ldots (2.8)$$

$$\frac{d^2T}{dy^2} + RP\frac{dT}{dy} = -\frac{1}{4} R^2 PST \qquad \ldots (2.9)$$

and

$$\frac{d^2C}{dy^2} + RP\frac{dC}{dy} - K_1 S_c C = 0 \qquad \ldots (2.10)$$

with the modified boundary conditions

$$y = 0 : u = 0,\ T = C = 1$$
$$y = 1 : u = 0,\ T = C = 0 \qquad \ldots (2.11)$$

Solution of equations

Now equation (2.8) is a third order differential equation for $R_c \neq 0$ and it reduces to the case of Newtonian fluid when $R_c = 0$. That means the pressure of the elastic property of the fluid increases the order of the governing equations from two to three and therefore, for a unique solution of non-dimensional momentum equation three boundary conditions are necessary. But the present problem provides only two boundary conditions. To overcome this difficulty, we follow the small perturbation technique given by Beard and Walters [31] to obtain the approximate solution of equation (2.8) and expand u in powers of R_e, as $R_c << 1$. Thus, we write

$$u = \sum_{i=0}^{\infty} R_c^i\, u_i \qquad \ldots (3.1)$$

where $i = 0, 1, 2$………. etc.

Substituting (3.1) in (2.8), equating the co-efficient of R_c^0, R_c^i and neglecting those of R_c^2 we obtain

Zeroth order equation:

$$\frac{d^2u_0}{dy^2}+R\frac{du_0}{dy}-\frac{1}{K^*}u_0=-G_T T-G_c C \qquad \ldots (3.2)$$

and first order equation :

$$R\frac{d^2u_0}{dy^3}+\frac{d^2u_1}{dy^2}+R\frac{du_1}{dy}-\frac{1}{K^*}u_1=0 \qquad \ldots (3.3)$$

With modified boundary conditions

$$y = 0 : u_0 = 0,\ u_1 = 0,\ T = C = 1$$
$$y = 1 : u_0 = 0,\ u_1 = 0,\ T = C = 0 \qquad \ldots (3.4)$$

Solution of equation of energy (2.9) :

This is a second order homogeneous differential equation which can be solved by the help of the boundary conditions given in equation (3.4) to give us

$$T=\frac{1}{e^{-Ra_1}-e^{-Ra_2}}\left[e^{-R(a_1+a_2y)}-e^{-R(a_2+a_1y)}\right] \text{ for } S\neq P \qquad \ldots (3.5)$$

and
$$T=\frac{1}{e^{-RP/2}}\left[e^{-RPY/2}-e^{-RP/2}\right]\text{for } S=P \qquad \ldots (3.6)$$

where,
$$a_1=\frac{1}{2}\left(P+\sqrt{P^2-PS}\right)$$
$$a_2=\frac{1}{2}\left(P-\sqrt{P^2-PS}\right) \qquad \ldots (3.7)$$

The parameters $S > 0$ and $S < 0$ represent source and sink respectively. When the source strength $S > P$, the temperature distribution will have an imaginary part. Subsequently, it will affect the velocity field also. Further, for $S < P$, temperature distribution will have only real part. But in case of sink there is no such restriction.

Solution of equation of concentration (2.10):

This is also a second order homogeneous differential equation which can also be solved with the aid of equation (3.4) to give

$$C = e^{-\alpha_1 y} \qquad \ldots (3.8)$$

where
$$\alpha_1=\frac{1}{2}\left[S_c+\sqrt{S_c^2+4K_1S_c}\right]$$

Solution of Zeroth order equation (3.2) :

This is second order in homogeneous differential equation whose complete solution is obtained by finding out the complimentary function (C.F.) and particular integral (P.I.), taking the help (3.5) and (3.8).

Hence,

$$\text{C.F.} = C_1\, e^{\left[\frac{R}{2}-\frac{1}{2}\sqrt{R^2+4\frac{1}{K^*}}\right]y}+C_2\, e^{\left[\frac{R}{2}-\frac{1}{2}\sqrt{R^2+4\frac{1}{K^*}}\right]y}$$

and
$$\text{P.I.} = \frac{G_T}{1-e^{R\left(P^2-PS\right)^{1/2}}} \times \frac{e^{Ra_2 y}}{R^2\left(a_2^2-a_2\right)-\dfrac{1}{K^*}}$$

$$+\frac{G_T e^{R(P^2-PS)^{1/2}}}{1-e^{R(P^2-PS)^{1/2}}} \times \frac{e^{-Ra_1 y}}{R^2(a_1^2-a_2)-\dfrac{1}{K^*}}$$

$$+\frac{G_c}{\dfrac{1}{K^*}(1-e^{RS})}+\frac{G_c e^{RS_c}}{(1-e^{RS_c})} \times \frac{e^{-RS_c y}}{R^2(S_c^2-S_c)-\dfrac{1}{K^*}}$$

The complete solution of (3.2) can be presented as

$$U_0 = \text{C.F.} + \text{P.I.}$$

The constants C_1 and C_2 are evaluated by the help of equation (3.4). Thus, we have

$$u_0 = A_3 + A_1 e^{-Ra_1 y} - A_2 e^{-Ra_2 y} + A_4\ e^{-RS_c y} - (A_1 - A_2 + A_4)\ e^{-Ra_1 y}$$

$$+\frac{1}{e^{-a_1}-e^{-a_2}}[(A_1 - A_2 + A_3 + A_4)\{e^{-(a_1+a_1 y)} - e^{-(a_1+a_2 y)}\}$$

$$+\ (A_2 e^{-a_2 R} - A_1 e^{-a_1 R} - A_3 - A_4 e^{-RS_c})(e^{-a_1 Y} - e^{-a_2 Y})] \quad \ldots (3.9)$$

Solution of first order equation (3.3) :

The value of $\dfrac{d^3U_0}{dy^3}$ is first obtained from (3.9) and is then put in (3.3) to get the in homogeneous from of his second order differential equation. Following the same procedure as is followed in the zeroth order equation, we have the C.F. and P.I. of the equation (3.3) as

$$\text{C.F.} = C_3\ e^{-a_1 Y} - C_4\ e^{-a_2 Y}$$

and
$$\text{P.I.} = A_5\ e^{-Ra_1 Y} - A_6\ e^{-Ra_2 Y}\ +\ e^{-RS_c Y}$$

$$+\frac{4R\alpha_1^3(A_1 - A_2 + A_3 + A_4)e^{-\alpha_1 y}}{\left(R^2+4\left(\dfrac{1}{K^*}\right)^2\right)} - \frac{R}{e^{-a_1}-e^{a_2}}(A_4 - A_2 + A_3 + A_4)\times$$

$$\left\{\frac{4\alpha_1^3 e^{a_1}}{R^2+4\left(\dfrac{1}{K^*}\right)}e^{-a_1 y} - \frac{4\alpha_2^3 e^{a_1}}{R^2+4\left(\dfrac{1}{K^*}\right)}e^{-a_2 y}\right\}\ +\ (A_2 e^{-a_2 R} - A_2 e^{-a_1 R} - A_3 - A_4 e^{-RS_c})$$

$$\times\left\{\frac{4\alpha_1^3 e^{a_1 y}}{R^2+4\left(\dfrac{1}{K^*}\right)} - \frac{4\alpha_2^3 e^{a_2 y}}{R^2+4\left(\dfrac{1}{K^*}\right)}\right\}$$

The complete solution of (3.3) is now given by

$$u_1 = \text{C.F.} + \text{P.I.}$$

The constants C_3 and C_4 are evaluated from the boundary conditions (3.4)

Thus we have,

$$u_1 = A_{11}\ e^{-a_1 Y} - A_{10}\ e^{-a_2 Y}\ - A_5\ e^{-a_1 RY} - A_6\ e^{-a_2 RY}$$
$$+ A_7\ e^{-RS_c y}\ + A_8\ e^{-a_1 Y}\ - A_{13}\ e^{-a_1 y}\ + A_{14}\ e^{-a_2 R} \quad \ldots (3.10)$$

Finally, the velocity of the fluid is given by

$$u = u_0 + R_c\, u_1$$
$$= A_3 + A_1\ e^{-Ra_1 Y} - A_2\ e^{-Ra_2 Y} + A_4\ e^{-RS_c Y} - (A_1 - A_2 + A_4)\ e^{-Ra_1 Y}$$
$$+ \frac{(e^{a_1 Y} - e^{a_2 Y})}{e^{-a_1} - e^{-a_2}}[(A_1 - A_2 + A_3 + A_4)e^{-a_1} + (A_2 e^{a_2 R} - A_1 e^{-a_1 R} - A_3 - A_4 e^{-RS_c})]$$
$$+ R_c\,[A_{11}\ e^{-a_1 Y} - A_{10}\ e^{-a_2 Y}\ + A_5\ e^{-a_1 RY} - A_6\ e^{-a_2 RY}$$
$$A_7 -\ e^{-RS_c Y} + A_8\ e^{-a_1 Y} -\ e^{-a_1 y}\ + A_{14}\ e^{-a_2 R}\,] \quad \ldots (3.11)$$

After substitution from (3.9) and (3.10)

The values of the constants involved in the above equations are

$$\alpha_1 = \frac{1}{2}\left(R - \sqrt{R^2 + 4\left(\frac{1}{K^*}\right)}\right)$$

$$\alpha_2 = \frac{1}{2}\left(R - \sqrt{R^2 + 4\left(\frac{1}{K^*}\right)}\right)$$

$$A_1 = \frac{G_T e^{R\left(P^2 - PS\right)^{1/2}}}{\left[R^2\left(a_1^2 - a_1\right) - \frac{1}{K^*}\right]\left[1 - e^{R\left(P^3 - PS\right)^{1/2}}\right]}$$

$$A_2 = \frac{G_T}{\left[R^2 (a_2^2 - a_2) - \frac{1}{K^*}\right]\left[1 - e^{R(P^3 - PS)^{1/2}}\right]}$$

$$A_3 = \frac{G_c}{\frac{1}{K^*}(1 - e^{RS_c})}$$

$$A_4 = \frac{G_c e^{RS_c}}{\left[R^2 (S_c^2 - S_c) - \frac{1}{K^*}\right](1 - e^{RS_c})}$$

$$A_5 = \frac{R^4 a_2^3 A}{R^2 (a_1^3 - a_1) - \frac{1}{K^*}}$$

$$A_6 = \frac{R^4 a_2^3 A_2}{R^2(a_2^2 - a_2) - \frac{1}{K^*}}$$

$$A_7 = \frac{R^4 S_c^3 A_4}{R^2(S_c^2 - S_c) - \frac{1}{K^*}}$$

$$A_8 = \frac{4Ra_1^3(A_1 - A_2 + A_3 + A_4)}{R^2 + 4\frac{1}{K^*}}$$

$$A_9 = \frac{4R\left(R^2 + 4\frac{1}{K^*}\right)}{(e^{-a_1} - e^{-a_2}) - \left(R^2 + 4\left(\frac{1}{K^*}\right)\right)^{1/2}} [A_1(e^{-a_1} - e^{-Ra_1}) - A_2(e^{-a_1} - e^{-Ra_2})$$
$$+ A_3\ (e^{-a_1} - 1)\ + A_4\ (e^{-a_1} - e^{-RS_c})]$$

$$A_{10} = \frac{1}{e^{-a_1} - e^{-a_2}}$$
$$[(A_5 - A_6 + A_7 + A_8 + A_9)\, e^{-a_1} - A_5\, e^{-Ra_1} + A_6\, e^{-Ra_2} - A_7\, e^{-RS_c} - A_8\, e^{-a_1}$$
$$+ \frac{4R(a_1^3 e^{-a_1} - a_2^3 e^{-a_2})}{(e^{-a_1} - e^{-a_2})\left(R^2 + 4\left(\frac{1}{K^*}\right)\right)} \{(A_1 - A_2 + A_3 + A_4)\ e^{-a_1}$$
$$+ A_2\, e^{-Ra_2}\, A_1\, e^{-Ra_1} - A_3 - A_4\, e^{-RS_c}\ )\}]$$

$$A_{11} = A_{10} - (A_5 - A_6 + A_7 + A_8 + A_9)$$

$$A_{12} = \frac{4R}{(e^{-a_1} - e^{-a_2}) - \left(R^2 + 4\left(\frac{1}{K^*}\right)\right)^{1/2}} [A_1 - A_2 + A_3 + A_4)\ e^{-a_1}$$
$$+ (A_2\, e^{-Ra_2}\, A_1\, e^{-Ra_1} - A_3 - A_4\, e^{-RS_c}\ )]$$

$$A_{13} = A_{12}\ \alpha_1^3$$

$$A_{14} = A_{12}\ \alpha_2^3$$

$$C_1 = -\,C_2 - A_1 + A_2 - A_3 - A_4$$

$$C_2 = \frac{1}{e^{-a_1} - e^{-a_2}} [(A_1 - A_2 + A_3 + A_4)\, e^{-a_1} - A_1\, e^{-Ra_1} + A_2\, e^{-Ra_2} - A_3 - A_4\, e^{-RS_c}\ ]$$

$$C_3 = A_{11}$$

and $C_4 = -\,A_{10}$

The skin friction τ_1 and rate of heat transfer Nu_i (I = 1.2) at the plates are obtained from the following expressions by putting $Y = 0$ for lower plate and $Y = 1$ for the upper plate.

$$\tau_1 = \frac{du}{dy} + R_c \frac{d^2u}{dy^2} \quad \dots (3.12)$$

$$Nu_i = -\frac{dT}{dy} \quad \dots (3.13)$$

In the variety of industrial processes it is essential to know the rate of mass flux. The mass flux per time and per unit are can be obtained by evaluating the following concentration gradient at the surface of the plates by substituting $Y = 0$ and $Y = 1$

$$CG_1 = D\frac{dC'}{dY'} = -\frac{dC}{dY} \text{ (where } i = 1, 2) \quad \dots (3.14)$$

Hence, the expression for skin-friction, rate of heat transfer and concentration gradient at the walls are given by

$$\tau_1 = A_2Ra_2 - A_1Ra_1 - A_4RS_c + A_{15}\, a_1 + \frac{\alpha_2 - \alpha_1}{e^{-\alpha_1} - e^{-\alpha_2}}[A_{15}e^{-\alpha_1} + A_{16}]$$
$$+ R_c\, [(A_{13} - A_8 - A_{11})\, \alpha_1 + \alpha_2\, (A_{10} + A_{14}) - A_5Ra_1 + A_6Ra_2 - A_7RS_c$$
$$+ A_1Ra_1^{\,2} - A_2R^2\, a_2^{\,2} - A_4\, RS_c^{\,2} - A_{15}\, \alpha_1^{\,2}\,]$$
$$+ \frac{\alpha_1^2 - \alpha_2^2}{e^{-\alpha_1} - e^{-\alpha_2}}[A_{15}e^{-\alpha_1} + A_{16}] \quad \dots (3.15)$$

$$\tau_2 = A_1Ra_1\, e^{-Ra_1} - 1A_2\, e^{-Ra_2} - A_4RS_c\, e^{-RS_c} + A_{15}\, \alpha_1\, e^{-\alpha_1}$$
$$- \frac{\alpha_1e^{-\alpha_1} - \alpha_2e^{-\alpha_2}}{e^{-\alpha_1} - e^{-\alpha_2}}[A_{15}e^{-\alpha_1} + A_{16}] + R_c\, [(A_{13} - A_8 - A_{11})\, \alpha_1\, e^{-\alpha_1} + (A_{10} - A_{14})\, \alpha_2\, e^{-\alpha_2}$$
$$- A_5\, Ra_1\, e^{-Ra_1} + A_6\, Ra_2\, e^{-Ra_2} - A_7\, RS_c\, e^{-RS_c}$$
$$+ A_1\, R^2\, a_1^{\,2}\, e^{-Ra_1} - A_2\, R^2\, a_2^{\,2}\, e^{-Ra_2} + A_4\, R^2\, S_c^{\,2}\, e^{-RS_c} - A_{15}\, \alpha_1^{\,2}\, e^{-\alpha_1}\,]$$
$$+ \frac{\alpha_1^2e^{-\alpha_1} - \alpha_2^2e^{-\alpha_2}}{e^{-\alpha_1} - e^{-\alpha_2}}[A_{15}e^{-\alpha_1} + A_{16}] \quad \dots (3.16)$$

$$Nu_1 = \frac{Ra_2e^{-Ra_1} - Ra_1e^{-Ra_2}}{e^{-\alpha_1} - e^{-\alpha_2}} \quad \dots (3.17)$$

$$Nu_2 = \frac{e^{-R(a_1+a_2)} - (Ra_2 - Ra_1)}{e^{-Ra_1} - e^{-Ra_2}} \quad \dots (3.18)$$

$$CG_1 = \left(\frac{RS_c}{1 - e^{-RS_c}}\right)e^{RS_c} \quad \dots (3.19)$$

$$CG_2 = CG_1\, e^{-RS_c} = -\left(\frac{RS_c}{1 - e^{RS_c}}\right) \quad \dots (3.20)$$

where $A_{15} = A_1 - A_2 + A_3 + A_4$

$$A_{16} = A_2\, e^{-a_2 R} - A_1\, e^{-a_1 R} - A_3 - A_4\, e^{-RS_c}$$

RESULTS AND DISCUSSIONS

The present investigation brings out interesting results about flow behaviour with mass transfer. The influence of the elastic parameter (R_c), modified Grashof number (G_c), free convection parameter (G_T), permeability parameter K^*, suction parameter (R), Prandtl number (P), source parameter (S) and the Schmidt number (S_c) on free convection flow of a visco-elastic fluid in vertical porous channel with a constant suction and heat sources, have been studied with variation of species concentration. The results are discussed here with the help of graphs and tables. Such results of this particular problem have neither been theoretically reported nor been experimentally observed so far. Neglecting the effects of mass transfer and chemical reaction, our results tally with the results of Dash and Ojha [19]. Neglecting the effects of Hall current and mass transfer, our results coincide with the results of Mishra, Ray, Biswal and Jena [29].

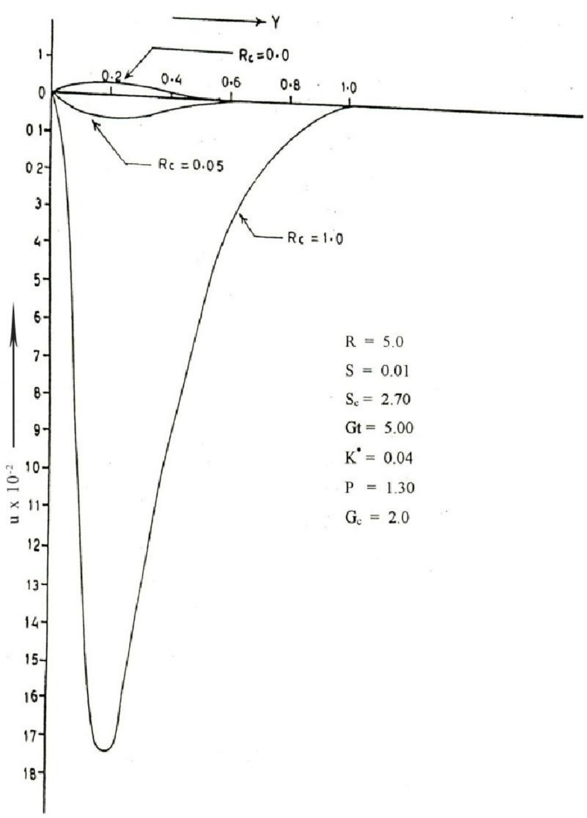

Fig. 1 : Effect of R_c on Velocity Field

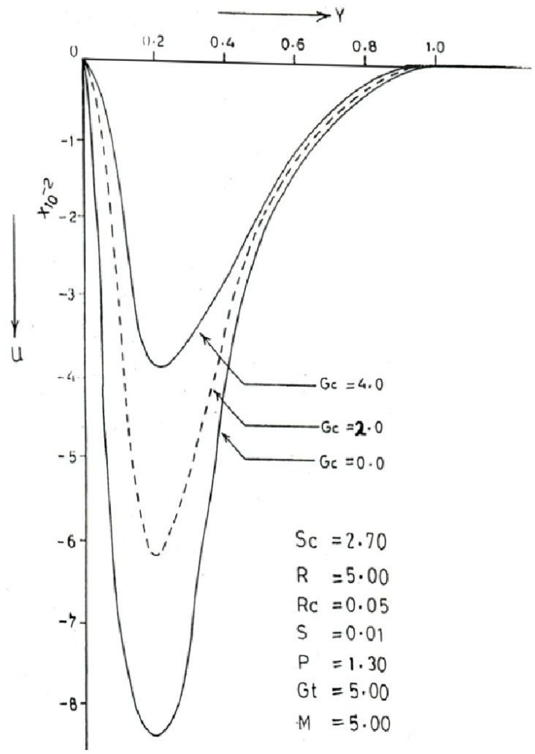

Fig. 2 : Effect of G_c on Velocity Field

Fluid motion :

The velocity profiles, the concentration profiles, the values of skin friction at the walls, the rates of heat transfer and the concentration gradient characterize the behaviour of flow phenomenon.

The influence of the combined action of the porosity of the medium and the species concentration modifies flow behaviour.

The effect of elastic parameter (R_c) on the flow with mass transport is investigated first. It is observed from Fig. 1 that as R_c increases the velocity of fluid flow decreases and consequently flow reversal occurs. For Newtonian fluid ($R_c = 0.0$) flow reversal is not noticed. Fig. 2 explains the effect of G_c on the velocity field. It is seen that the velocity increases with the increase of G_c.

The influence of free-convection term (G_T) on the velocity distribution is indicated in Fig. 3. It is clear that with an increase in the free-convection parameter $G_T > 0$, the magnitude of the fluid velocity rises in the reverse direction across the entire channel width.

The effect of permeability parameter on the velocity field is studied with the aid of Fig. 4. It is observed that the fluid motion of modified greatly with the nature of the porous medium, *i.e.* with the value of the permeability parameter K^*.

Tables 1 to 6 illustrate the effects of K^*, S, S_c, R_c, G_T and G_c on skin-friction respectively. It is evident that the skin-friction at the walls of the channel first falls and then rises with the rise of magnetic field strength (Table 1). From Table 2 it is observed that the skin-friction (τ_1) at the first wall ($Y = 0$) increases with the increase in source-strength and the skin-friction (τ_2)

at the second wall (Y = 1) decreases with the increase in source-strength. The 3 shows that skin friction τ_1 decreases with the increase in the value of S_c whereas τ_2 increases with S_c. Table 4 discovers similar result in case of R_c.

Table 5 depicts that the skin-friction τ_1 decreases with increases of G_T while τ_2 increases with G_T. A reverse effect is observed in case of G_c (Table 6).

Table : 1. Effect of K^* on Skin-friction for
$R_c = 0.05, R = 5.0, G_T = 5.0, S_c = 2.5, G_c = 2.0, P = 1.3, S = 0.5$

K^*	0.04	0.01	0.004
τ_1	–0.278768	–7.884780	–0.443665
τ_2	0.010.18	–0.029893	0.034170

Table : 2. Effect of S on Skin-friction for
$R_c = 0.05, R = 5.0, G_T = 5.0, G_c = 2.0, S_c = 2.7, P = 1.3, K^* = 0.04$

S	0.1	0.5	1.0
τ_1	–1.145020	–0.369479	0.318174
τ_2	0.012906	0.010510	0.008122

Table : 3. Effect of S_c on Skin-friction for
$R_c = 0.05, R = 5.0, G_T = 5.0, G_c = 2.0, P = 1.3, S = 0.5, K^* = 0.04$

S_c	2.5	3.0	5.0
τ_1	–0.278768	–0.456910	–0.647157
τ_2	0.010318	0.010678	0.010957

Table : 4. Effect of R_c on Skin-friction for
$R = 5.0, G_T = 5.0, S_c = 2.0, P = 1.3, S = 0.01, K^* = 0.04$

R_c	0.05	1.0
τ_1	–1.367360	–39.523700
τ_2	0.01350	0.324266

Table : 5. Effect of G_T on Skin-friction for
$R = 5.0, R_c = 0.05, S = 0.01, G_c = 2.0, S_c = 2.7, P = 1.3, K^* = 0.04$

S	0.1	0.5	1.0
τ_1	–1.367360	–3.551310	–5.735260
τ_2	0.013580	0.027745	0.041901

Table : 6. Effect of G_c on Skin-friction for
$R = 0.05, S = 0.01, S_c = 2.7, G_T = 5.0, P = 1.3, K^* = 0.04$

S	0.1	0.5	1.0
τ_1	–2.183950	–1.367360	–0.550767
τ_2	0.014161	0.013580	0.013005

Heat transfer :

Figs. 5 and 6 illustrate the effects of heat generating term (S) and the Prandtl number (P) on the temperature profiles. From Fig. 5 it is observed that the heat generating parameter considerably increases the magnitude of temperature. T rises with S. Fig. 6 shows that the temperature falls as Prandtl number (P) rises. In the absence of dissipation, it is marked that the net effect of internal heat generation and wall temperature variation is to enhance heat transfer.

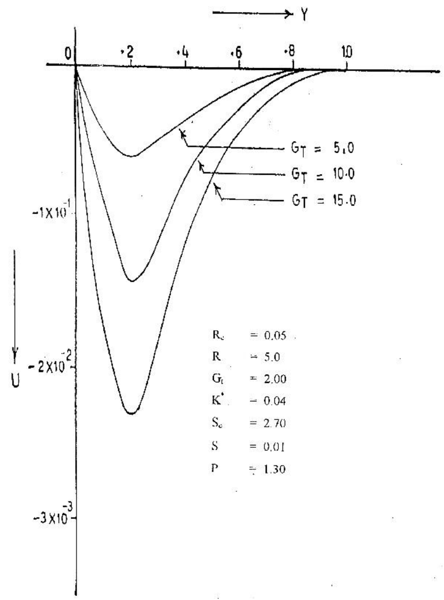

Fig. 3 : Effect of *GT* on Velocity Field

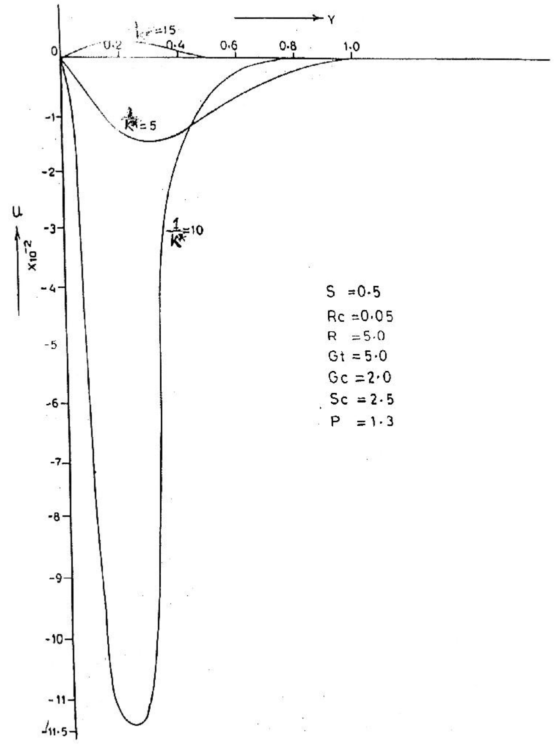

Fig. 4 : Effect of 1/K^* on Velocity Field

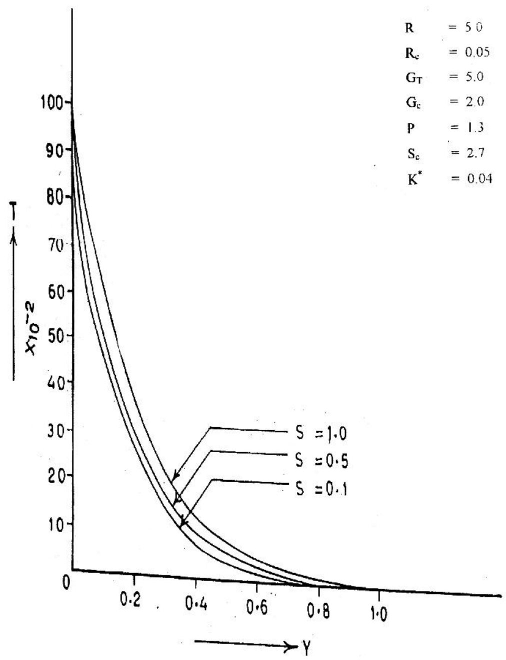

Fig. 5 : Effect of *S* on Temperature

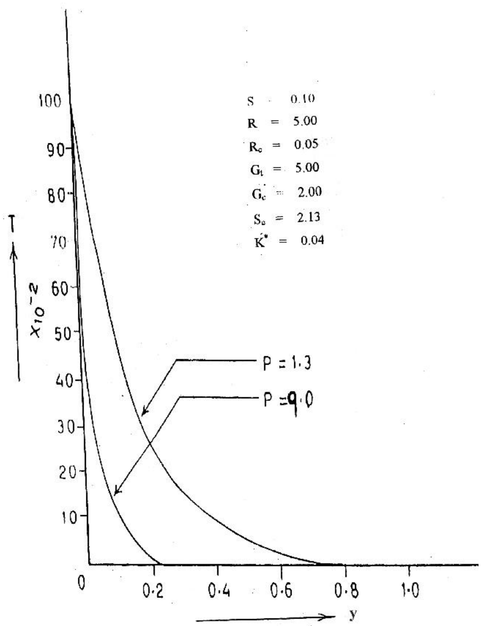

Fig. 6 : Effect of *P* on Temperature

The effect of Reynolds number on temperature field has been shown in Fig. 7, which shows that the increase in R decreases the temperature uniformly in the entire domain of the channel.

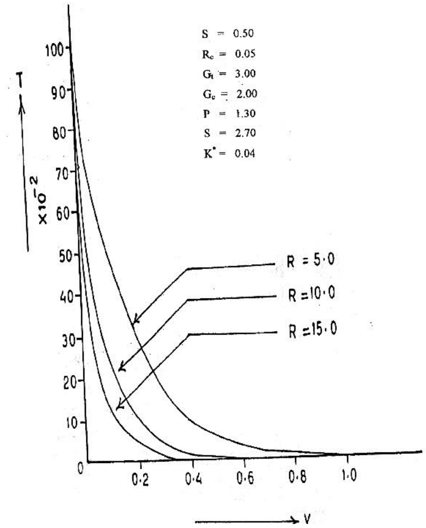

Fig. 7 : Effect of *R* on Temperature

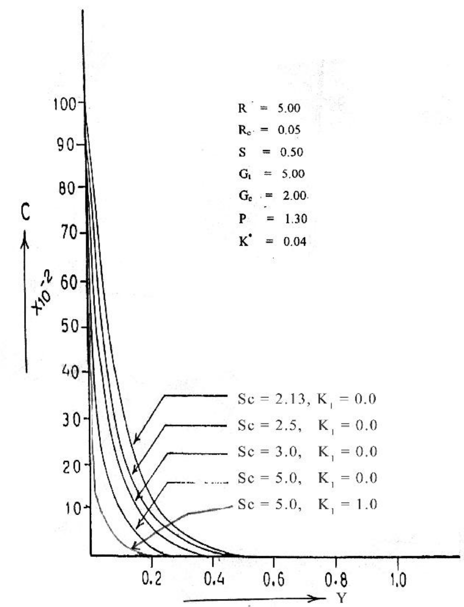

Fig. 8 : Effect of K_1 and S_c on Concentration

The rates of heat transfer at both the walls of the channel are recorded in Tables 7 to 9 in order to study the effects of *R.S* and *P* on heat transfer. It is observed from Table 7 that the rate of heat transfer at the first wall ($Y = 0$) increases with the increase Reynolds number (R) and that at the second wall cooled due to flow of visco-elastic fluid in the vertical porous channel with variation of species concentration.

The effect of source parameter on the rates of heat transfer at the walls is studied from Table 8. As the source strength increases. Nu_1 decreases while Nu_2 increases. The reverse effect is noticed in case of Prandtl number, i.e. the rate of heat transfer rises at the first and falls at the second wall with the rise in P (Table 9).

Table : 7. Effect of R on Nusselt's number for
$R_c = 0.05$, $S_c = 2.7$, $S = 0.01$, $G_c = 2.0$, $G_T = 5.0$, $P = 1.3$, $K^* = 0.04$

R	– 10	–5.0	5.0	10.0
Nu_1	– 0.05017	– 0.002527	6.497470 1	2.975000
Nu_2	13.278400	6.566680	0.009872	0.000030

Table : 8. Effect of S on Nusselt's number for
$R = 0.05$, $R_c = 0.05$, $G_T = 5.0$, $G_c = 2.0$, $P = 1.3$, $S_c = 2.7$, $K^* = 0.04$

S	0.1	0.5	1.0
Nu_1	6.384640	5.830820	4.955120
Nu_2	0.010686	0.015540	0.026580

Table : 9. Effect of P on Nusselt's number for
$R = 0.05$, $R_c = 0.05$, $G_T = 5.0$, $G_c = 2.0$, $P = 1.3$, $S_c = 2.7$, $K^* = 0.04$

P	1.3	9.0	12.0
Nu_1	6.3846	44.874600	59.874700
Nu_2	0.010686	0.145202×10^{-17}	0.593014×10^{-24}

Table : 10. Effect of S_c on Conc. Gradient for
$R = 5.0$, $R_c = 0.05$, $G_T = 5.0$, $G_c = 2.0$, $P = 1.3$, $S = 0.5$, $K^* = 0.04$

S_c	2.13	2.5	3.0	5.0
CG_1	10.6503	12.5	15.0	25.0
CG_2	0.000252	0.000046	0.000004	0.0

Mass transfer :

Fig. 8 explains the influence of S_c on the concentration. It is seen that the concentration decreases at all points of the channel with the increase in the value of S_c from 2.13 to 5.0, without chemical reaction. But chemical reaction parameter reduces the concentration further.

Effect of S_c on concentration gradient is studied with the aid of Table 10. It is marked that the value of concentration gradient near the first wall increases and that at the second wall decreases with the increase in Schmidt number.

Conclusions

From the above studies on the mass transfer effects of free convection flow of a visco-elastic fluid inside a porous vertical channel with constant suction and heat sources, the following conclusions are drawn.

1. The reversal of flow direction occurs with the imposition of porosity along with mass transfer of visco-elastic fluids. But, flow reversal is not resulted in case of viscous fluid.
2. Flow rate decelerates rapidly with the rise of free convection parameter.
3. Fluid flow is modified with the behaviour of porous medium.

4. Temperature rises with the rise in source strength.
5. The concentration decreases with increase in Schmidt number.
6. Chemical reaction parameter reduces the concentration.

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