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EFFECT OF BUOYANCY RATIO ON MHD FREE CONVECTION FLOW WITH HEAT AND MASS TRANSFER PAST A VERTICAL POROUS PLATE OF SLIP FLOW REGION IN THE PRESENCE OF HALL CURRENT AND CHEMICAL REACTION

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The effect of buoyancy ratio on unsteady flow of an electrically conducting fluid past a vertical porous plate in the presence of slip condition, chemical reaction, transverse magnetic field, heat transfer and mass transfer is studied. The dimensionless governing equations are solved using exponential accelerate flow conditions. The results are obtained for velocity, Temperature, mass concentration and shearing stress for different parameters like Schmidt number, time, magnetic parameter, buoyancy ratio, chemical reaction parameter, magnetic parameter, rarefaction parameter, etc. The flow characteristics are discussed and shown by means of graphs.

KEYWORDS : Chemical reaction, Mass Transfer, Heat transfer, MHD, Hall current and Porous Medium.

INTRODUCTION

Convection flow in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is the working medium. MHD has attracted the attention of many scholars due to its diverse application in geophysics and astrophysics. It is applied to study the stellar and solar structure, interstellar matter, radio propagation through the ionosphere, design of MHD generators and accelerators in geophysics, in design of underground water energy storage system, soil-sciences, nuclear power reactors and so on. The phenomena of mass transfer is very common in theory of stellar structure, burning a pool of oil, spray drying, adsorption, leaching and mass transport process in animal and plant life. The growing need for chemical reaction in industries and engineering requires the study of heat and mass transfer in the presence of different condition and parameters with chemical reaction. There are many transfer processes are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of chemical reaction. It has many applications in nuclear reactor and combustion, solar collectors, drying, dehydration operations in chemical and food processing plants, polymer production, etc. Further, the effect of Hall current on the fluid flow with variable concentration has many applications in MHD power generation, in several astrophysical and meteorological studies as well as in plasma flow through MHD power generators. Datta, *et al* [1] have studied the Oscillatory magnetohydrodynamic flow past a flat plate with Hall effects. Hall effect on oscillatory hydromagnetic free convective flow of a visco-elastic fluid past an infinite vertical porous flat plate with mass transfer have been discussed by Biswal *et al* [2]. Acharya *et al* [4] considered the Hall Effect with simultaneous thermal and mass diffusion on unsteady hydromagnetic flow near an accelerated vertical plate. Aboeldahab, *et al* [5] have studied the Hall current effect on magnetohydrodynamics free convection flow past a semi-infinite vertical plate with mass transfer. Rahman *et al* [3] discussed Convective-Diffusive Transport chemical reaction in Natural convection Flows. Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects analyzed by Kandasamy *et. al* [6]. Senapati *et. al* [10] have studied the mass transfer effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Sivaiah *et.al* highlighted Heat and mass transfer effects on MHD free convective flow past a vertical porous plate. Senapati *et. al* [8] have studied magnetic effect on mass and heat transfer of a hydrodynamic flow past a vertical oscillating plate in presence of chemical reaction. Agrawal *et. al* [9] have stuided the effect of Hall current on MHD free convection flow with heat and mass transfer past a vertical porous plate. Devika *et. al* [11] have discussed the MHD oscillatory flow of a visco elastic fluid in a porous channel with chemical reaction.

It is proposed to study the Effects of buoyancy ratio on MHD free convection flow with heat and mass transfer past a vertical porous plate of slip flow region in the presence of Hall Current and Chemical reaction.

FORMULATION OF THE PROBLEM

We consider a two dimensional unsteady flow of an incompressible viscous electrically conducting fluid past an infinite vertical porous plate immersed in porous medium with Hall current such that x'-axis is along the plate and y-axis is normal to it. A uniform magnetic field B_0 is applied in the direction of y'-axis and plate is non-conducting. Initially the plate and surrounding fluid are at rest and are in same temperature T'_∞ and having mass concentration C'_∞ at all points. After the process starts with first order chemical reaction, it is assumed that the velocity of the flow, temperature and concentration of the fluid is made to rise with time exponentially. The fluid has constant properties and the variation in density and mass concentration is considered only in the body force term. Then neglecting viscous dissipation and Ohmic dissipation by usual Boussinesq's approximation the unsteady flow is governed by the following equations:

The equation of continuity gives suction velocity $v' = -U_0$

$$\frac{\partial u'}{\partial t'} - U_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho(1+m^2)} + g\beta(T' - T'_\infty) + g\beta_c(C' - C'_\infty) - \frac{\nu u'}{K} \qquad \ldots (1)$$

$$\frac{\partial T'}{\partial t'} - U_0 \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \qquad \ldots (2)$$

$$\frac{\partial C'}{\partial t'} - U_0 \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - R'(C' - C'_\infty) \qquad \ldots (3)$$

with the following boundary conditions

$$\left.\begin{array}{l} u' = h'\frac{\partial u'}{\partial y'} \, , T' = T'_w \, , C' = C'_w \quad at\, y' = 0 \\ u' = 0 \;\; T' = T'_\infty \, , C' = C'_\infty \quad at\, y' \to \infty, \end{array}\right\} \qquad \dots (4)$$

where g is the acceleration due to gravity, *T'* is the temperature of the fluid within the boundary layer, *C'* is the species concentration, ρ is the fluid density of boundary layer, ν is the kinematic viscosity, *k* is the thermal conductivity, c_p is the specific heat at constant pressure, *D* is the mass diffusivity, *K* is the permeability of porous medium, R' is the chemical reaction parameter, *h'* is refraction parameter and $m = \omega_e \tau_e$ is the Hall parameter.

Let us introduce the non-dimensional variables

$$\left.\begin{array}{l} u = \frac{u'}{U_0} \, , t = \frac{t' U_0^2}{\nu} \, , y = \frac{y' U_0}{\nu} \, , T = \frac{T' - T'_\infty}{T'_w - T'_\infty} \, , C = \frac{C' - C'_\infty}{C'_w - C'_\infty} \, , K = \frac{K' U_0^2}{\nu^2} \, , Pr = \frac{\rho \nu c_p}{k} \\ \mathrm{Sc} = \frac{\nu}{\mathrm{D}} \, , \mathrm{M} = \frac{\sigma B_0^2 \nu}{\rho U_0^2} \, , N = \frac{\beta_c}{\beta}\left(\frac{C'_w - C'_\infty}{T'_w - T'_\infty}\right) , Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0^3} \, , R = \frac{R' \nu}{U_0^2} \, , h = h' \frac{U_0^2}{\nu} \end{array}\right\} \qquad \dots (5)$$

Equations (1) to (3) are transformed to their corresponding non-dimensional forms as

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = Gr(T + NC) + \frac{\partial^2 u}{\partial y^2} - \left(\frac{M}{1+m^2} + \frac{1}{K}\right) u \qquad \dots (6)$$

$$Pr\frac{\partial T}{\partial t} - Pr\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \qquad \dots (7)$$

$$Sc\frac{\partial C}{\partial t} - Sc\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - ScRC \qquad \dots (8)$$

where *Pr* is the Prandtl number, *Gr* is the Grashof number, *N* is the buoyancy ratio, *Sc* is the Schmidt number, *M* is the magnetic parameter, *K* is the non-dimensional permeability parameter of porous medium, *R* is the non-dimensional chemical reaction parameter and h is the non-dimensional refraction parameter.

With the following boundary conditions

$$\left.\begin{array}{l} u = h\frac{\partial u}{\partial y} \, , \; T = 1 \, , C = 1 \quad at\;\, y = 0 \\ u = 0, \; T = 0 \; , \; C = 0 \quad as\, y \to \infty \end{array}\right\} \qquad \dots (9)$$

METHOD OF SOLUTION

To solve the governing equations from equation (6) to (8), we assume

$$u(y,t) = u_0(y)e^{-at} \qquad \dots (10)$$

$$T(y,t) = T_0(y)e^{-bt} \qquad \dots (11)$$

$$C(y,t) = C_0(y)e^{-ct} \qquad \dots (12)$$

Then equation (6) to (8) reduce to

$$u_0'' + u_0' + \left(a - \frac{M}{1+m^2} - \frac{1}{K}\right) u_0 = -Gr(bT_0 + cNC_0) \qquad \dots (13)$$

$$T_0'' + PrT_0' + bPrT_0 = 0 \qquad \dots (14)$$

$$C_0'' + ScC_0' + (c - R)ScC_0 = 0 \qquad \dots (15)$$

with the boundary condiions

$$\left.\begin{aligned} & u_0 = h\frac{\partial u_0}{\partial y}, T_0 = e^{bt}, C_0 = e^{ct} \quad at\ \ y = 0 \\ & u_0 = 0\ \ T_0 = 0\ , C_0 = 0 \quad as\ y \to \infty \end{aligned}\right\} \qquad \ldots (16)$$

Solving equations (13) to (15) and using in equations (10) to (12), we have

$$u = \left(A_{16}e^{-A_{13}y} - \frac{Grb}{A_{14}}e^{-A_{11}y} - \frac{GrcN}{A_{15}}e^{-A_{12}y}\right)e^{-at} \qquad \ldots (17)$$

$$T = e^{-A_{11}y} \qquad \ldots (18)$$

$$C = e^{-A_{12}y} \qquad \ldots (19)$$

where $A_{11} = \frac{\left(Pr+\sqrt{Pr^2-4Prb}\right)}{2}$, $A_{12} = \frac{\left(Sc+\sqrt{Sc^2-4Sc(c-R)}\right)}{2}$, $A_{13} = \frac{1+\sqrt{1-4\left(a-\frac{M}{1+m^2}-\frac{1}{K}\right)}}{2}$

$$A_{14} = A_{11}^2 - A_{11} + \left(a - \frac{M}{1+m^2} - \frac{1}{K}\right), A_{15} = A_{12}^2 - A_{12} + \left(a - \frac{M}{1+m^2} - \frac{1}{K}\right)$$

And $A_{16} = \left[\frac{hGrbA_{11}e^{bt}}{A_{14}} + \frac{hGrcNA_{12}e^{ct}}{A_{15}} + \frac{Grbe^{bt}}{A_{14}} + \frac{GrcNe^{ct}}{A_{15}}\right]/(1 + hA_{13})$

The non-dimensional shearing stress

$$\tau = \left(\frac{du}{dy}\right)_{y=0} = \left(-A_{13}A_{16} + \frac{GrbA_{11}}{A_{14}} + \frac{GrcNA_{12}}{A_{15}}\right)e^{-at} \qquad \ldots (20)$$

The non-dimensional rate of heat transfer

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0} = A_{11} \qquad \ldots (21)$$

The non-dimensional rate of mass transfer

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0} = A_{12} \qquad \ldots (22)$$

RESULT AND DISCUSSION

In this paper we have studied Effects of buoyancy ratio on MHD free convection flow with heat and mass transfer past a vertical porous plate of slip flow region in the presence of Hall current and chemical reaction. The effect of the parameters *Gr*, *N*, *M*, *m*, *R*, *h*, *Pr*, *K* and *Sc* on flow characteristics have been studied and shown by means of graphs. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of velocity, temperature and mass concentration are taken w.r.t *y* and the graphs of Nusselt number and Shearing stress are taken w.r.t. time.

Velocity profiles: The velocity profiles are depicted in Figs. 1-4. Figure-(1) shows the effect of the parameters *h*, *m*, *N* on velocity at any point of the fluid, when $K = 1$, $M = 1$, $a = 0.1$, $b = 0.1$, $c = 0.1$, $Gr = 1$, $Sc = 0.22$, $Pr = 3$, $R = 1$ and $t = 1$. It is noticed that the velocity increases with the increase of Hall parameter (*m*) and buoyancy ratio (*N*), whereas decreases with the increase of rarefaction parameter (*h*).

Figure (2) shows the effect of the parameters *Sc*, *Pr*, *R* and *t* on velocity at any point of the fluid, when $K = 1$, $M = 1$, $a = 0.1$, $b = 0.1$, $c = 0.1$, $Gr = 1$, $h = 1$, $m = 1$ and $N = 1$. It is noticed that the velocity decreases with the increase of time (*t*), chemical reaction parameter (*R*), Prandtl number (*Pr*) and Schmidt number (*Sc*).

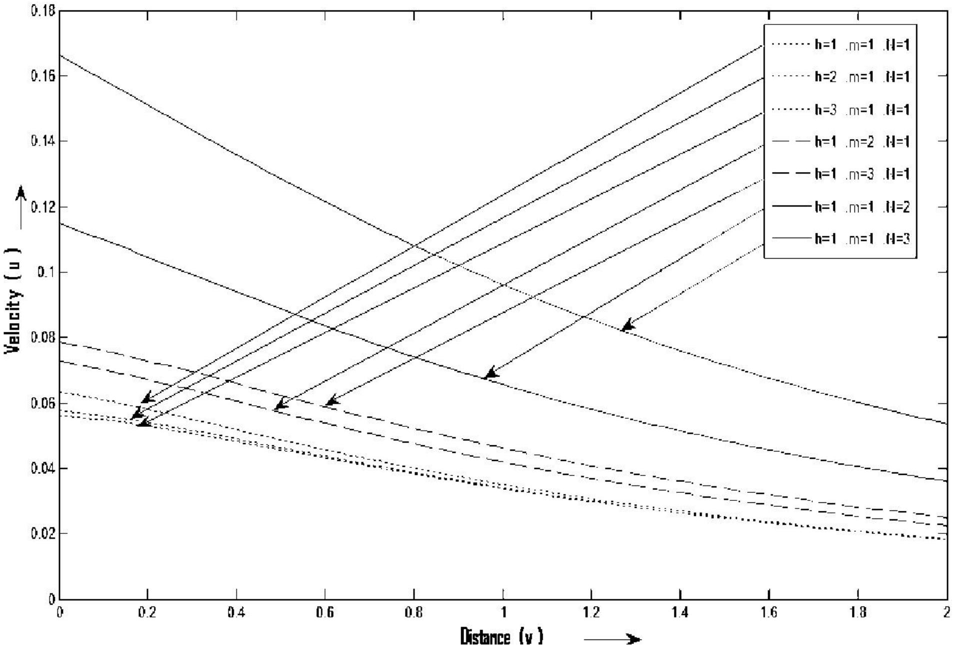

Fig. 1. Effect of *h*, *m* and *N* on velocity profile when
$K = 1$, $M = 1$, $a = 0.1$, $b = 0.1$, $c = 0.1$, $Gr = 1$, $Sc = 0.22$, $Pr = 3$, $R = 1$ and $t = 1$.

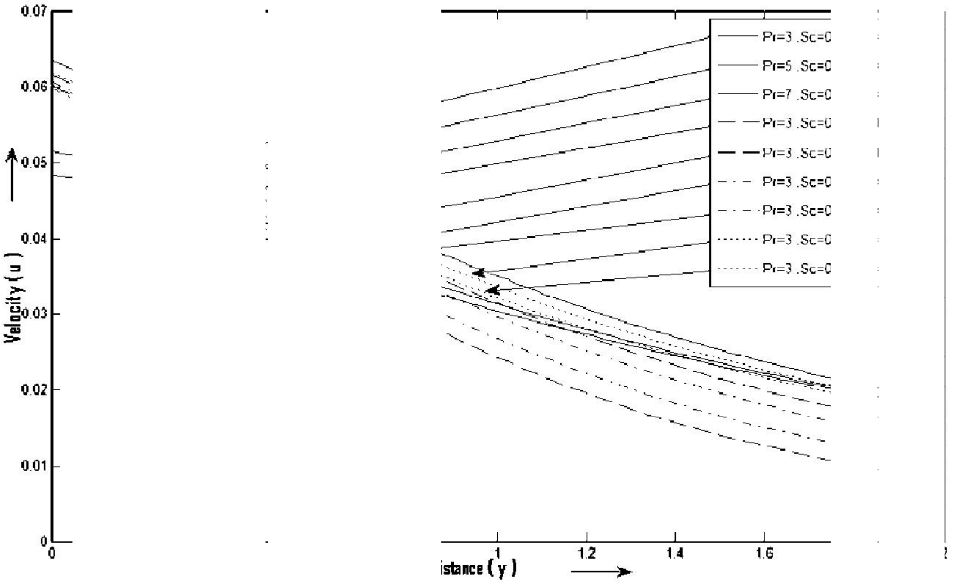

Fig. 2. Effect of *Sc*, *Pr*, *R* and *t* on velocity profile when
$m = 1$, $K = 1$, $M = 1$, $a = 0.1$, $b = 0.1$, $c = 0.1$, $Gr = 1$, $h = 1$ and $N = 1$.

Figure (3) shows the effect of the parameters a, b and c on velocity at any point of the fluid when $K = 1$, $M = 1$, $Sc = .22$, $Pr = 3$, $t = 1$, $Gr = 1$, $h = 1$, $R = 1$ and $N = 1$. It is noticed that the velocity increases with the increase of exponential parameters (b) and (c) and decreases with the increase of (a).

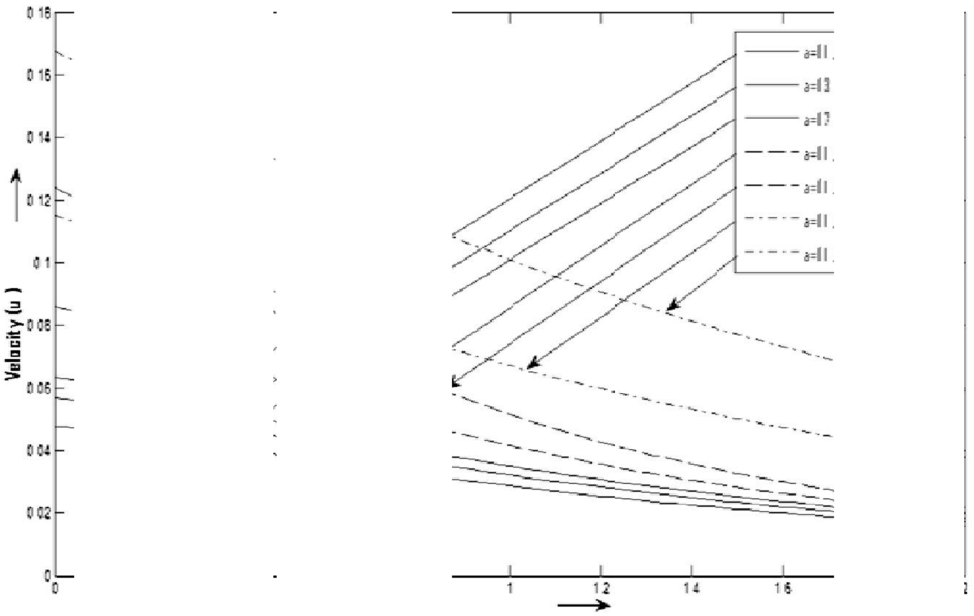

Fig. 3. Effect of *a*, *b* and *c* on velocity profile when
K **= 1,** ***M*** **= 1,** ***Sc*** **= 0.22,** ***Pr*** **= 3,** ***t*** **= 1,** ***Gr*** **= 1,** ***h*** **= 1,** ***R*** **= 1 and** ***N*** **= 1.**

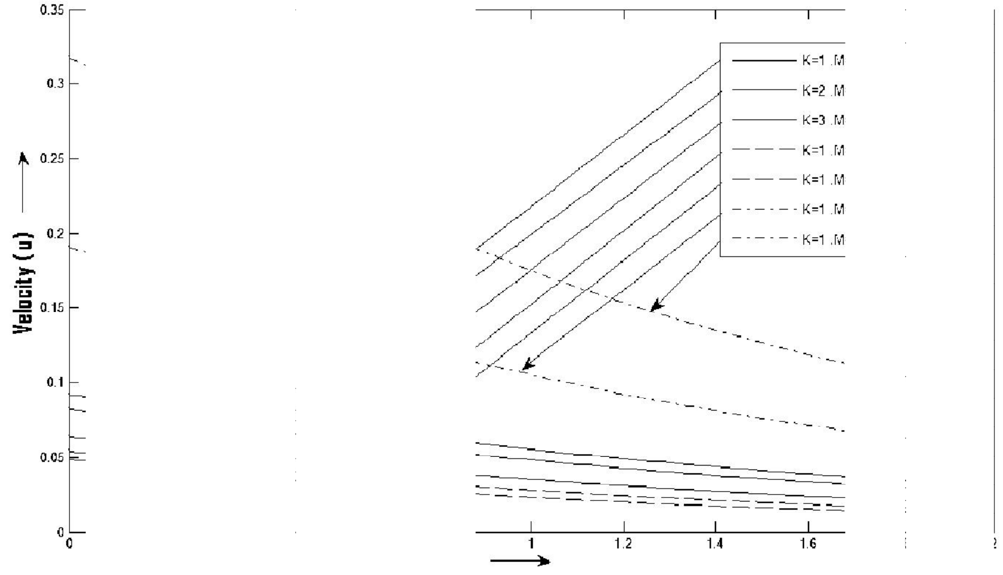

Fig. 4. Effect of *K*, *M* and *Gr* on velocity profile when
Sc **= 0.22,** ***Pr*** **= 3,** ***m*** **= 1,** ***t*** **= 1,** ***a*** **= 0.1,** ***b*** **= 0.1,** ***c*** **= 0.1,** ***Gr*** **= 1,** ***h*** **= 1 and** ***N*** **= 1.**

Figure (4) shows the effect of the parameters K, M and Gr on u at any point of the fluid, when $h = 1$, $m = 1$, $N = 1$, $a = 0.1$, $b = 0.1$, $c = 0.1$, $Sc = 0.22$, $Pr = 3$, $R = 1$ and $t = 1$. It is noticed that the velocity increases with the increase of both parameters of porous medium (K) and Grashoff number (Gr), whereas decreases with the increase of Hartmann number (M).

Temperature profile: The temperature profiles are depicted in Fig. 5. Figure (5) shows the effect of the parameters *Pr* and *h* on Temperature profile at any point of the fluid in the absence of other parameters. It is noticed that the temperature rises in the increase of exponential parameter (*b*) whereas decreases with the increase of Prandtl number (*Pr*).

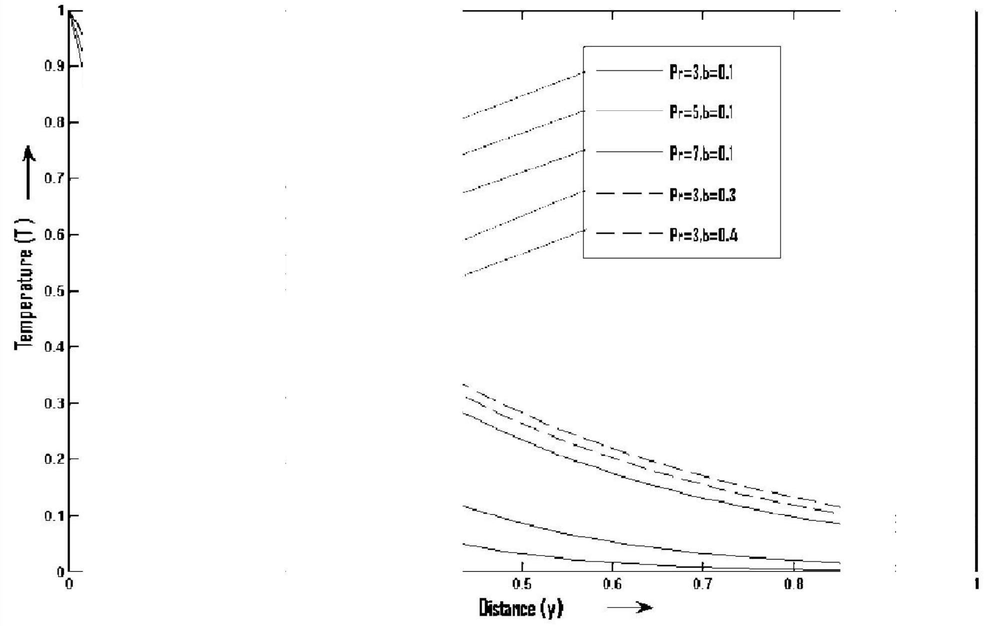

Fig. 5. Effect of *Pr* and *b* on Temperature profile (*T*).

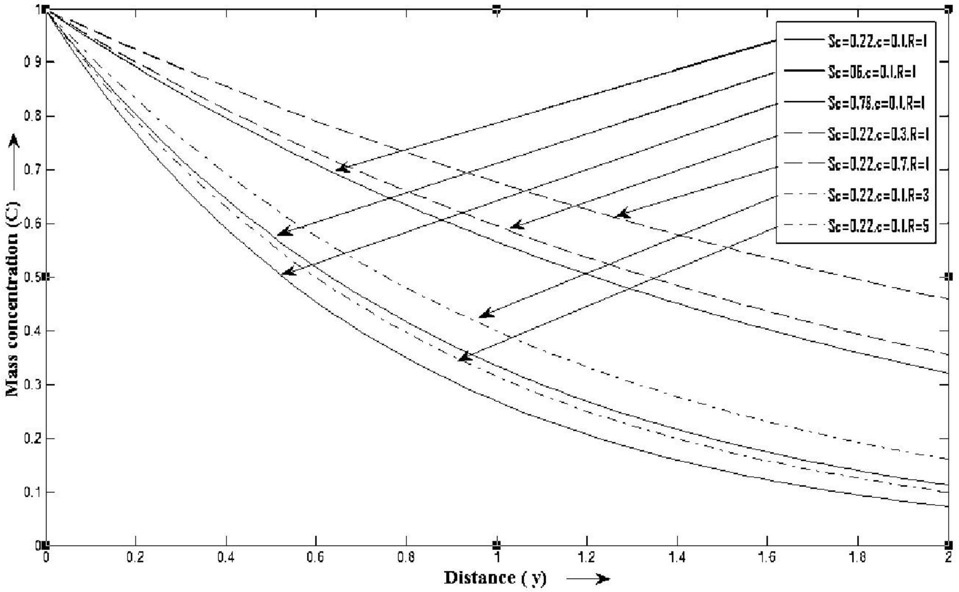

Fig. 6. Effect of *Sc*, *R* and *c* on mass concentration.

Mass concentration profile: The mass concentration profiles are depicted in Fig. 6 only. Figure (6) shows the effect of the parameters R, c and Sc on mass concentration profile at any point of the fluid in the absence of other parameters. It is noticed that the mass concentration decreases with the increase of both reaction parameter (R) and Schmidt number (Sc) whereas increases with the increase of exponential parameter (c).

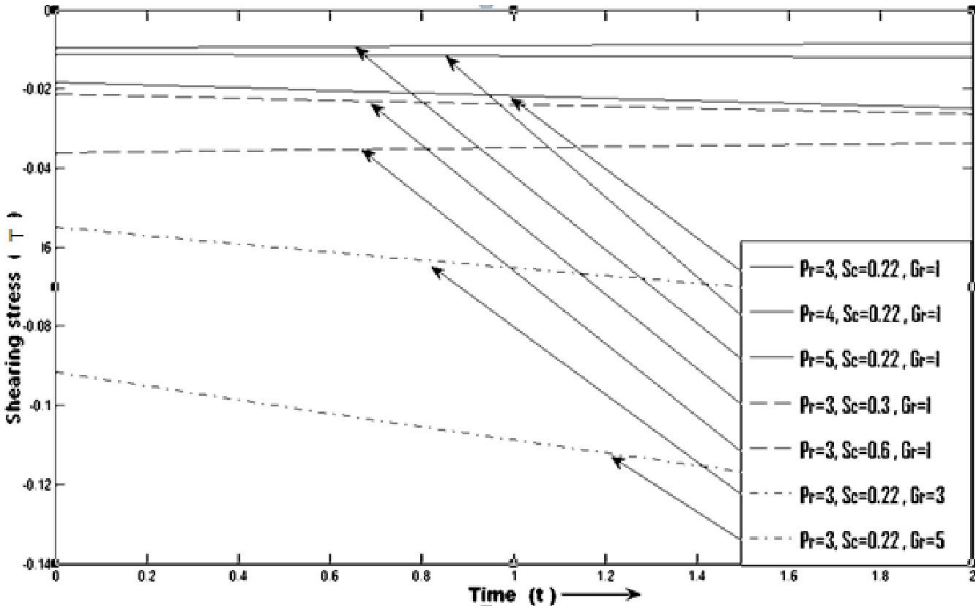

Fig. 7. Effect of *Pr*, *Sc* and *Gr* on shearing stress when
$a = 0.1$, $b = 0.1$, $c = 0.1$, $K = 1$, $M = 1$, $m = 1$, $h = 1$, $R = 1$, and $N = 1$.

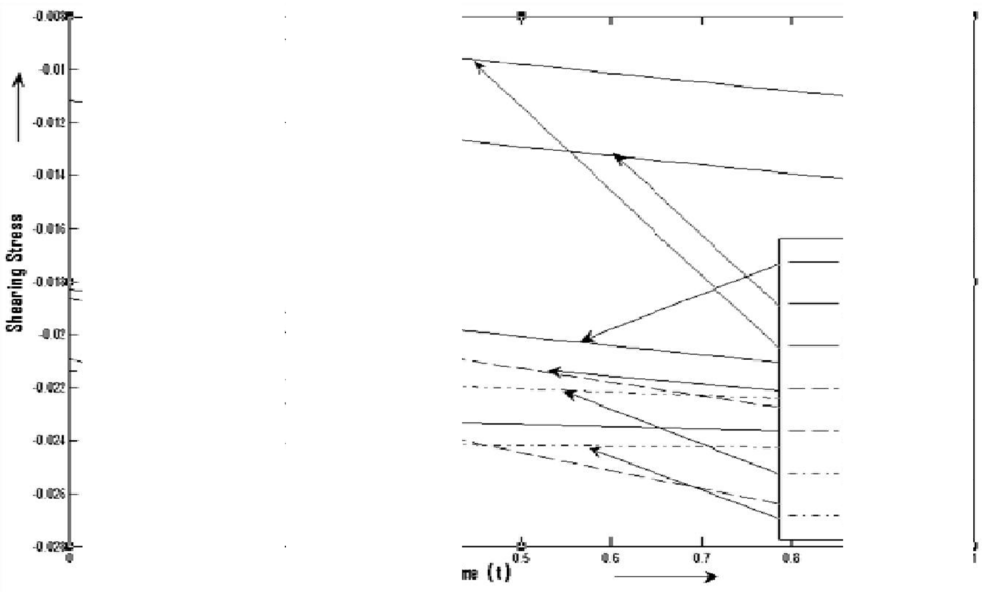

Fig. 8. Effect of *h*, *M* and *K* on shearing stress when
$a = 0.1$, $b = 0.1$, $c = 0.1$, $Gr = 1$, $Pr = 3$, $m = 1$, $Sc = 0.22$, $R = 1$ and $N = 1$.

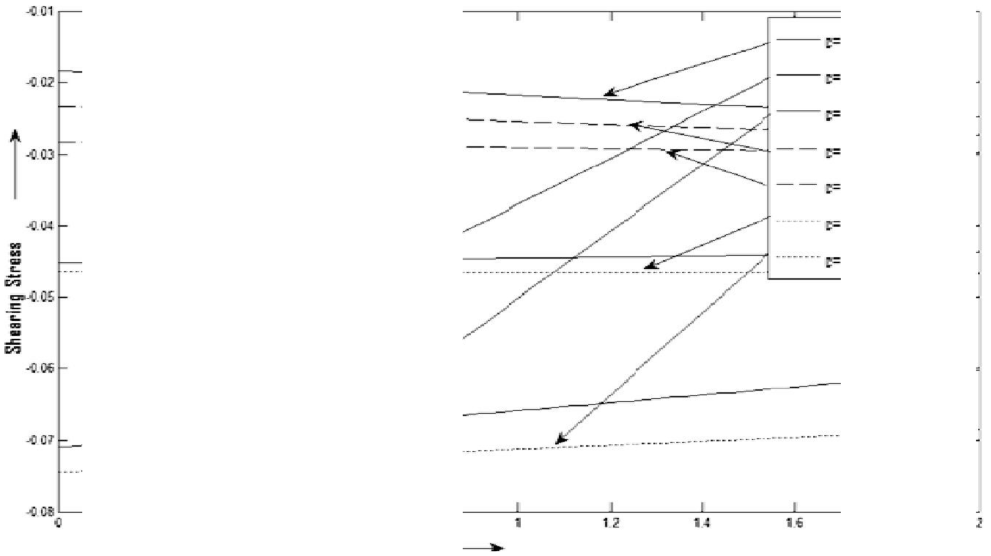

Fig. 9. Effect of *c*, *R* and *N* on shearing stress when
***a* = 0.1, *b* = 0.1, *K* = 1, *Gr* = 1, *Pr* = 3, *m* = 1, *Sc* = 0.22, *M* = 1 and *h* = 1.**

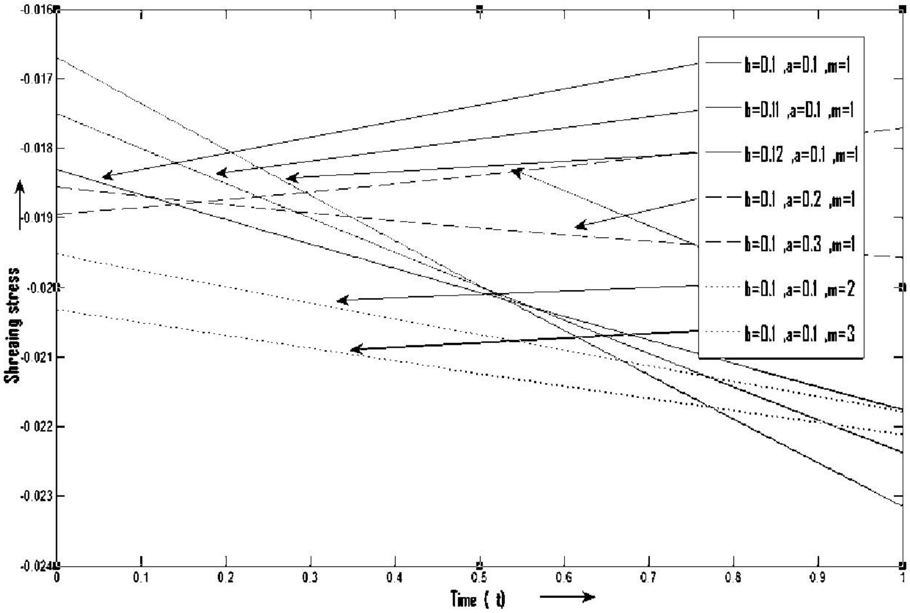

Fig. 10. Effect of *b*, *a* and *m* on shearing stress when
***c* = 0.1, *R* = 1, *N* = 1, *K* = 1, *Gr* = 1, *Pr* = 3, *m* = 1, *Sc* = 0.22, *M* = 1 and *h* = 1.**

Shearing stress: The Shearing stresses of velocity are depicted in Figs. 7-10. Figure (7) shows the effect of the parameters *Sc*, *Pr* and *Gr* on shearing, when $h = 1$, $m = 1$, $N = 1$, $a = 0.1$, $b = 0.1$, $c = 0.1$, $M = 1$, $K = 1$ and $R = 1$. It is noticed that shearing stress at surface decreases with the increase of both Prandtl number (*Pr*), where as increases with the increase of Schmidt number (*Sc*) and Grashoff number (*Gr*).

Figure (8) shows the effect of the parameters h, M and K on Shearing stress when $Sc = 0.22$, $Pr = 3$, $Gr = 1$, $R = 1$, $a = 0.1$, $b = 0.1$, $c = 0.1$, $m = 1$ and $N = 1$. It is noticed that the shearing stress at the surface increases with the increase of rarefaction parameter (h), where as decreases with the increase of Hartmann number (M), and porous medium (K).

Figure (9) shows the effect of the parameters c, R and N on Shearing stress when $Sc = 0.22$, $Pr = 3$, $Gr = 1$, $M = 1$, $a = 0.1$, $b = 0.1$, $K = 1$, $m = 1$ and $h = 1$. It is noticed that the shearing stress at the surface decreases with the increase of exponential parameter (c), Reaction number (R), and buoyancy ratio (N).

Figure (10) shows the effect of the parameters b, a and m on Shearing stress when $Sc = 0.22$, $Pr = 3$, $Gr = 1$, $M = 1$, $R = 1$, $N = 1$, $K = 1$ and $h = 1$. It is noticed that the shearing stress at the surface increases with the decrease of exponential parameter (a, b), where as decreases with the increase of Hall parameter (m).

Conclusion

In this paper, Effect of buoyancy ratio on MHD free convection flow with Heat and Mass Transfer past a vertical porous plate of slip flow region in the presence of Hall Current and Chemical reaction is presented. Results are presented graphically to illustrate the variation of velocity, temperature and skin-friction with various parameters. In this study, the following conclusions are set out:

1. In case of cooling of the plate when ($Gr > 0$) has been discussed. Velocity of the fluid increases with all exponential parameters, buoyancy ratio, void space, porous and Hall parameters, whereas decrease with magnetic parameter, chemical reaction parameter and all others.
2. In case of cooling of the plate when ($Gr > 0$) has been discussed. Both Temperature and mass concentration increase for their respective parameters and mass concentration decreases for the increase of reactive species.
3. The mass concentration increases with the exponential parameter (c), whereas decreases with chemical reaction parameter and Schmidt number.

In this paper, the same facts can be placed in different fluids, different flows in the presence or absence of different parameters with different boundary conditions.

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