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MATHEMATICAL MODELING OF AIR POLLUTION DUE TO TRANSPORTATION, CASE STUDY ON RANCHI, INDIA

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This paper provides to develop a geo-spatial mathematical model relating land use, road type and air quality. The model shows how spatial elements and issues were quantified to accurately represent the usual and unusual urban environment in the development of residential land-use. The mathematical relationship was based on the optimum distance between residential area and urban transportation network. This mathematical analysis would provide a better planning for urban transportation. The spatial data (urban land-use and urban network development) were generated using aerial photos and land use maps. Geospatial analyses were performed to find the effect and impact of urban air quality with respect to urban transportation networks. The output of the study would assist the task to reduce negative transport environmental impacts particularly in the field of air pollution. It would also be useful in identifying the potential residential area with respect to urban transportation network towards achieving sustainable development.

KEYWORDS: Transportation, Model, Air pollution, urban environment, land use.

Introduction

Air quality, regarded a main infrastructure element in urban transportation system, is considered as a major criterion for human settlement. Therefore transport emitted air pollution appears related to the establishment of urban land use in proportion to urban transportation network. Increasing demand for residential areas, along with this development of cities, has given rise to some environmental issues (Bell and Blake, 2000; Ranjan, 2001). More than half of the world's population lives in urban areas (Colesca S., 2009), therefore increasing urban land-uses resulted in several impacts on various fields such as air quality, accessibility and land use. Air quality has been considered as one of the major environmental elements by many urban planners (Colvile *et al*. 2000; Nicolas *et al*. 2005). Therefore, there is a need to look into urban transportation planning together with land use development. With respect to above note, paper presents development of a mathematical model to find suitable locations for landuses and urban transportation network for the urban transportation system of Ranchi. The models sentially investigated the best air quality for residential area. It was based on the optimum distance from residential zones to urban networks as main element avoiding transport emitted air pollution. Ranch, a developing city in India, was chosen as study area. Its based on choices of environmental factors such as: several routes to access to important public facilities, High transportation activities, Efforts to maintain its "Hill city" concept, Efforts to establish well-developed infrastructures and excellent investment opportunities.

RESEARCH APPROACH

The works undertaken includes several statistical and mathematical analysis of urban planning, focusing on land use and air quality using Geographic Information System (GIS) as visualization plateform. Distance of residential land use from urban transportation network is modeled based on determining factors of Carbon Monoxide (CO) emission from vehicles movement. This emission is useful for modeling suitable locations of residential land use in urban areas. The determination of factors is based on their analyzing individual role in CO emission. Relevant factors such as plume rise of CO, average atmospheric temperature and pressure, stack exiting velocity, estimated stack diameter, average wind speed and traffic volume are determined based on historical data (Highway Planning Unit, 2004) and standard definition of road types. Mathematical analysis methods, used to determine quantitative role of factors in the model, include programming by software of Excel and power function analyzing.

The air quality expected with respect to CO distribution is modeled using the basic theory of Primary Standard of CO Pollutant in 8 hours averaging time (WHO, 2000), which shows that the threshold of air pollution for human health in India is considered equal to 10 mg/m The model development method is based on the maximum ground level concentration of CO, considering the spread of a plume in vertical and horizontal directions, which is assumed to occur by simple diffusion along the direction of the mean wind. CO like other pollutants accumulates based on stability time, independently or in conjunction with atmospheric temperature and pressure, wind speed, curb length, or area. Curb length of CO, as indicator of suitable distance to locate residential zones, can be a function of plume rise of CO, average atmospheric temperature and pressure, stack exiting velocity, estimated stack diameter, average wind speed and traffic volume. This function can be restructured for the study area to plan future urban development and improvement.

2.1 MATHEMATICAL MODELING TRAJECTORY

The base of mathematical trajectory is combination of several relative formulas and models to calibrate a new model in relation to statement problem. The basic steps of the mathematical trajectory are summarized in this section. Assuming that the optimum Euclidean distance (Dmin) from the residential area to the urban transportation network is defined based on good air quality, the element to be constructed in the model is listed as indicator of good air quality for residential area, the role of transportation in production of air pollution and acceptable distance between roads and residential area to meet good air quality. The main portion of the modeling is to determine main variables of these elements.

2.1.1. ELEMENT DESCRIPTION

There are some elements which are emitted by vehicle transportation. But the statistics of emissions of air pollution elements attributed to transportation shows main contribution (70-90% of total emission rate) of transportation producing CO (Clean Water Action Council, 2008; Department of Transport, 1996; Haughton *et al*. 2003; Meszaros *et al*. 2005; Rodrigue *et al*. 2006). Hence in this research CO has been considered as indicator of transportation air pollution. According to this consideration concentration level of CO for human health (10 mg/m 3 for 8 hours) is known as indicator of good air quality for residential area (WHO, 2000). The role of transportation in production of with respect to consideration of CO, air pollution can be quantified as CO emission rate for total vehicles. it is calculated by CO emission rate of one vehicle and number of vehicles passed during specific time and road length, called traffic volume (Vos, 2002) With respect to above explanation, the minimum safe distance between roads and residential area to avoid emitted air pollution of vehicles (Dmin) is the distance in which total CO emission rate of vehicles is reduced to 10 mg/m 3 (acceptable concentration level of CO, for human health). This safe distance depends on a plume in vertical and horizontal directions is assumed to occur by simple diffusion along the direction of the mean wind, total CO emission rate of vehicles as expressed in Equation (1), developed by Turner (1995).

$$C_x = \frac{Q}{\pi \sigma_y \sigma_z U} e^{-1/2}\left(\frac{H}{\sigma_z}\right) e^{-1/2}\left(\frac{Y}{\sigma_y}\right)^{-1} \qquad \ldots (1)$$

Hence for calculating of this distance, it is required to apply total CO emission rate of vehicles, rise distance of emitted CO, mean wind speed and standard deviation of vertical and horizontal wind direction.

2.1.2. QUANTIFICATION OF ELEMENTS

CO pollution is very sensitive and traffic volume changes over times are considerably unpredictable. Therefore, for calculating total CO emission rate of vehicles, it is better to consider road capacity (maximum possible traffic volume) replacing traffic volume, as expressed in Equation (2)

$$Q = (RC) q_c \qquad \ldots (2)$$

where Q = Total CO emission rate of vehicles

RC = Possible road capacity in stability time of CO

q_c = Possible average CO emission rate for one vehicle

A simple equation to calculate road capacity is developed by Li (1998) as follow:

$$RC = \left(\frac{(W L_p)/V_t}{C_m}\right) \qquad \ldots (3)$$

where, W = Road width based on road type (m),

L_p = Passed road length by vehicle,

Cm = One vehicle's normal Average space time usage (m^2) and

Vt = Average Vehicle speed based on road type (m/s)

= Average passed length by car per second (m).

Since, the concentration of pollutant reaches a peak value within 5 minutes of the gas injection, the maximum reasonable specific time for calculating the number of passed car is considered as less than 15 minutes (Colorado Department of Public Health and Environment, 2006) Cm is calculated for the study area, based on percentage of vehicle types (according to historical data of traffic volume) and their actual size. W and Vt were applied based on road types (table1). Lp (passed road length by vehicle) refers to selected greed size to investigate and study of air pollution. And qc was obtained trough mathematical process, considering some important elements like percentage of vehicle types, average fuel consumption of foreign and domestic employing vehicles, average normal age of employing vehicles, percentage of different ages of employing vehicles. For computing rise distance of emitted CO by vehicle transportation (Δh), Equations (4) and (5) developed by Wayson (2000) were used as follows:

$$\Delta h = 1.6\left(\frac{F_0\, t^2}{U}\right)^{1/3} \quad \ldots (4)$$

$$F_0 = g\, v_s\, r_s^2\left[1-\left(\frac{T_a}{T_s}\right)\right] \quad \ldots (5)$$

where, Δh = Rise distance (m),

F_0 = Buoyancy factor (m^4/s_3),

t = Time (s),

U = Ambient horizontal wind speed (m/s),

g = Gravitational constant = 9.81 m/s^2,

v_s = Exit velocity (m/s), r_s = Exit radius (m),

Ta = Ambient temperature (K) and

Ts = Exit temperature (K).

The parameters and also mean wind speed could easily be obtained through annual reports of the study area (DOE, 2004) actual measurement with respect to various vehicle types, standards and guidelines. Horizontal and vertical dispersion (σy and σz) are determined from the graphs found in the Figure1 and in the attributes written in Table 1. In these figures vertical and horizontal dispersion coefficients for different areas have been categorized in 6 weather stability classes (A, B, C for day and D, E, F for night). It is based on 3 atmospheric factors of wind speed, incoming solar radiation and thinly overcast.

2.1.3 FINALIZATION OF MODEL

By applying amount of Cx = 0.01g/m^3 (primary standard of CO pollutant in 8 hours), equation (1) can be rewritten as follow:

$$\sigma_y\left(\sigma_z\right)^3 = \left(\frac{19.3 Q\Delta h^2}{U}\right) \quad \ldots (6)$$

where is Dmin? In Equation (7), σy and σz were replaced by a mathematical function of Dmin.

$$F\left(D_{\min}\right) = \sigma_y\left(\sigma_z\right)^3 \quad \ldots (7)$$

Therefore, based on Equation (6)

$$F\left(D_{\min}\right)=\left(\frac{19.3Q\Delta h^{2}}{U}\right) \quad \ldots (8)$$

So, it was necessary to develop *F* (Dmin). Since 85-88% of vehicle transportation was done during the day (FHWA, 2000), thin overcast as weather indicator for night (Table 1) was ignored.

Table 1 - Key To Weather Stability Classes

Wind Speed (m/s)	Day			Night	
	Incoming Solar Radiation			Thinly Overcast	
	Strong	Moderate	Slight	More than 50 % Cloud	Less than 50 % Cloud
Less than 2	A	A-B	B	E	F
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
More than 6	C	D	D	D	D

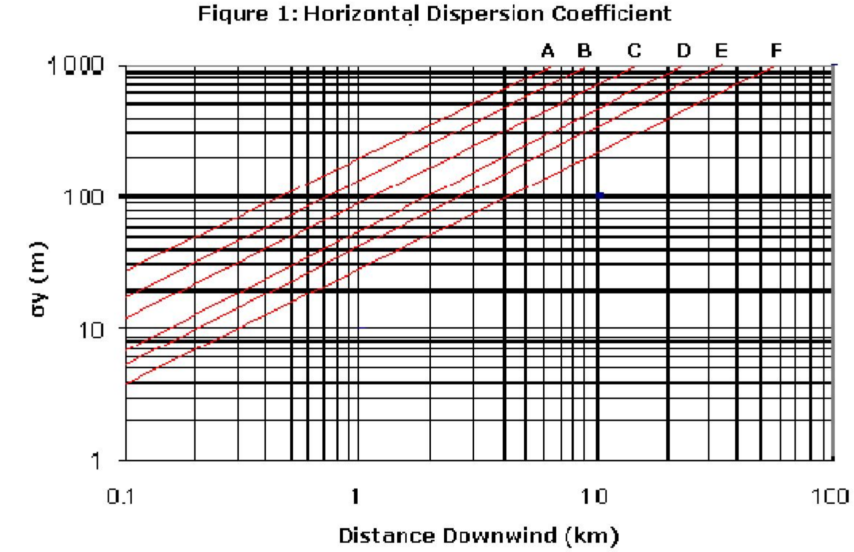
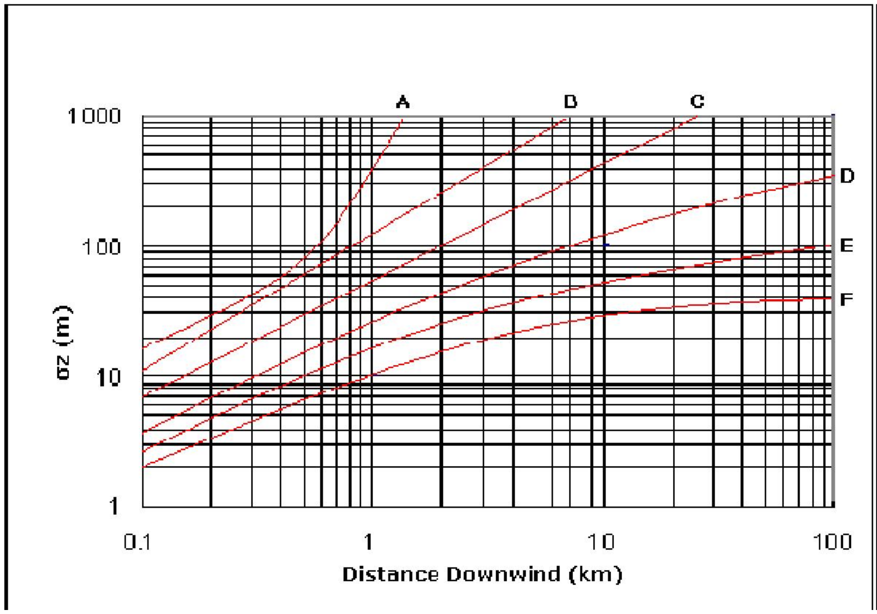


Figure 1: Horizontal Dispersion Coefficient



Vertical dispersion coefficient σ_z

Solar radiation and average wind speed in the study area played main role in this process. Based on historical data and Table 1, weather conditions in the study area were more close to situation A in The graph (A) was used to develop F (Dmin). Crowl and Louvar (2002) developed following equations for graph A:

$$\sigma_z = 0.24\,D\left(1+0.0001D\right)^{-1/2} \quad \ldots (9)$$

$$\sigma_y = 0.32\,D\left(1+0.0004D\right)^{1/2} \quad \ldots (10)$$

Since the numerical values of 0.0004 Dmin and 0.0001 Dmin which given in Equations (9) and (10) are very small, they can be eliminated and these equations can be rewritten as:

$$\sigma_y = 0.32\left(D_{\min}\right) \quad \ldots (11)$$

$$\sigma_z = 0.24\left(D_{\min}\right) \quad \ldots (12)$$

Based on equation (7)

$$F\left(D_{\min}\right) = 0.32\left(D_{\min}\right)\left[0.24\left(D_{\min}\right)\right]^3 \quad \ldots (13)$$

Then:

$$F\left(D_{\min}\right) = 4.42\times10^{-3}\left(D_{\min}\right)^4 \quad \ldots (14)$$

and then based on equation (8) and (14)

$$\left(\frac{19.3\Delta h^2 Q}{U}\right) = 4.42\times10^{-3}\left(D_{\min}\right)^4 \quad \ldots (15)$$

$$D_{\min} = 8.13\Delta h^{1/2}\left(\frac{Q}{U}\right)^{1/4} \qquad \ldots (16)$$

Generally final model can be written as :

$$D_{\min} = k\frac{\Delta h^{1/2} Q^{1/4}}{U^{1/4}} \qquad \ldots (17)$$

2.3 MODEL CALIBRATION

Comparing equations (16) and (17), the value of k can be taken as 8.13. But it must be calibrated for the study area, using field data. Therefore, the model calibration should include comparisons between model-calculated conditions and field conditions, using available data on wind speed (U), total vehicle emission rate for CO (Q), and rise distance of CO (Hh). These four parameters (Hh, Q, U, and calculated Dmin) can be computed by using applied primary data in the fieldwork such as samples are shown in Table 2.

Table 2: Some Samples of Calculated Parameters by Fieldwork Data

U	Δh	Q	Calculated D_{min}	$(\Delta h)^{1/2} (Q)^{1/4} (U)^{-1/4}$	Field D_{min}	Justified K
10.53	0.15	1717.65	11.07	1.38	9.83	7.12
10.80	0.15	1717.65	11.11	1.39	8.72	6.27
11.08	0.15	1717.65	10.98	1.37	12.62	9.21

Field Dmin also is interpolated using amount of *Q* and *D* (distance of sampling). In Table 2 amount of justified *k* is computed by Equation (18).

$$K_j = \frac{Field\ D_{\min}}{\left(U^{-1/4}\,\Delta h^{1/2}\,Q^{1/4}\right)} \qquad \ldots (18)$$

where Δh = Rise distance of *CO*,

Q = Total vehicle emission rate for *CO*,

U = Wind speed, and

Kj = Justified constant value for every sample.

After calculating *Kj* for all the samples, justified constant value for final model is calculated by averaging total amounts of *Kj*. The results show average total amount of *Kj* is equal to 8.68. Hence the model for study area can be written as follow:

$$D_{\min} = 8.68\frac{\Delta h^{1/2} Q^{1/4}}{U^{1/4}} \qquad \ldots (19)$$

2.4. MODEL VALIDATION

The model validation is done by comparing its predictions with field data. With respect to basic knowledge of sensitivity analysis and model calibration, the validation is done by applying values of parameters of calculated D_{min} (after calibration) and field D_{min} and analyzing mathematical relationship of these two parameters. This process was done for two series of observations, which were randomly selected from field data. Some samples are shown Table 3. Fortunately this mathematical relationship shows acceptable level of accuracy, 87.89% (Figure 2) and 87.10% (Figure 3).

Table 3: Some samples of comparing field D_{min} and calculated D_{min} after calibration

Sample Numbers	Calculated D_{min}	Field D_{min}	Error (%)
1	16.27	14.38	13.15
2	15.44	15.21	1.57
3	15.86	17.98	11.76

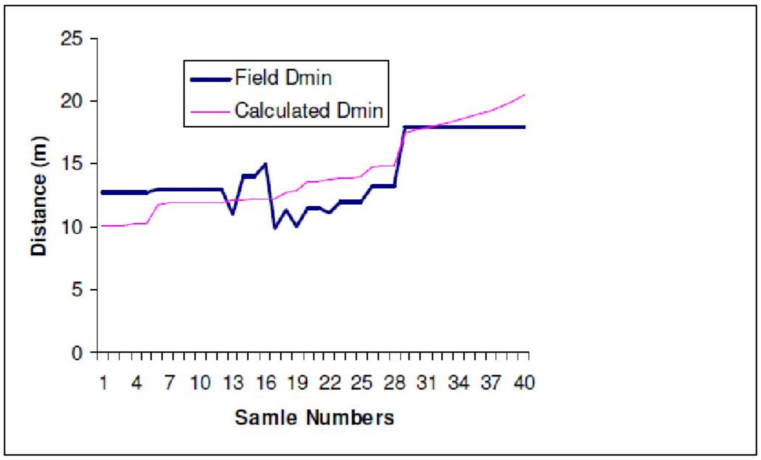


Figure 2 - Mathematical error between field and calculated D_{min} by model (Seri 1)

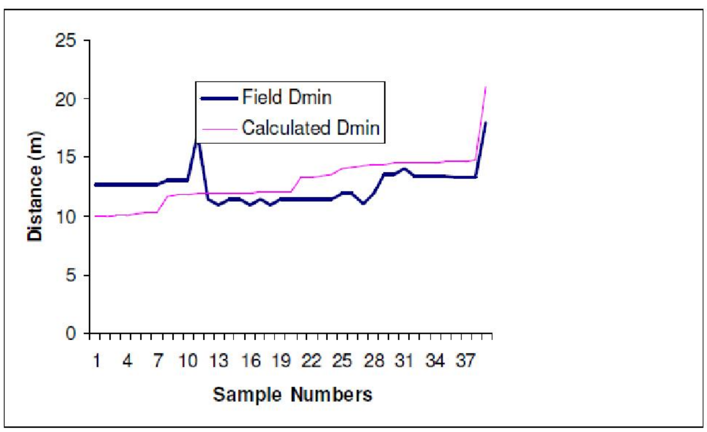


Figure 3. Mathematical error between field and calculated D_{min} by model (Seri 2)

Conclusion

This research has successfully managed to identify and develop a scientific based method understanding the relationship between land use and urban network location, by modeling for transportation air pollution and analyzing the successful and non-successful development of land uses and urban networks based on the developed model. The research strategy is able to support urban planners with a range of options. Implementation of the model suggests that some areas can be more suitable than others for residential land use and urban transportation network development, if performances and criteria are considered carefully. This suitability largely depends on the goals of the transportation projects, but importance of the main negative element (air quality) cannot be ignored in all of transportation projects. The operational model can handle multiple types of transportation sources, by corresponding and averaging conditions, where each condition includes some important elements with respect to human health, all economic. The model was not sensitive to small changes of the values of input parameters. Therefore under specific meteorological conditions, areas far away from the emission sources (roads) can also be highly polluted.

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