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RADIATIVE EFFECTS ON TRANSIENT FREE CONVECTIVE HEAT TRANSFER PAST AN EXPONENTIALLY ACCELERATED HOT VERTICAL SURFACE IN POROUS MEDIUM IN THE PRESENCE OF MAGNETIC FIELD

J.C. MUDULI

Department of Mathematics, Orissa Engineering College, Bhubaneswar

R.K. DHAL

J.N.V. Paralakhemundi, Gajapati, Orissa-761201(India)

B. JENA

J.N.V. Khiriyadevat, Ashoknagar, M.P.-473335 (India)

AND

M. SAHOO

Sai Shibani International School, Pallaspur, Bhubaneswar-752054

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Radiative effects on transient free convective heat transfer past an exponentially accelerated hot vertical surface in porous medium in the presence of magnetic field has been studied. The fluid is assumed to be gray, emitting-absorbing with non-scattering medium having optically thick radiation limit. The dimensionless governing equations are solved using Laplace transform technique. The results are obtained for velocity, temperature, Nusselt number, and shearing stress for different parameters like time, magnetic parameter, radiation parameter, etc. The flow characteristics are shown by means of graphs with vivid discussion.

KEYWORDS: *Transient, Heat Transfer, free convection, radiation, MHD and Porous Medium*

INTRODUCTION

The effects of radiation on temperature have become more important industrially. Many processes in engineering areas occur at high temperature and acknowledge radiation heat transfer which becomes very important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for air craft, missiles, satellites and space vehicles are example of such engineering areas. Radiative-convective heat transfer flows find numerous applications in glass manufacturing, furnace technology, high temperature aerodynamics, fire dynamics and spacecraft. Many studies have appeared concerning the interaction of radiative flux with thermal convection flows with applications in geophysics and geothermal reservoirs, on rocket combustion thrust chamber walls where significant radiation heat transfer is imparted from the hot propellant to the chamber walls.

The above studies did not consider transient effects or incorporate porous media effects. Both unsteady flows and porous convective-radiative flows have important applications in geophysics, geothermics, chemical and ceramic processing. The conventional approach in porous media transport modeling has been considered to simulate the pressure drop across the porous regime using Darcy's linear model. This basically adds an extra body force to the momentum boundary layer equation. Mohammadein *et al*. [1] studied the radiative flux effects on free convection in Darcian porous media with the Rosseland model. Satapathy *et al*. [2] and El-Hakiem *et al*. [3] have also analyzed radiative-convection flows in non- Darcian porous media using asymptotic and numerical methods. Ganesan *et al*. [4] studied theoretically the thermal radiation effects on unsteady flow past an impulsively started plate. Muthucumaraswamy and Ganesan [5] analyzed transient radiation-convection impulsively-started flow with variable temperature effects. Raptis and Perdikis [6] have also studied analytically the transient convection in a highly porous medium with unidirectional radiative flux. Ghosh and Pop [8] studied indirect radiation effects on convective gas flow. Very recently Mazumdar *et al*, [7] have studied the MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Ghosh *et. al*, [9] have discussed the theoretical Analysis of Radiative Effects on Transient Free Convection Heat Transfer past a Hot Vertical Surface in Porous Media. Vasu *et al*., [10] presented radiation and mass transfer effects on transient free convection flow of a dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. N. Ahmed, [11] has discussed about Soret and Radiation effects on Transient MHD Free Convection from an Impulsively Started Infinite Vertical Plate. N. Ahmed and M. Dutta [12] studied about Analytical Analysis of Magneto-hydrodynamics (MHD) Transient Flow Past a suddenly started Infinite Vertical Plate with Thermal Radiation and Ramped Wall Temperature.

This paper deals with the study of Radiative effects on transient free convective heat transfer past an exponentially accelerated hot vertical surface in porous medium in the presence of magnetic field.

FORMULATION OF THE PROBLEM

We consider the unsteady flow of a viscous incompressible electrically conducting fluid occupying semi-infinite region of the space past an exponentially accelerated infinite hot vertical plate. The plate starts accelerating with a velocity $u' = U_0 \exp(a't')$ in its own plane. x′-axis is taken along the vertical plate in upward direction and the y′-axis is taken normal to the plate in the direction of applied uniform magnetic field of strength H_o. The magnetic permeability μ_e is constant throughout the field. All fluid properties are considered constant except the influence of density variation in the body force term. The radiation heat flux in the x′-direction is considered negligible in comparison to the *y*′-direction. The fluid is gray, absorbing emitting but non-scattering. Gravity acts in the opposite direction to the positive *x*′-axis. The porous regime is assumed to be in local thermal equilibrium and thermal dispersion effects are ignored. Then by usual Boussinesq's approximation, the unsteady flow is governed by following equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) - \frac{\nu u'}{K'} - \frac{\sigma B_0^2 u'}{\rho} \quad \dots (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \quad \dots (2)$$

with the following boundary conditions

$$\left.\begin{array}{l} t \leq 0 : u' = 0 \ , T' = T'_\infty \ \text{ for } y' \geq 0 \\ t' > 0 \ : u' = U_0 e^{a't'}, \ T' = T'_w \ \text{ at } y' = 0 \\ u' = 0 \ , \ T' \to T'_\infty \ \text{ as } \ y' \to \infty \end{array}\right\} \qquad \ldots (3)$$

where $u', t', \nu, g, \beta, T', T'_w, T'_\infty, k, C_p, \rho, q_r$ *and* K' are respectively, the velocity component along the plate, the velocity component normal to the plate, dimensional time, the kinematic coefficient of viscosity, the gravitational acceleration, the coefficient of thermal expansion, the temperature of the fluid, the temperature at the plate, the temperature of the fluid far away from the plate (in the free stream), the thermal conductivity, the specific heat at constant pressure, the density of the fluid, the radiative heat flux and the permeability of the porous medium (dimensions, m^2).

The radiation flux on the basis of the Rosseland diffusion model for radiation heat transfer is expressed as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \qquad \ldots (4)$$

in which σ^* and k^* are Stefan-Boltzmann constant and the spectral mean absorption coefficient of the medium. It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as linear function of the temperature. It can be established by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher order terms, T'^4 can be expressed in the following way:

$$T'^4 = 4T'^3_\infty T' - 3T'^4_\infty \qquad \ldots (5)$$

We introduce the following non-dimensional quantities:

$$\left.\begin{array}{l} u = \frac{u'}{U_0}, y = \frac{y' U_0}{\nu}, t = \frac{t' U_0^2}{\nu}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, Pr = \frac{\rho \nu C_p}{k}, M = \frac{\sigma \nu B_0^2}{\rho U_0^2} \\ R = \frac{16\sigma^* T'^3_\infty}{3k^* k}, a = \frac{a' \nu}{U_0^2}, \frac{1}{K} = \frac{\nu^2}{K' U_0^2}, Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U_0^3} \end{array}\right\} \qquad \ldots (6)$$

where *Gr* is Grashof number, *M* is magnetic field strength, *Pr* is Prandtl number, *R* is radiation parameter, *K* is permeability parameter of the porous medium and a is exponential parameter.

Using equations (4) to (5) in equations (1) to (3), we get the following:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - \left(\frac{1}{K} + M\right) u \qquad \ldots (7)$$

$$(1 + R)\frac{\partial^2 \theta}{\partial y^2} - Pr\frac{\partial \theta}{\partial t} = 0 \qquad \ldots (8)$$

With the boundary conditions

$$\left.\begin{array}{l} t \leq 0: u = 0, \quad \theta = 0 \ \text{ for } y \geq 0 \\ t > 0 : u = e^{at}, \ \theta = 1 \ \text{ at } y = 0 \\ u = 0, \quad \theta \to 0 \ \text{ as } \ y \to \infty \end{array}\right\} \qquad \ldots (9)$$

METHOD OF SOLUTION

We solve the governing equations in an exact form by using Laplace transform technique. Taking Laplace transform of equations (7) to (8) using condition (9), we get

$$\frac{d^2\bar{u}}{dy^2} - (M' + s)\bar{u} = -Gr\bar{\theta} \qquad \dots (10)$$

$$(1+R)\frac{d^2\bar{\theta}}{dy^2} - Prs\bar{\theta} = 0 \qquad \dots (11)$$

With the boundary conditions

$$\left.\begin{aligned} &\bar{u} = \frac{1}{s-a}, \bar{\theta} = \frac{1}{s} \quad at \ y = 0 \\ &\bar{u} \to 0, \bar{\theta} \to 0 \quad as \ y \to \infty \end{aligned}\right\} \qquad \dots (12)$$

where $M' = M + \frac{1}{K}$ and s is the Laplace transformation parameter.

Solving equations (10) to (11) using condition (12), we have

$$\bar{u} = \left(\frac{1}{s-a} + \frac{Gr(R+1)}{Pr-R-1}\frac{1}{s\left(s-\frac{M'(R+1)}{Pr-R-1}\right)}\right)e^{-\sqrt{M'+s}y} - \frac{Gr(R+1)}{Pr-R-1}\frac{1}{s\left(s-\frac{M'(R+1)}{Pr-R-1}\right)}e^{-\sqrt{\frac{\Pr s}{R+1}}y} \qquad \dots (13)$$

$$\bar{\theta} = \frac{1}{s}e^{-\sqrt{\frac{\Pr s}{R+1}}y} \qquad \dots (14)$$

Taking the inverses Laplace transforms of equations (13) and (14), we get

$$u = \frac{1}{2}\left[e^{at-2\eta\sqrt{(M'+a)t}}erfc\left(\eta - \sqrt{(M'+a)t}\right) + e^{at+2\eta\sqrt{(M'+a)t}}erfc\left(\eta + \sqrt{(M'+a)t}\right)\right]$$
$$+\frac{Gr}{2M'}\begin{bmatrix} e^{\frac{M't(R+1)}{Pr-R-1}-2\eta\sqrt{\frac{M'tPr}{Pr-R-1}}}.erfc\left(\eta - \sqrt{\frac{tM'Pr}{Pr-R-1}}\right) \\ +e^{\frac{M't(R+1)}{Pr-R-1}+2\eta\sqrt{\frac{M'tPr}{Pr-R-1}}}.erfc\left(\eta + \sqrt{\frac{tM'Pr}{Pr-R-1}}\right) \\ -e^{-2\eta\sqrt{tM'}}erfc\left(\eta - \sqrt{M't}\right) - e^{2\eta\sqrt{tM'}}erfc\left(\eta + \sqrt{M't}\right) \end{bmatrix}$$
$$-\frac{Gr}{2M'}\left[-2erfc\left(\eta\sqrt{\frac{Pr}{R+1}}\right) + e^{\frac{M't(R+1)}{Pr-R-1}-2\eta\sqrt{\frac{M'tPr}{Pr-R-1}}}.erfc\left(\eta\sqrt{\frac{Pr}{R+1}} - \sqrt{\frac{M't(R+1)}{Pr-R-1}}\right) + e^{\frac{M't(R+1)}{Pr-R-1}+2\eta\sqrt{\frac{M'tPr}{Pr-R-1}}}.erfc\left(\eta\sqrt{\frac{Pr}{R+1}} + \sqrt{\frac{M't(R+1)}{Pr-R-1}}\right)\right] \qquad \dots (15)$$

and

$$\theta = erfc\left(\eta\sqrt{\frac{Pr}{R+1}}\right) \qquad \dots (16)$$

where $\eta = \frac{y}{2\sqrt{t}}$

The dimensionless shearing stress/skin friction:

$$\tau = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} = \frac{1}{2}\Big[2e^{at}\sqrt{t(M'+a)}\big(erfc(t(M'+a)) - erfc(-t(M'+a))\big) - \frac{4}{\sqrt{\pi}}e^{-M't}\Big]$$
$$+\frac{Gr}{2M'}\left[2\sqrt{\frac{M'tPr}{Pr-R-1}}\,.\,\mathrm{e}^{\left(\frac{M't(R+1)}{Pr-R-1}\right)}\left(\mathrm{erfc}\left(\sqrt{\frac{M'tPr}{Pr-R-1}}\right) - \mathrm{erfc}\left(-\sqrt{\frac{M'tPr}{Pr-R-1}}\right)\right) -\right.$$
$$\left. 2\sqrt{\mathrm{tM'}}\big(\mathrm{erfc}(\sqrt{\mathrm{M't}}) - \mathrm{erfc}(-\sqrt{\mathrm{M't}})\big)\right] - \frac{Gr}{M'}\left[\left(2\sqrt{\frac{M'tPr}{Pr-R-1}}\,.\,\mathrm{e}^{\left(\frac{M't(R+1)}{Pr-P-1}\right)}\left(\mathrm{erfc}\left(\sqrt{\frac{M't\,(R+1)}{Pr-R-1}}\right) -\right.\right.\right.$$
$$\left.\left.\left.\mathrm{erfc}\left(-\sqrt{\frac{M't(R+1)}{Pr-R-1}}\right)\right)\right) - \sqrt{\frac{Pr}{(R+1)\pi}}\right] \quad \ldots (17)$$

The non-dimensional rate of heat transfer (Nusselt number):

$$Nu = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{Pr}{R+1}} \quad \ldots (18)$$

Discussion and graphs

In this paper, we have studied the effect of radiation on transient free convective heat transfer past an exponentially accelerated hot vertical surface in porous medium in the presence of magnetic field. The effect of the parameters *a*, *R*, *K*, *M*, *Gr*, *t* and *Pr* on flow characteristics have been studied and shown by means of graphs. To obtain the graphs, velocity and temperature are plotted against η and shearing stress is taken w.r.t. time (t). Effects of different parameters are discussed for each graph.

Figure-(1) illustrates the effects of the parameters *Gr*, *a* and *M* on velocity at any point of the fluid, when $K = 1$, $R = 2$, $Pr = 7$ and $t = 1$. It is noticed that the velocity increases with the increase of Grashof number (*Gr*) and exponential parameter a, whereas it decreases with the increase of magnetic field strength (*M*).

Figure-(2) illustrates the effects of the parameters *K* and *R* on velocity at any point of the fluid, when $a = 1$, $M = 1$, $Gr = 2$, $Pr = 7$ and $t = 1$. It is noticed that the velocity increases with the increase of both permeability parameter of the porous medium (*K*) and radiation parameter (*R*).

Figure-(3) illustrates the effects of the parameters *Pr* and t on velocity at any point of the fluid, when $a = 1$, $M = 1$, $Gr = 2$, $K = 1$ and $R = 2$. It is noticed that the velocity at any point of the fluid increases with the increase in time (*t*), whereas decreases with the increase of Prandtl number (*Pr*).

Figure-(4) illustrates the effects of the parameters *Pr* and *R* on temperature at any point of the fluid. It is noticed that the temperature falls with the increase of Prandtl number (*Pr*), whereas rises with the increase of radiation parameter (*R*).

Figure-(5) illustrates the effects of the parameters *Gr*, *a* and *M* on shearing stress, when $K = 1$, $R = 2$ and $Pr = 7$. It is noticed that the shearing stress increases with the increase of Grashof number (*Gr*) and also decreases with the increase of magnetic field strength (*M*). But shearing stress increases for a few moment after few times.

Figure-(6) illustrates the effects of the parameters *K*, *R* and *Pr* on shearing stress, when $a = 1$, $M = 1$ and $Gr = 2$. It is noticed that the shearing stress increases with the increase of permeability parameter of the porous medium (*K*) and Prandtl number (*Pr*), whereas decreases with the increase of radiation parameter (*R*).

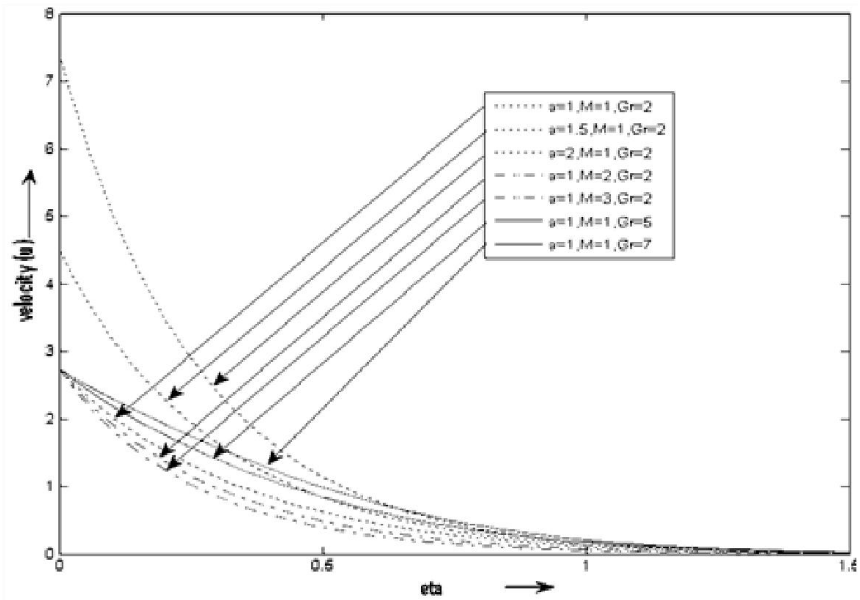

Fig. 1. Effects of *a, M* and *Gr* on velocity profile, when *K* = 1, *R* = 2, *Pr* = 7 and *t* = 1.

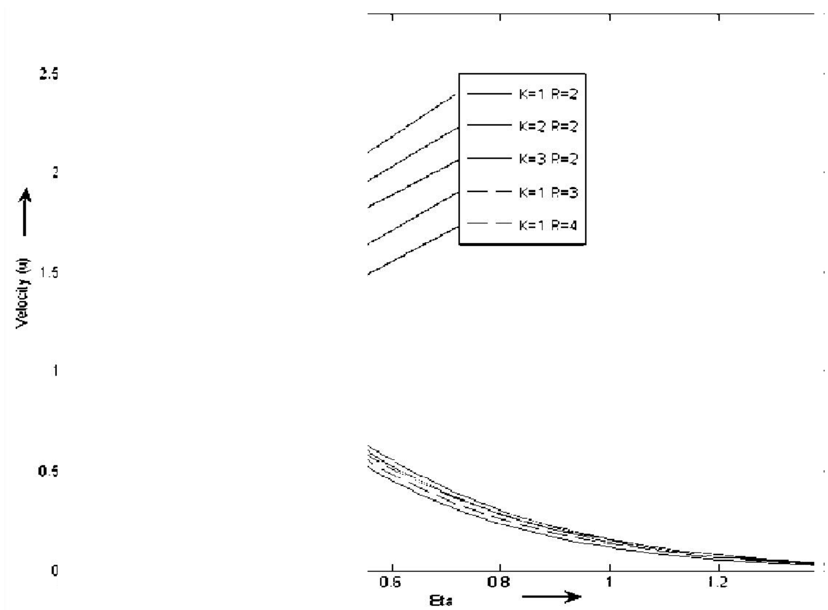

Fig. 2. Effects of *K* and *R* on velocity profile, when *a* = 1, *M* = 1, *Gr* = 2, *Pr* = 7 and *t* = 1.

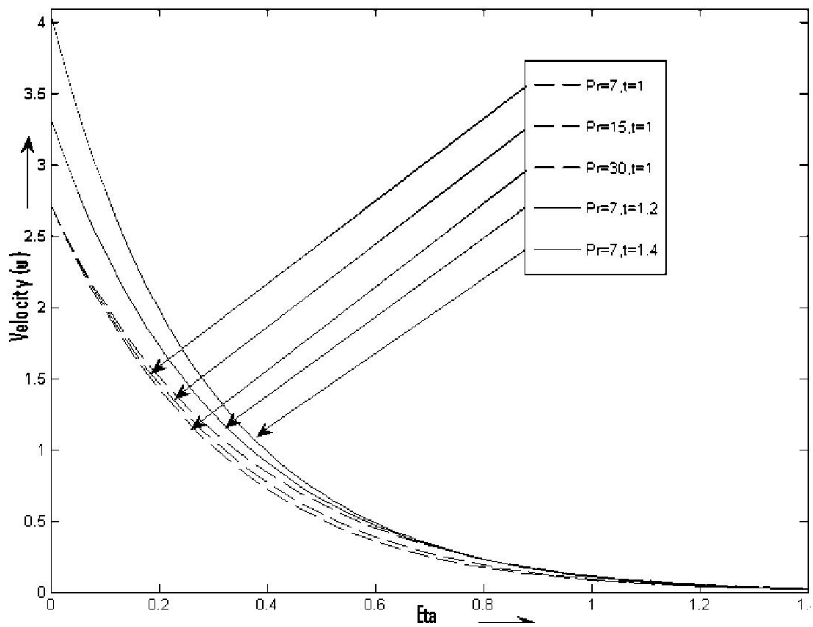

Fig. 3. Effects of *Pr* and *t* on velocity profile, when *a* = 1, *M* = 1, *Gr* = 2, *K* = 1 and *R* = 2.

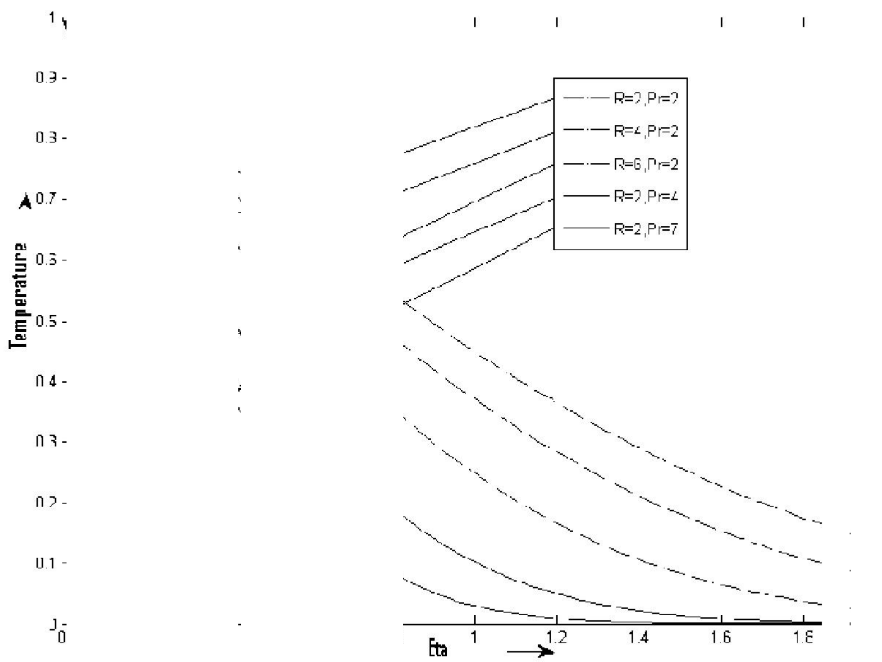

Fig. 4. Effects of *Pr* and *R* on temperature profile.

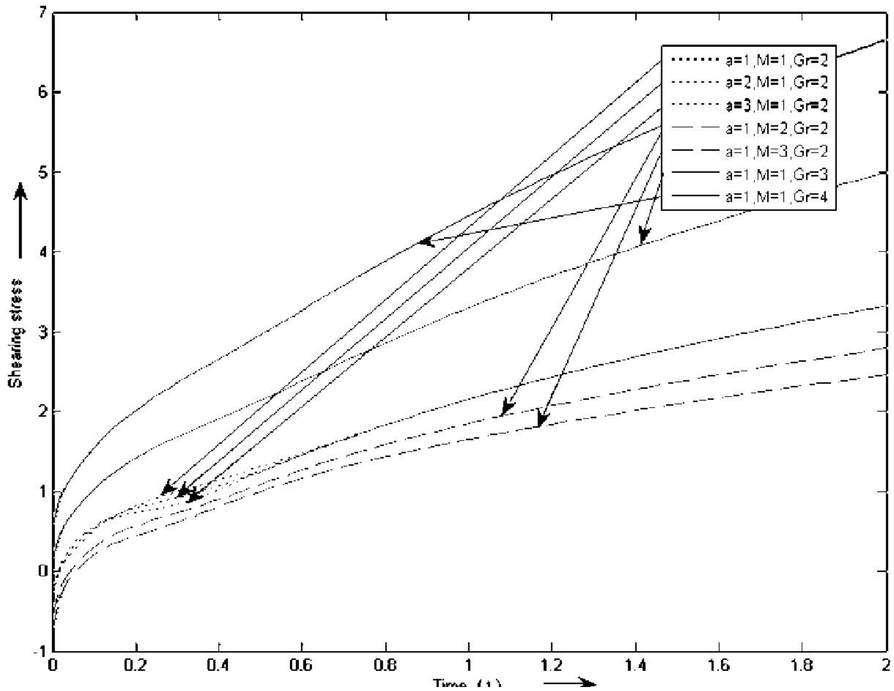

Fig-5- Effects of a, M and Gr on Shearing stress, when K=1, R=2 and Pr=7.

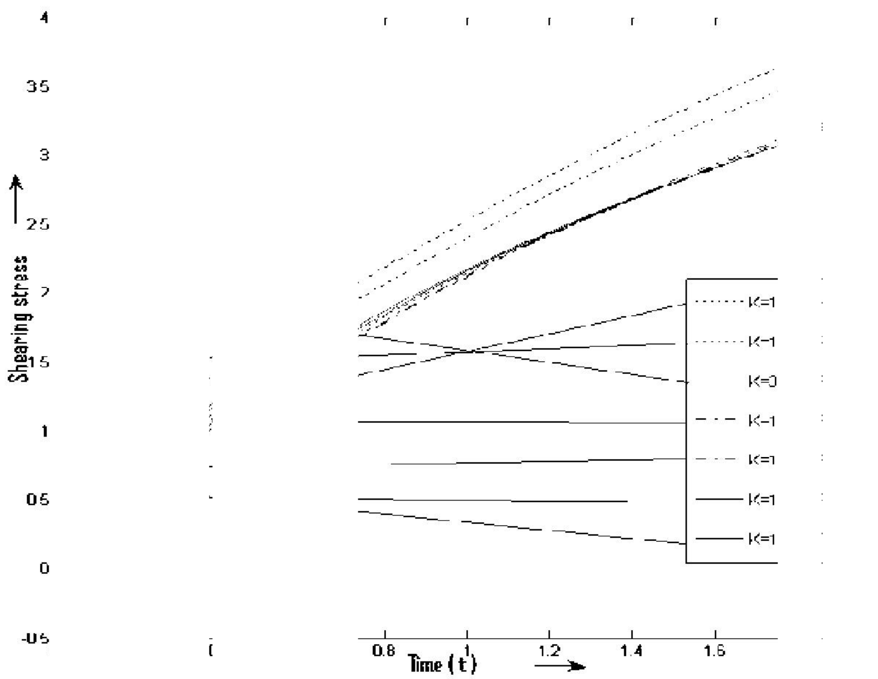

Fig. 6. Effects of *K*, *R* and *Pr* on Shearing stress, when *a* = 1, *M* = 1 and *Gr* = 2.

Conclusion

In this paper, Radiative effects on transient free convective heat transfer past an exponentially accelerated hot vertical surface in porous medium in the presence of magnetic field is presented. Results are represented graphically to illustrate the variation of velocity, temperature and skin-friction with various parameters. In this study, the following conclusions are set out:

(1) In case of cooling of the plate ($Gr > 0$), the velocity decreases with an increase in magnetic parameter and Prandtl number. On the other hand, it increases with an increase in the value of Grashof number, Radiation parameter, Exponential parameter and permeability parameter.

(2) In case of cooling of the plate ($Gr > 0$), the temperature drops for the increase of Prandtl number, but rises for the increase of Radiation parameter.

(3) The skin-friction increases with an increase in Prandtl number, permeability parameter and Grashof number, whereas, it decreases with an increase in the value of magnetic parameter and Radiation parameter.

Facts discussed in this paper can be applicable for different fluids, for different flows in the presence/absence of different parameters with or without mass transfer.

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