Acta Ciencia Indica, Vol. XL M, No. 2, 153 (2014)

Λ_p^*-SEPARATION AXIOMS

P. GNANACHANDRA

Aditanar College of Arts and Science, Tiruchendur -628216, India

RECEIVED : 16 September, 2013

Separation axioms are among the most **common** and important and interesting concepts in topology as well as in bitopologies. In this paper, we introduce Λ_p^* -sets and some weak separation axioms using Λ_p^* -open sets and Λ_p^* -closure operator. The aim of this paper is to introduce Λ_p^*-T_i and Λ_p^*-R_j, for i = 0, 1, 2 and j = 0, 1 spaces using Λ_p^*-open and Λ_p^*-closed sets. Some existing lower separation axioms are characterized by using these spaces.

KEYWORDS AND PHRASES: pre*-open, pre*-closed sets, Λ_p^*-open sets and Λ_p^* -closed sets.

MSC 2010: 54A20, 54C10, 54D10.

INTRODUCTION AND PRELIMINARY

The separation axioms R_0 and R_1 in topological spaces were introduced by Shanin [16] in 1943. Murdeshwar and Naimpally [12, 13] investigated the properties of R_0 topological spaces and many interesting results have been obtained. Caldas *et. al*. [3] introduced Λ_α-sets and V_α -sets characterize some of their properties. Navaneethakrishnan [14] used regular-open sets to define V_r-sets and Λ_r-sets and investigate some separation axioms using these sets in topological spaces. Using semi-open sets, Caldas and Dontchev [1] extended Maki's work by introducing and studying Λ_s-sets and V_s-sets. The purpose of this paper is to continue the research along these directions but this time by utilizing Λ_p^* -open sets. For details see ([2], [3], [4], [8], [9], [11], [11] and [12]). In this paper, we introduce some Λ_p^* -separation axioms in topological spaces. To define and investigate the axioms, we use Λ_p^* -open sets.

Throughout this paper (X, τ) denotes a topological space on which no separation axioms are assumed unless explicitly stated. Standard definitions and notations in point set topology are used in this paper.

A subset A of a topological space (X, τ) is said to be pre*-*open* [15] if $A \subseteq \text{int}^*(\text{cl}(A))$, where int*($A$) and cl($A$) respectively denote the g-interior and the closure of A. The complement of a pre*-open set is pre*-closed. We shall denote the families of all pre*-open sets in a space (X, τ) by $P^*O(X, \tau)$. Also A subset A is called a Λ_p^* -closed set [6] if $A = S \cap C$

where S is a Λ_p^*-set and C is a closed set. The complement of a Λ_p^*-closed set is called a Λ_p^*-open set. The collection of all Λ_p^*-open sets in (X, τ) is denoted by $\Lambda_p^* O(X, \tau)$ and the collection of all Λ_p^*-closed sets in (X, τ) is denoted by $\Lambda_p^* C(X, \tau)$. Recall that a subset S of a space (X, τ) is called a *pre**-Λ-set (briefly Λ_p^*-set [6]) if $S = \Lambda_p^*(S)$

where $\Lambda_p^*(S) = \cap\{G: S \subseteq G, G \in P^*O(X, \tau)\}$.

Definition 1.1: [6] Let X be a space and $A \subseteq X$. Then a point $x \in X$ is called a Λ_p^*-cluster point of A if for every Λ_p^*-open set U containing x, $A \cap U \neq \emptyset$. The collection of all Λ_p^*-cluster points of A is called the Λ_p^*-closure of A and is denoted by Λ_p^*-cl(A).

Proposition 1.2: [6] *(i)* $A \subseteq \Lambda_p^*\text{-}cl(A)$.

(ii) $\Lambda_p^*\text{-}cl(A) = \cap\{F : A \subseteq F$ and F is Λ_p^*-closed$\}$,

(iii) If $A \subseteq B$, then $\Lambda_p^*\text{-}cl(A) \subseteq \Lambda_p^*\text{-}cl(B)$, *(iv)* A is Λ_p^*-closed if and only if $A = \Lambda_p^*\text{-}cl(A)$ and

(v) $\Lambda_p^*\text{-}cl(A)$ is Λ_p^*-closed.

Λ_p^*-T_K (K=0,1,2) SPACES

Definition 2.1: A space X is said to be Λ_p^*-T_0 if for each pair of distinct points x, y of X, there exists a Λ_p^*-open set containing one of the points but not the other.

For the existence of Λ_p^*-T_0 space, consider a topological space (X, τ) where $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$.

Clearly, (X, τ) is Λ_p^*-T_0.

The following theorem characterizes Λ_p^*-T_0 spaces.

Theorem 2.2: A space X is Λ_p^*-T_0 if and only if for each pair of distinct points x, y of X, $\Lambda_p^*\text{-}cl(\{x\}) \neq \Lambda_p^*\text{-}cl(\{y\})$.

Proof. Suppose X is a Λ_p^*-T_0 space. Let $x, y \in X$ such that $x \neq y$. By using Definition 2.1, there exists a Λ_p^*-open set V containing one of the points but not the other, say $x \in V$ and $y \notin V$ and so $X \backslash V$ is a Λ_p^*-closed set containing y but not x. It follows that $y \in \Lambda_p^*\text{-}cl(\{y\}) \subseteq X \backslash V$ and so $X \notin \Lambda_p^*\text{-}cl(\{y\})$ which implies that $\Lambda_p^*\text{-}cl(\{x\}) \neq \Lambda_p^*\text{-}cl(\{y\})$.

Conversely, let x, $y \notin X$, $x \neq y$ such that Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$. Suppose there is an element $z \in X$ such that $z \in \Lambda_p^*$-$cl(\{x\})$ and $z \notin \Lambda_p^*$-$cl(\{y\})$. If $x \in \Lambda_p^*$-$cl(\{y\})$, then Λ_p^*-$cl(\{x\}) \subseteq \Lambda_p^*$-$cl(\{y\})$ that implies $z \in \Lambda_p^*$-$cl(\{y\})$, a contradiction. Thus $x \notin \Lambda_p^*$-$cl(\{y\})$ which implies that $x \in X \backslash \Lambda_p^*$-$cl(\{y\})$, $y \notin X \backslash \Lambda_p^*$-$cl(\{y\})$ and $X \backslash \Lambda_p^*$-$cl(\{y\})$ is Λ_p^*-open. This shows that X is Λ_p^*-T_0.

Corollary 2.3. A space X is Λ_p^*-T_0 if and only if for each pair of distinct points x, y of X, either $x \notin \Lambda_p^*$-$cl(\{y\})$ or $y \notin \Lambda_p^*$-$cl(\{x\})$.

Theorem 2.4. A space X is Λ_p^*-T_0 if and only if for each pair of distinct points x, y of X, Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$.

Proof. Suppose X is a Λ_p^*-T_0 space. By Theorem 2.2, Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$ and so by Theorem 3.8 of [6], Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$.

Conversely, suppose for x, $y \in X$ with $x \neq y$, Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$, so by Theorem 3.8 of [6], Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$ and by Theorem 2.2, X is a Λ_p^*-T_0 space.

Definition 2.5. A space X is said to be Λ_p^*-T_1 if for any pair of distinct points x, y of X, there is a Λ_p^*-open set U in X such that $x \in U$ and $y \notin U$ and there is a Λ_p^*-open set V in X such that $y \in V$ and $x \notin V$.

For the existence of Λ_p^*-T_1 space, consider a topological space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Clearly, (X, τ) is Λ_p^*-T_1.

The following theorem characterizes Λ_p^*-T_1 spaces.

Theorem 2.6. For a space X, the following are equivalent :

(i) X is Λ_p^*-T_1.

(ii) For every $x \in X$, $\{x\} = \Lambda_p^*$-$cl(\{x\})$.

(iii) For each $x \in X$, the intersection of all Λ_p^*-open sets containing x is $\{x\}$.

Proof. *(i)* $\Rightarrow$ *(ii)*. Suppose X is a Λ_p^*-T_1 space. Let $x \in X$ and $y \neq x$ in X. By Definition 2.5, there exists a Λ_p^*-open set V in X such that $x \notin V$ and $y \in V$. If $y \in \Lambda_p^*$-$cl(\{x\})$, then by using Definition 1.1, y is a Λ_p^*-cluster point of $\{x\}$ which implies that for every Λ_p^*-open set U containing y, $\{x\} \cap U \neq \emptyset$. Now V is a -open set containing y and so $\{x\} \cap V \neq \emptyset$ which implies that $x \in V$, a contradiction. Hence $y \notin \Lambda_p^*$-$cl(\{x\})$. That is $y \notin \Lambda_p^*$-$cl(\{x\})$ for every $y \neq x$. This shows that $\{x\} = \Lambda_p^*$-$cl(\{x\})$.

(ii) $\Rightarrow$ *(iii)*. Suppose for every $x \in X$, $\{x\} = \Lambda_p^*$-$cl(\{x\})$. By using Lemma 3.7(1) of [6], we have $\{x\} \subseteq \Lambda_p^*$-$ker(\{x\})$. If $y \in \Lambda_p^*$-$ker(\{x\})$, then by By Lemma 3.7(4) of [6], $x \in \Lambda_p^*$-$cl(\{y\})$ and so by hypothesis, $x \in \{y\}$, that is, $y \in \{x\}$ which implies that Λ_p^*-$ker(\{x\}) \subseteq \{x\}$. Thus we get $\{x\} = \Lambda_p^*$-$ker(\{x\})$ and so $\{x\} = \cap \{G : G \in \Lambda_p^* O(X, \tau)$ and $\{x\} \subseteq G\}$.

(iii) $\Rightarrow$ *(i)*. Suppose that for each $x \in X$, the intersection of all Λ_p^*-open sets containing x is $\{x\}$. Let $x, y \in X$ with $x \neq y$. Then by hypothesis, $\{x\} = \cap \{G : G \in \Lambda_p^* O(X, \tau)$ and $\{x\} \subseteq G\}$. From this, we can find one Λ_p^*-open set V containing x but not y. In the same manner, we can find one Λ_p^*-open set U containing y but not x and so X is Λ_p^*-T_1.

Theorem 2.7. A space X is Λ_p^*-T_1 if and only if the singletons are Λ_p^*-closed sets.

Proof. Suppose X is Λ_p^*-T_1. Then Λ_p^*-$cl(\{x\}) = \{x\}$ for every $x \in X$ and so $\{x\}$ is Λ_p^*-closed. Conversely, suppose $\{x\}$ is Λ_p^*-closed for every $x \in X$. By Proposition using 3.2(4) of [6], Λ_p^*-$cl(\{x\}) = \{x\}$. By using Theorem 2.6, X is a Λ_p^*-T_1 space.

Definition 2.8. A space X is said to be Λ_p^*-T_2 if for each pair of distinct points x and y in X, there Λ_p^*-open sets U and V in X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

For the existence of Λ_p^*-T_2 space, consider a topological space (X, τ) where $X = \{a, b\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, X\}$. It can be verified that, (X, τ) is Λ_r-T_2.

Theorem 2.9 characterizes Λ_p^*-T_2 spaces.

Theorem 2.9. For a space X, the following are equivalent:

(i) X is Λ_p^*-T_2.

(ii) If $x \in X$, then for each $y \neq x$, there is a Λ_p^*-open set U containing x such that

$$Y \notin \Lambda_p^*\text{-}cl(U).$$

(iii) For each $x \in X$, $\{x\} = \cap \{\Lambda_p^*$-$cl(U) : U$ is a Λ_p^*-open set containing $x\}$.

Proof. *(i)* $\Rightarrow$ *(ii)*. Suppose X is a Λ_p^*-T_2 space. Let $x \in X$. By Definition 2.8, for each $y \neq x$, there exist Λ_p^*-open sets A and B such that $x \in A$, $y \in B$ and $A \cap B = \emptyset$. Take $X \backslash B = F$.

Then it follows that F is Λ_p^*-closed, $A \subseteq F$ and $y \notin F$ which implies that $y \notin \cap\{F : F$ is Λ_p^*-closed and $A \subseteq F\}$ and so by Proposition 1.2 *(ii)*, we have $y \notin \Lambda_p^*\text{-}cl(A)$.

(ii) $\Rightarrow$ *(i)*. Suppose for each $y \neq x$ in X, there is a Λ_p^*-open set U containing x such that $y \notin \Lambda_p^*\text{-}cl(U)$. Then $y \in X \backslash \Lambda_p^*\text{-}cl(U))$ and by using Proposition 1.2*(i),* Proposition 1.2*(v)*, $x \in U \subseteq \Lambda_p^*\text{-}cl(U)$ and $X \backslash(\Lambda_p^*\text{-}cl(U))$ is Λ_p^*-open which implies that $U \cap (X \backslash(\Lambda_p^*\text{-}cl(U))) =$ Ø.

This shows that X is Λ_p^*-T_2.

The proof of *(ii)* $\Leftrightarrow$ *(iii)* is clear and so it is omitted.

Λ_p^*-R_0 SPACES

Definition 3.1. A topological space X is said to be Λ_p^*-R_0 if for each Λ_p^*-open set G, $x \in G$ implies $\Lambda_p^*\text{-}cl(\{x\}) \subseteq G$.

For the existence of Λ_p^*-R_0 space, consider a topological space (X, τ)

where $X = \{a, b, c, d\}$ and $\tau = \{$Ø$, \{a\}, \{b, c, d\}, X\}$. It is easy to check that (X, τ) is Λ_p^*-R_0.

Theorem 3.2. A space X is Λ_p^*-R_0 if and only if every Λ_p^*-open subset of X is the union of Λ_p^*-closed sets.

Proof. Suppose X is a Λ_p^*-R_0 space. If $A \subseteq X$ is Λ_p^*-open, then by using 3.1, for each $x \in A$, $\Lambda_p^*\text{-}cl(\{x\}) \subseteq A$ which implies $\cup\{\Lambda_p^*\text{-}cl(\{x\}): x \in A\} \subseteq A$, and hence $A = \cup\{\Lambda_p^*\text{-}cl(\{x\}) : x \in A\}$. By Proposition 1.1($v$), A is the union of Λ_p^*-closed sets.

Conversely, suppose A is Λ_p^*-open and $x \in A$. Then by hypothesis, there exist Λ_p^*-closed sets B_i in X such that $A = \cup\{B_i : i \in I\}$. Now $x \in A$ implies $x \in B_i$ for some $i \in I$. Then $x \in \Lambda_p^*\text{-}cl(\{x\}) \subseteq B_i \subseteq A$ and so X is Λ_p^*-R_0.

Theorem 3.3. For a space X, the following statements are equivalent:

(i) X is Λ_p^*-R_0.

(ii) For any Λ_p^*-closed set F and a point $x \notin F$, there exists $U \in \Lambda_p^* O(X, \tau)$ such that $x \notin U$ and $F \subseteq U$.

(iii) For any Λ_p^*-closed set F and a point $x \notin F$, $\Lambda_p^*\text{-}cl(\{x\}) \cap F =$ Ø.

Proof. Suppose *(i)* holds. If F is a Λ_p^*-closed set and $x \notin F$, then $X\backslash F$ is Λ_p^*-open and $x \in X\backslash F$. By Definition 3.1, $\Lambda_p^*\text{-}cl(\{x\}) \subseteq X\backslash F$ and so $F \subseteq X\backslash(\Lambda_p^*\text{-}cl(\{x\}))$. Thus by Proposition 1.2(v) and (i), $X\backslash(\Lambda_p^*\text{-}cl(\{x\}))$ is the required Λ_p^*-open set containing F and $x \notin X\backslash(\Lambda_p^*\text{-}cl(\{x\}))$. This proves ($ii$).

Suppose (ii) holds. If F is a Λ_p^*-closed set and $x \notin F$, then by hypothesis, there exists $U \in \Lambda_p^* O(X, \tau)$ such that $x \notin U$ and $F \subseteq U$. If $U \cap \Lambda_p^*\text{-}cl(\{x\}) \neq \emptyset$, then there exists $y \in X$ such that $y \in U$ and $y \in \Lambda_p^*\text{-}cl(\{x\})$. By Definition 1.1, y is a Λ_p^*-cluster point of $\{x\}$ and so for every Λ_p^*-open set G containing y, $G \cap \{x\} \neq \emptyset$, that is, $x \in G$. Now U is a Λ_p^*-open set containing y and so $x \in U$, a contradiction. Hence $U \cap \Lambda_p^*\text{-}cl(\{x\}) = \emptyset$ and $F \cap \Lambda_p^*\text{-}cl(\{x\}) = \emptyset$.

This proves (iii).

Suppose (iii) holds. If G is a Λ_p^*-open set and $x \in G$, then $X\backslash G$ is Λ_p^*-closed and $x \notin X\backslash G$. By hypothesis, $\Lambda_p^*\text{-}cl(\{x\}) \cap (X\backslash G) = \emptyset$ which implies that $\Lambda_p^*\text{-}cl(\{x\}) \subseteq G$. This proves ($i$).

Theorem 3.4. A space X is $\Lambda_p^*\text{-}R_0$ if and only if for each pair of points x, y of X, $\Lambda_p^*\text{-}cl(\{x\}) \neq \Lambda_p^*\text{-}cl(\{y\})$ implies $\Lambda_p^*\text{-}cl(\{x\}) \cap \Lambda_p^*\text{-}cl(\{y\}) = \emptyset$.

Proof. Assume that X is $\Lambda_p^*\text{-}R_0$. Let $x, y \in X$ such that $\Lambda_p^*\text{-}cl(\{x\}) \neq \Lambda_p^*\text{-}cl(\{y\})$. Then there exists $z \in X$ such that $z \in \Lambda_p^*\text{-}cl(\{x\})$ and $z \notin \Lambda_p^*\text{-}cl(\{y\})$. Since $z \notin \Lambda_p^*\text{-}cl(\{y\})$, there exists a Λ_p^*-open set V containing z such that $\{y\} \cap V = \emptyset$ and so $y \notin V$. Since $z \in \Lambda_p^*\text{-}cl(\{x\})$, for every Λ_p^*-open set G containing z, $\{x\} \cap G \neq \emptyset$, that is $x \in G$ which implies that $x \in V$. Since V is a Λ_p^*-open set containing x and $y \notin V$, $x \notin \Lambda_p^*\text{-}cl(\{y\})$ and so $x \in X\backslash \Lambda_p^*\text{-}cl(\{y\})$. Now by using Definition 3.1, $\Lambda_p^*\text{-}cl(\{x\}) \subseteq X\backslash \Lambda_p^*\text{-}cl(\{y\})$ and so $\Lambda_p^*\text{-}cl(\{x\}) \cap \Lambda_p^*\text{-}cl(\{y\}) = \emptyset$.

Conversely, suppose for each pair of points x, y of X, $\Lambda_p^*\text{-}cl(\{x\}) \neq \Lambda_p^*\text{-}cl(\{y\})$ implies $\Lambda_p^*\text{-}cl(\{x\}) \cap \Lambda_p^*\text{-}cl(\{y\}) = \emptyset$. Let G be a Λ_p^*-open set such that $x \in G$. If $y \notin G$, then $x \neq y$ and so $x \notin \Lambda_p^*\text{-}cl(\{y\})$ which implies that $\Lambda_p^*\text{-}cl(\{x\}) \neq \Lambda_p^*\text{-}cl(\{y\})$. By hypothesis, $\Lambda_p^*\text{-}cl(\{x\}) \cap \Lambda_p^*\text{-}cl(\{y\}) = \emptyset$ and so $y \notin \Lambda_p^*\text{-}cl(\{x\})$. This shows that $\Lambda_p^*\text{-}cl(\{x\}) \subseteq G$ and so X is a $\Lambda_p^*\text{-}R_0$ space.

Theorem 3.5. A space X is Λ_p^*-R_0 if and only if for each pair of points x, y of X, Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$ implies Λ_p^*-$ker(\{x\}) \cap \Lambda_p^*$-$ker(\{y\}) = \emptyset$.

Proof. Suppose X is a Λ_p^*-R_0 space. Let $x, y \in X$ such that Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$. Let $z \in \Lambda_p^*$-$ker(\{x\}) \cap \Lambda_p^*$-$ker(\{y\})$. Then $z \in \Lambda_p^*$-$ker(\{x\})$ and $z \in \Lambda_p^*$-$ker(\{y\})$. By Lemma 3.7 (4) of [6], we have $x \in \Lambda_p^*$-$cl(\{z\})$ and $y \in \Lambda_p^*$-$cl(\{z\})$ and so Λ_p^*-$cl(\{x\}) \cap \Lambda_p^*$-$cl(\{z\}) \neq \emptyset$ and Λ_p^*-$cl(\{y\}) \cap \Lambda_p^*$-$cl(\{z\}) \neq \emptyset$. By Theorem 3.4, we have Λ_p^*-$cl(\{x\}) = \Lambda_p^*$-$cl(\{z\})$ and Λ_p^*-$cl(\{y\}) = \Lambda_p^*$-$cl(\{z\})$ which implies that Λ_p^*-$cl(\{x\} = \Lambda_p^*$-$cl(\{y\})$. Then by Theorem 3.8 of [6], Λ_p^*-$ker(\{x\}) = \Lambda_p^*$-$ker(\{y\})$, a contradiction. Hence Λ_p^*-$ker(\{x\}) \cap \Lambda_p^*$-$ker(\{y\}) = \emptyset$.

Conversely, suppose that for $x, y \in X$, Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$ implies Λ_p^*-$ker(\{x\}) \cap \Lambda_p^*$-$ker(\{y\}) = \emptyset$. Let $x, y \in X$ such that Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$. Suppose $z \in \Lambda_p^*$-$cl(\{x\}) \cap \Lambda_p^*$-$cl(\{y\})$. Then $z \in \Lambda_p^*$-$cl(\{x\})$ and $z \in \Lambda_p^*$-$cl(\{y\})$. By Lemma 3.7(4) of [66], $x \in \Lambda_p^*$-$ker(\{z\})$, $y \in \Lambda_p^*$-$ker(\{z\})$ and Λ_p^*-$ker(\{x\}) \cap \Lambda_p^*$-$ker(\{z\}) \neq \emptyset$ and so Λ_p^*-$ker(\{y\}) \cap \Lambda_p^*$-$ker(\{z\}) \neq \emptyset$. By hypothesis, Λ_p^*-$ker(\{x\}) = \Lambda_p^*$-$ker(\{z\})$, Λ_p^*-$ker(\{y\}) = \Lambda_p^*$-$ker(\{z\})$ and so Λ_p^*-$ker(\{x\}) = \Lambda_p^*$-$ker(\{y\})$. Again by using Theorem 3.8 of [6].

Λ_p^*-$cl(\{x\}) = \Lambda_p^*$-$cl(\{y\})$, a contradiction. Therefore Λ_p^*-$cl(\{x\}) \cap \Lambda_p^*$-$cl(\{y\}) = \emptyset$ and so by Theorem 3.4, X is a Λ_p^*-R_0 space.

Theorem 3.6. For a space X, the following are equivalent:

(i) X is Λ_p^*-R_0.

(ii) For any nonempty set A and $G \in \Lambda_p^* O(X, \tau)$ such that $A \cap G \neq \emptyset$, there exists $F \in \Lambda_p^* C(X, \tau)$ such that $A \cap F \neq \emptyset$ and $F \subseteq G$.

(iii) For any $G \in \Lambda_p^* O(X, \tau)$, $G = \cup \{F : F \in \Lambda_p^* C(X, \tau)$ and $F \subseteq G\}$.

(iv) For any $F \in \Lambda_p^* \mathrm{C}(\mathrm{X}, \tau)$, $\mathrm{F} = \cap \{G : G \in \Lambda_p^* \mathrm{O}(X, \tau)$ and $F \subseteq G\}$.

(v) For any $x \in X$, Λ_p^*-$cl(\{x\}) \subseteq \Lambda_p^*$-$ker(\{x\})$.

(vi) For any $x, y \in X$, $y \in \Lambda_p^*$-$cl(\{x\}) \Leftrightarrow x \in \Lambda_p^*$-$cl(\{y\})$.

Proof. Suppose (*i*) holds. Let A be any nonempty subset of X and G be a Λ_p^*-open set in X such that $A \cap G \neq \emptyset$. Let $x \in A \cap G$. Then by Definition 3.1, $x \in G$ implies $\Lambda_p^*\text{-}cl(\{x\}) \subseteq G$. Since $x \in A$, we have $\Lambda_p^*\text{-}cl(\{x\}) \cap A \neq \emptyset$. Thus $\Lambda_p^*\text{-}cl(\{x\})$ is the required Λ_p^*-closed set contained in G such that $A \cap \Lambda_p^*\text{-}cl(\{x\}) \neq \emptyset$. This proves (*ii*).

Suppose (*ii*) holds. If $G \in \Lambda_p^* O(X, \tau)$ and $x \in G$, then by hypothesis, there exists $F \in \Lambda_p^* C(X, \tau)$ such that $\{x\} \cap F \neq \emptyset$ and $F \subseteq G$. Then it follows that $x \in F$ and so $x \in \cup \{F : F \in \Lambda_p^* C(X, \tau)$ and $F \subseteq G\}$ and so $G \subseteq \cup \{F : F \in \Lambda_p^* C(X, \tau)$ and $F \subseteq G\}$. Also $\cup \{F : F \in \Lambda_p^* C(X, \tau)$ and $F \subseteq G\} \subseteq G$. This proves (*iii*).

Suppose (*iii*) holds. If $F \in \Lambda_p^* C(X, \tau)$, then $X \setminus F \in \Lambda_p^* O(X, \tau)$ and so by hypothesis, $X \setminus F = \cup \{X \setminus G : X \setminus G \in \Lambda_p^* C(X, \tau)$ and $X \setminus G \subseteq X \setminus F\}$ which implies that $F = \cap \{G : G \in \Lambda_p^* O(X, \tau)$ and $F \subseteq G\}$. This proves (*iv*).

Suppose (*iv*) holds. If $y \notin \Lambda_p^*\text{-}ker(\{x\})$, then by Lemma 3.7 (*iv*) of [6], $x \notin \Lambda_p^*\text{-}cl(\{y\})$. So there exists a Λ_p^*-open set V containing x such that $V \cap \{y\} = \emptyset$ which implies that $\Lambda_p^*\text{-}cl(\{y\}) \cap V = \emptyset$. Since $\Lambda_p^*\text{-}cl(\{y\})$ is Λ_p^*-closed, by hypothesis, $\Lambda_p^*\text{-}cl(\{y\}) = \cap \{G : G \in \Lambda_p^* O(X, \tau)$ and $\Lambda_p^*\text{-}cl(\{y\}) \subseteq G\}$. Since $x \in V$, we have $x \notin \Lambda_p^*\text{-}cl(\{y\})$ and so there exists $G \in \Lambda_p^* O(X, \tau)$ such that $\Lambda_p^*\text{-}cl(\{y\}) \subseteq G$ and $x \notin G$ which implies that $\Lambda_p^*\text{-}cl(\{x\}) \cap G = \emptyset$. Hence $y \notin \Lambda_p^*\text{-}cl(\{x\})$ and so $\Lambda_p^*\text{-}cl(\{x\}) \subseteq \Lambda_p^*\text{-}ker(\{x\})$. This proves *(v)*.

Suppose (*v*) holds. If $y \in \Lambda_p^*\text{-}cl(\{x\})$, then by hypothesis, $y \in \Lambda_p^*\text{-}ker(\{x\})$ and so by Lemma 3.7(*iv*) of [6], $x \in \Lambda_p^*\text{-}cl(\{y\})$. In the same manner, if $x \in \Lambda_p^*\text{-}cl(\{y\})$, then by using hypothesis, $x \in \Lambda_p^*\text{-}ker(\{y\})$ and so $y \in \Lambda_p^*\text{-}cl(\{x\})$. This shows that $x \in \Lambda_p^*\text{-}cl(\{y\}) \Leftrightarrow y \in \Lambda_p^*\text{-}cl(\{x\})$. This proves (*vi*).

Suppose (*vi*) holds. Let $G \in \Lambda_p^* O(X, \tau)$ and $x \in G$. If $y \notin G$, then $y \in X \setminus G$ and, $y \in \Lambda_p^*\text{-}cl(\{y\}) \subseteq X \setminus G$. Then $\Lambda_p^*\text{-}cl(\{y\}) \cap G = \emptyset$ which implies that $x \notin \Lambda_p^*\text{-}cl(\{y\})$. Then by hypothesis, $y \notin \Lambda_p^*\text{-}cl(\{x\})$. This shows that $\Lambda_p^*\text{-}cl(\{x\}) \subseteq G$. This proves (*i*).

Theorem 3.7. For a space X, the following properties are equivalent:

(*i*) X is $\Lambda_p^*\text{-}R_0$.

(*ii*) If F is Λ_p^*-closed, then $F = \Lambda_p^*\text{-}ker(F)$.

(*iii*) If F is Λ_p^*-closed and $x \in F$, then Λ_p^*-$ker(\{x\}) \subseteq F$.

(*iv*) If $x \in X$, then Λ_p^*-$ker(\{x\}) \subseteq \Lambda_p^*$-$cl(\{x\})$.

Proof. Suppose (*i*) holds. Let F be Λ_p^*-closed and $x \notin F$. Then $X \backslash F$ is a Λ_p^*-open set containing x. By Definition 3.1, Λ_p^*-$cl(\{x\}) \subseteq X \backslash F$ and so Λ_p^*-$cl(\{x\}) \cap F = \emptyset$. By Lemma 3.7(v) of [6], $x \notin \Lambda_p^*$-$ker(F)$. This shows that Λ_p^*-$ker(F) \subseteq F$. Also by Lemma 3.7 (i) of [6], $F \subseteq \Lambda_p^*$-$ker(F)$. This proves (*ii*).

Suppose (*ii*) holds. Let F be Λ_p^*-closed and $x \in F$. By using Lemma 3.7 of [6] (*ii*), Λ_p^*-$ker(\{x\}) \subseteq \Lambda_p^*$-$ker(F)$ and by hypothesis, Λ_p^*-$ker(\{x\}) \subseteq F$. This proves (*iii*).

Suppose (*iii*) holds. By Also by Lemma 3.7 (i and v) of [6], $x \in \Lambda_p^*$-$cl(\{x\})$ and Λ_p^*-$cl(\{x\})$ is Λ_p^*-closed. By hypothesis, Λ_p^*-$ker(\{x\}) \subseteq \Lambda_p^*$-$cl(\{x\})$. This proves (*iv*).

Suppose (*iv*) holds. Let $x \in \Lambda_p^*$-$cl(\{y\})$. Then by Lemma 3.7(iv) of [6], $y \in \Lambda_p^*$-$ker(\{x\})$ and by hypothesis, $y \in \Lambda_p^*$-$cl(\{x\})$. Conversely, let $y \in \Lambda_p^* \Lambda_r$-$cl(\{x\})$. Then by Lemma 3.7(*iv*), $x \in \Lambda_p^*$-$ker(\{y\})$ and by hypothesis, $x \in \Lambda_p^*$-$cl(\{y\})$. This shows that $x \in \Lambda_p^*$-$cl(\{y\}) \Leftrightarrow y \in \Lambda_p^*$-$cl(\{x\})$ and so by Theorem 3.6, X is Λ_p^*-R_0. This proves (*i*).

Corollary 3.8. A space X is Λ_r-R_0 if and only if for any $x \in X$, Λ_p^*-$cl(\{x\})= \Lambda_p^*$-$ker(\{x\})$.

Λ_p^*-R_1 SPACES

Definition 4.1. A space X is said to be Λ_p^*-R_1 if for each pair of points $x, y \in X$ with Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$, there exists Λ_p^*-open sets U and V in X such that Λ_p^*-$cl(\{x\}) \subseteq U$, Λ_p^* $cl(\{y\}) \subseteq V$ and $U \cap V = \emptyset$.

For the existence of Λ_p^*-R_1 space, consider a topological space (X, τ) where $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$. This space (X, τ) is Λ_p^*-R_1.

Theorem 4.2. Every Λ_p^*-R_1 space is Λ_p^*-R_0.

Proof. Suppose X is a Λ_p^*-R_1 space. Let U be Λ_p^*-open in X and $x \in U$. Then for each $y \in X \backslash U$, $x \neq y$ and so Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$. By Definition 4.1, there exist disjoint Λ_p^*-open sets U_y and V_y such that Λ_p^*-$cl(\{x\}) \subseteq U_y$ and Λ_p^*-$cl(\{y\}) \subseteq V_y$. Take $V = \cup \{V_y : y \in$

$X \backslash U\}$. Then by Proposition 3.5 of [1], V is Λ_p^*-open. Now $x \in \Lambda_p^*$-$cl(\{x\}) \subseteq U_y$ and $U_y \cap V_y = \emptyset$ for each $y \in X \backslash U$ and so $x \notin V_y$ for each $y \in X \backslash U$ which implies that $x \notin V$. Thus we have V is Λ_p^*-open, $x \in X \backslash V$ and $X \backslash U \subseteq V$. By Remark 3.3 of [6], $x \in \Lambda_p^*$-$cl(\{x\}) \subseteq X \backslash V \subseteq U$.

Theorem 4.3. If X is a Λ_p^*-R_0 space and for each pair of points x and y of X with Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$, there exists Λ_p^*-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$, then X is a Λ_p^*-R_1 space.

Proof. Let X be a Λ_p^*-R_0 space and for each pair of points x and y of X with Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$, there exists Λ_p^*-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. By Definition 3.1, Λ_p^*-$cl(\{x\}) \subseteq U$ and Λ_p^*-$cl(\{y\}) \subseteq V$ and so by Definition 4.1, X is Λ_p^*-R_1.

The following theorem characterizes Λ_p^*-R_1 spaces.

Theorem 4.4. A topological space X is Λ_p^*-R_1 if and only if for each points $x, y \in X$ such that Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$, there exist Λ_p^*-open sets U and V in X such that Λ_p^*-$cl(\{x\}) \subseteq U$, Λ_p^*-$cl(\{y\}) \subseteq V$ and $U \cap V = \emptyset$.

Proof. Suppose X is Λ_p^*-R_1. Let $x, y \in X$ such that Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$. By Theorem 4.2, X is Λ_p^*-R_0 and so by Corollary 4.3, Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$. Then by Definition 4.1, there exist disjoint Λ_p^*-open sets U and V such that Λ_p^*-$cl(\{x\}) \subseteq U$ and Λ_p^*-$cl(\{y\}) \subseteq V$.

Conversely, suppose for each of points $x, y \in X$ such that Λ_p^*-$ker(\{x\}) \neq \Lambda_{p\,r}^*$-$ker(\{y\})$, there exist Λ_p^*-open sets U and V in X such that Λ_p^*-$cl(\{x\}) \subseteq U$, Λ_p^*-$cl(\{y\}) \subseteq V$ and $U \cap V = \emptyset$. Let $x, y \in X$ such that Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$. Then by Theorem 3.8 of of [6], Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$ and by hypothesis, there exist disjoint Λ_p^*-open sets U and V such that Λ_p^*-$cl(\{x\}) \subseteq U$ and Λ_p^*-$cl(\{y\}) \subseteq V$. This shows that X is a Λ_p^*-R_1 space.

Theorem 4.5. For a space X, the following are equivalent :

(*i*) X is a Λ_p^*-T_2 space.

(*ii*) X is both a Λ_p^*-R_1 space and Λ_p^*-T_1 space.

(*iii*) X is both a Λ_p^*-R_1 space and Λ_p^*-T_0 space.

Proof. (***i***) $\Rightarrow$ (***ii***). Suppose X is a Λ_p^* -T_2 space. Then by Definition 2.8 itself, X is Λ_p^* -T_1. If $x, y \in X$ with Λ_p^* -$cl(\{x\}) \neq \Lambda_p^*$ -$cl(\{y\})$, then by Theorem 2.6, $x \neq y$ and so by Definition 2.8, there exists Λ_p^* -open sets G and H such that $x \in G$, $y \in H$ and $G \cap H = Ø$. Then it follows that $\{x\} = \Lambda_p^*$ -$cl(\{x\}) \subseteq G$ and $\{y\} = \Lambda_p^*$ -$cl(\{y\}) \subseteq H$ and so by Definition 4.1, X is Λ_p^* -R_1.

(ii) $\Rightarrow$ ***(iii)***. Suppose X is both Λ_p^* -R_1 and Λ_p^* -T_1 spaces. Then by Definition 2.1 itself, X is Λ_p^* -T_0.

(iii) $\Rightarrow$ ***(i)***. Suppose X is both a Λ_p^* -R_1 space and Λ_p^* -T_0 space. Let $x, y \in X$ with $x \neq y$. By Theorem 2.2, Λ_p^* -$cl(\{x\}) \neq \Lambda_p^*$ -$cl(\{y\})$ and so by Definition 4.1, there exist disjoint Λ_p^* -open sets G and H such that Λ_p^* -$cl(\{x\}) \subseteq G$ and Λ_p^* -$cl(\{y\}) \subseteq H$ which implies that $x \in G$, $y \in H$ and $G \cap H = Ø$ and so X is Λ_p^* -T_2.

Theorem 4.6. A space X is Λ_p^* -R_1 if and only if for each points $x, y \in X$ such that Λ_p^* -$cl(\{x\}) \neq \Lambda_p^*$ -$cl(\{y\})$, there exists Λ_p^* -closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $x \notin F_2$, $y \in F_2$ and $X = F_1 \cup F_2$.

Proof. Suppose X is a Λ_p^* -R_1 space. Let $x, y \in X$ such that Λ_p^* -$cl(\{x\}) \neq \Lambda_p^*$ -$cl(\{y\})$. By Definition 4.1, there exist disjoint Λ_p^* -open sets U and V in X such that Λ_p^* -$cl(\{x\}) \subseteq U$ and Λ_p^* -$cl(\{y\}) \subseteq V$. Then $F_1 = X \backslash V$ and $F_2 = X \backslash U$ are Λ_p^* -closed sets such that $x \in F_1$, $y \notin F_1$, $x \notin F_2$, $y \in F_2$ and $X = F_1 \cup F_2$.

Conversely, let $x, y \in X$ such that Λ_p^* -$cl(\{x\}) \neq \Lambda_r$-$cl(\{y\})$. Then by hypothesis, there exists Λ_p^* -closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $x \notin F_2$, $y \in F_2$ and $X = F_1 \cup F_2$. Then $U = X \backslash F_2$ and $V = X \backslash F_1$ are Λ_p^* -open sets, $x \in U$, $y \in V$ and $U \cap V = Ø$. This shows that X is Λ_p^* -T_2 and so by Theorem 4.5, X is Λ_p^* -R_1.

The above discussions lead to the following implications but none of the reverse implications is true.

Diagram 4.7.

$$\begin{array}{ccccc} \Lambda_p^*\text{-}T_2 & \Rightarrow & \Lambda_p^*\text{-}T_1 & \Rightarrow & \Lambda_p^*\text{-}T_0 \\ \Downarrow & & \Downarrow & & \\ \Lambda_p^*\text{-}R_1 & \Rightarrow & \Lambda_p^*\text{-}R_0 & & \end{array}$$

Conclusion

Λ_p^*-sets and Λ_p^*-sets are used to introduce and investigate Λ_p^*-R_1 spaces, Λ_p^*-R_0 spaces, Λ_p^*-T_2 spaces, Λ_p^*-T_1 spaces and Λ_p^*-T_0 spaces. The further scope for research in this area is to characterize the existing concepts in topological spaces. For example the lower separation axioms namely Λ_s^*-T_i spaces, Λ_s^*-R_j spaces [7] for $i = 0, 1, 2$ and $j = 0, 1$ may be characterized by using Λ_p^*-T_i spaces, Λ_p^*-R_j spaces.

References

1. Caldas, M. and Dontchev, J.G., Λ_s-sets and V_s-sets, *Mem. Fac. Sci. Kochi Univ. Math.,* **21**, 21-30 (2002).
2. Caldas, M., Ganster, M., Jafari, S. and Noiri, T., On Λ_p–sets and functions, *Mem. Fac. Sci. Kochi Univ. Math.,* **25**, 1-8 (2004).
3. Caldas, M., Georgiou, D.N. and Jafari, S., Study of (Λ, α)-closed sets and the related notions in topological spaces, *Bull. Malays. Math. Sci. Soc.,* **(2)**, **30(1)**, 23-36 (2007).
4. Caldas, M. and Jafari, S., On some low separation axioms in topological space, *Houston Journal of Math*., **29**, 93-104 (2003).
5. Ganster, M. and Jafari, S., On *pre*-Λ-sets and *pre*-V-sets, *Acta Math. Hungar*, **95 (4)**, 337-343 (2002).
6. Gnanachandra, P., On *pre**-Λ-sets and *pre**-V-sets, *Acta Ciencia Indica.* (Preprint)
7. Gnanachandra, P., On *Semi**-Λ-sets and *Semi**-V-sets, *Ultra Scientist.* (Preprint)
8. Maki, H., Generalized Λ-sets and the associated closure operator, *The Special Issue in Commemoration of Prof. Kazusada IKEDA' Retirement,* **1(10)**, 139-146 (1986).
9. Cueva, Miguel Caldas and Dontchev, Julian, Λ_s-closure operator and the associated topology $\tau\, \Lambda_s$, *Journal of the Indian Math. Soc.,* **69 (1-4)**, 71-79 (2002).
10. Caldas, Miguel, Jafari, Saeid and Navalagi, Govindappa, More on λ-closed sets in topological spaces, *Revista Colombiana de Matemáticas,* **41(2)**, 355-369 (2007).
11. Caldas, Miguel and Jafari, Saeid, Generalized Λ_δ-sets and related topics, *Georgian Mathematical Journal,* **16(2)**, 247–256 (2009).
12. Murdeshwar, M.G. and Naimpally, S.A., R_1-topological spaces, *Canad. Math.Bull.,* **9**, 521–523 (1966).
13. Naimpally, S.A., On R_0-topological spaces, *Ann.Univ. Sci. Budapest Eötvös sect. Math.,* **10**, 53–54 (1967).
14. Navaneetha Krishnan, M., A study on ideal topological spaces, *Ph.D. Dissertation,* Manonmaniam Sundaranar University, Tirunelveli, India (2009).
15. Selvi, T. and Dharani, A. Punitha, Some new class of nearly closed and open sets, *Asian J. of Curr. Eng. and Maths,* **1(5)**, 305-307 (2012).
16. Shanin, N.A., On separation in topological spaces, *Dokl. Akad. Nauk SSSR,* **38**, 110–113 (1943).

❑