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ON PRE*-Λ-SETS AND PRE*-V-SETS

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In general topology, the arbitrary intersection of open sets is not open and the arbitrary union of closed sets is not closed. These properties motivated to introduce the concepts of Λ-sets and V-sets in topological spaces. In this paper we introduce the notions of a pre*- Λ -set and a pre*-V-set in a topological space. We study the fundamental properties of pre*-Λ-sets and pre*-V-sets and investigate the topologies defined by these families of sets. Also we introduce generalized pre*-Λ-sets and generalized pre*-V-sets and characterizes their properties.

KEYWORDS AND PHRASES: pre*-open, pre*-closed, pre*-Λ-set, pre*-V-set and $T_{1/2}$ space.

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INTRODUCTION AND PRELIMINARIES

In 1986, Maki [7] and Jin Han Park *et al*. [5] continued the work of Levine [6] and Dunham [3] on generalized closed sets and closure operators by introducing the notion of a generalized Λ-set in a topological space (X, τ) and by defining an associated closure operator, *i.e*. the Λ-closure operator. He studied the relationship between the given topology τ and the topology τ^{Λ} generated by the family of generalized Λ-sets. Das and Dontchev [2] built on Maki's work by introducing and studying so-called Λ_s-sets and V_s-sets, and also other forms called $g.\Lambda_s$-sets and $g.V_s$-sets. Maximilian Ganster *et al*. [4] studied the notions of a pre-Λ-set and a pre-V-set in a topological space.

The purpose of our paper is to continue research along these directions but this time by using pre*-open sets. We introduce pre*-Λ-sets and pre*-V-sets in a given topological space and thus obtain new topologies defined by these families of sets. We also consider some of the fundamental properties of these new topologies.

A subset S of a topological space (X, τ) is said to be *pre-open* [8] (resp. pre*-*open* [9]) if $S \subseteq int\,(cl\,(S)\,)$(resp. $S \subseteq (int^*(cl(S)\,)$, where $int(S)$, $int^*(S)$ and $cl(S)$ respectively denote the interior, g-interior and the closure of S. The complement of a pre-open set is pre-closed and that of pre*-open set is pre*-closed.

The intersection of all pre-closed (resp.pre*-closed) supersets of a subset S is called the pre-closure (resp. pre*-closure) of S and is denoted by pcl (S) (resp. $p^*cl(S)$). It is well known that a subset S is pre-closed (resp. pre*-closed) if and only if $cl(int\,(S)) \subseteq S$ (resp. $cl^*(int\,(S)) \subseteq S$). We shall denote the families of all pre open (resp. pre*-open, pre-closed and pre*closed) in a space (X, τ) by $PO\,(X, \tau)$ (resp. $P^*O\,(X, \tau)$, $PC\,(X, \tau)$ and $P^*C\,(X, \tau)$).

In the following X and Y (or (X, τ) and (Y, σ) will always denote topological spaces.

No separation axioms are assumed unless stated explicitly.

Definition 1.1. A subset S is a Λ-set (resp. a V-set) if and only if it is an intersection (resp. a union) of open (resp. closed) sets.

Definition 1.2. Let S be a subset of a space (X, τ). We define subsets $\Lambda p(S)$ and $Vp(S)$ as follows:

$$\Lambda_p(S) = \cap \{G : S \subseteq G, G \in PO(X, \tau)\} \text{and}$$

$$V_p(S) = \cup \{D : D \subseteq S, D \in PC(X, \tau)\}.$$

Recall that a space (X, τ) is said to be *pre*-*T_1 [10] if for each pair of distinct points x and y of X there exists a pre*-open set containing x but not y. Clearly a space (X, τ) is pre*-T_1 if and only if singletons are pre*-closed.

Recall that a subset A of a space (X, τ) is said to be generalized closed [6] (briefly g-closed), if cl $(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \tau$. A space (X, τ) is said to be a $T_{1/2}$ *space* if every g-closed subset of X is closed.

Recall that a space (X, τ) is called *resolvable* if it has two disjoint dense subsets and Alexandorff [1] if arbitrary intersection of open sets is open.

PRE*- Λ –SETS AND PRE*-V-SETS

Definition 2.1. Let S be a subset of a space (X, τ). We define subsets Λ_p^* (S) and V_p^* (S) as follows:

$$\Lambda_p^* (S) = \cap \{G : S \subseteq G, G \in P^*O(X, \tau)\}$$

and

$$V_p^* (S) = \cup \{D : D \subseteq S, D \in P^*C(X, \tau)\}.$$

In our first result, we summarize the fundamental properties of the sets Λ_p^* (S) and V_p^* (S).

Lemma 2.2. For subsets S, Q and Si, $I \in I$, of a space (X, τ) the following properties hold:

(1) $S \subseteq \Lambda_p^* (S)$,

(2) $Q \subseteq S$ implies that $\Lambda_p^* (Q) \subseteq \Lambda_p^* (S)$,

(3) $\Lambda_p^* (\Lambda_p^* (S)) = \Lambda_p^* (S)$,

(4) If $S \in P^*O(X, \tau)$, then $S = \Lambda_p^* (S)$,

(5) $\Lambda_p^* (\cup \{S_i : i \in I\}) = \cup \{\Lambda_p^* (S_i) : i \in I\}$,

(6) $\Lambda_p^* (\cap \{S_i : i \in I\}) \subseteq \cap \{\Lambda_p^* (S_i) : i \in I\}$,

(7) $\Lambda_p^* (X \setminus S) = X \setminus V_p^* (S)$.

Proof: (1) Let $x \notin \Lambda_p^*(S)$. Then there exists a pre*-open set G such that $S \subseteq G$ and $x \notin G$. Hence $x \notin S$ and so $S \subseteq \Lambda_p^*(S)$.

(2) Let $x \notin \Lambda_p^*(S)$. Then there exists a pre*-open set G such that $S \subseteq G$ and $x \notin G$. By our assumption $Q \subseteq S$, $Q \subseteq G$ and hence $x \notin \Lambda_p^*(Q)$. This shows (2).

(3) From (1) and (2), we have $\Lambda_p^*(S) \subseteq \Lambda_p^*(\Lambda_p^*(S))$. If $x \notin \Lambda_p^*(S)$, then there exists $G \in P^*O(X, \tau)$ such that $S \subseteq G$ and $x \notin G$. Hence $\Lambda_p^*(S) \subseteq G$, and so we have

$$x \notin \Lambda_p^*(\Lambda_p^*(S)). \text{ Thus } \Lambda_p^*(\Lambda_p^*(S)) = \Lambda_p^*(S).$$

(4) It directly follows from the definition of $\Lambda_p^*(S)$ and (1).

(5) Let $S = \cup\{S_i : i \in I\}$. By (2), we have that $\cup\{\Lambda_p^*(S_i) : i \in I\} \subseteq \Lambda_p^*(S)$. If $x \notin \cup\{\Lambda_p^*(S_i) : i \in I\}$, then, for each $i \in I$, there exists $G_i \in P^*O(X, \tau)$ such that $S_i \subseteq G_i$ and $x \notin G_i$. If $G = \cup\{G_i : i \in I\}$, then $G \in P^*O(X, \tau)$ with $S \subseteq G$ and $x \notin G$. Hence $x \notin \Lambda_p^*(S)$, and so (5) holds.

(6) From (2), $\Lambda_p^*(S) \subseteq \Lambda_p^*(S_i)$ for each $i \in I$ where $S = \bigcap_{i \in I} S_i$ and hence

$$\Lambda_p^*(S) = \Lambda_p^*\left(\bigcap_{i \in I} S_i\right) \subseteq \bigcap_{i \in I} \Lambda_p^*(S_i).$$

(7) Let $x \in \Lambda_p^*(X \backslash S)$. Then for every pre*-open set G containing $X \backslash S$, $x \in G$. Hence $x \notin X \backslash G$, for every pre*-closed set $X \backslash G \subseteq S$. Therefore $x \notin V_p^*(S)$ and hence $x \in X \backslash V_p^*(S)$. Similarly, $X \backslash V_p^*(S) \subseteq \Lambda_p^*(X \backslash S)$.

The following lemma is an immediate consequence of Lemma 2.2.

Lemma 2.3. For subsets S, Q and S_i, $i \in I$, of a space (X, τ) the following properties hold:

(1) $V_p^*(S) \subseteq S$,

(2) $Q \subseteq S$ implies that $V_p^*(Q) \subseteq V_p^*(S)$,

(3) $V_p^*(V_p^*(S)) = V_p^*(S)$,

(4) If $S \in P^*C(X, \tau)$ then $S = V_p^*(S)$,

(5) $V_p^*(\cap\{S_i : i \in I\}) = \cap\{V_p^*(S_i) : i \in I\}$,

(6) $\cup\{V_p^*(S_i) : i \in I\} \subseteq V_p^*(\cup\{S_i : i \in I\})$.

Definition 2.4. A subset S of a space (X, τ) is called a

(1) *pre**-Λ-set, briefly Λ_p^* -set if $S = \Lambda_p^* (S)$,

(2) *pre**-*V*-set, briefly V_p^* -set if $S = V_p^* (S)$.

Remark 2.5. Clearly, a subset S is a Λ_p^* -set (resp. a V_p^* -set) if and only if it is an intersection (resp. a union) of pre*-open (resp. pre*-closed) sets.

Hence Λ-sets, pre Λ-sets and pre*-open sets are Λ_p^* -set, and *V*-sets, pre-*V*-sets and pre*-closed sets are V_p^* -sets.

Observe also that a subset S is a Λ_p^* -set if and only if $X \backslash S$ is a V_p^* -set.

Proposition 2.6. For a space (X, τ) the following statements hold:

(1) ϕ and X are Λ_p^* -sets and V_p^* -sets.

(2) Every union of Λ_p^* -sets (resp. V_p^* -sets) is a Λ_p^* -set (resp. V_p^* -set).

(3) Every intersection of Λ_p^* -sets (resp. V_p^* -sets) is a Λ_p^* -set (resp. V_p^* -set).

Proof : (1) We shall only consider the case of Λ_p^* -sets. (1) is obvious.

(2) Let $\{S_i : i \in I\}$ be a family of Λ_p^* -sets in (X, τ). Then $\Lambda_p^* (S_i) = S_i$ for every $i \in I$.

If $S = \cup\{S_i : i \in I\}$, then by Lemma 2.2, we have $S = \cup \{\Lambda_p^* (S_i) : i \in I\} = \Lambda_p^* (S)$.

(3) Let $S_1 = \cap \{S_i : i \in I\}$, then by lemma 3.2(6), $\Lambda_p^* (S_1) = \Lambda_p^* (\cap \{S_i : i \in I\}) \subseteq \cap\{\Lambda_p^* (S_i) : i \in I\} = \cap\{S_i : i \in I\} = S_1$. Also by Lemma 3.2(1), $S \subseteq \Lambda_p^* (S_1)$. This proves (3).

Remark 2.7. Let $\tau_*^{\Lambda p}$ (resp. τ_*^{Vp}) denote the family of all Λ_p^* -sets (resp. V_p^* -sets) in (X, τ). Then $\tau_*^{\Lambda p}$ (resp. τ_*^{Vp}) is a topology on X containing all pre*-open (resp. pre*-closed) sets. Clearly, $(X, \tau_*^{\Lambda p})$) and (X, τ_*^{Vp}) and are Alexandroff spaces.

We now offer additional characterizations of pre*-T_1 spaces.

Theorem 2.8. For a space (X, τ) the following are equivalent:

(1) (X, τ) is pre*-T_1,

(2) Every subset of X is a Λ_p^* -set,

(3) Every subset of X is a V_p^* -set,

(4) Every open subset of X is a V_p^* -set.

Proof: Clearly (2) $\Leftrightarrow$ (3).

(1) $\Rightarrow$ (3): Let $A \subseteq X$. Since $A = \cup \{\{x\}: x \in A\}$, A is a union of pre*closed sets, hence a V_p^*-set.

(3) $\Rightarrow$ (4) : This is obvious.

(4) $\Rightarrow$ (1) : First observe that every singleton is open or pre*-closed. Let $x \in X$. If $\{x\}$ is open, then by assumption, $\{x\}$ is a V_p^*-set and so pre*-closed. Hence each singleton is pre*closed. That is $p^*cl(\{x\}) = \{x\}$. Therefore (X, τ) is pre*-T_1.

Lemma 2.9. Let (X, τ) be a topological space and $x \in X$. Then $y \in \Lambda_p^*(\{x\})$ if and only if $p^*cl(\{x\})$.

Proof: Suppose $y \in \Lambda_p^*(\{x\})$. Then for every pre*-open set $G \supseteq \{x\}$, $y \in G$.

If $x \notin p^*cl(\{y\})$, then there exists $H \in P^*C(X, \tau)$ such that $\{y\} \subseteq H$ and $x \notin H$. That implies $x \in X \backslash H$, $X \backslash H \in P^*O(X, \tau)$ and $y \notin X \backslash H$. Take $X \backslash H = G$. Then $G \in P^*O(X, \tau)$, $\{x\} \subseteq G$ and $y \notin G$. By this Contradiction we get, $x \in p^*cl(\{y\})$. Conversely, Suppose $x \in p^*cl(\{y\})$. Then for every pre*-closed set $G \supseteq \{y\}$, $x \in G$. If $y \notin \Lambda_p^*(\{x\})$, then there exists $H \in P^*O(X, \tau)$ such that $\{x\} \subseteq H$ and $y \notin H$. Take $X \backslash H = G$. Then $G \in P^*C(X, \tau)$, $y \in G$ and $x \notin G$. So there exists a pre*-closed set $G \supseteq \{y\}$ such that $x \notin G$. By this contradiction, we get $y \in \Lambda_p^*(\{x\})$.

Theorem 2.10. The following statements are equivalent for any points x and y in a topological space (X, τ)

(1) $\Lambda_p^*(\{x\}) \neq \Lambda_p^*(\{y\})$

(2) $p^*cl(\{x\}) \neq p^*cl(\{y\})$

Proof: (1) $\Rightarrow$ (2): Let $\Lambda_p^*(\{x\}) \neq \Lambda_p^*(\{y\})$. Then there exists $z \in X$ such that $z \in \Lambda_p^*(\{x\})$ and $z \notin \Lambda_p^*(\{y\})$ or $z \notin \Lambda_p^*(\{x\})$ and $z \in \Lambda_p^*(\{y\})$. If we consider the first case, we have $x \in p^*cl(\{z\})$, $y \notin p^*cl(\{z\})$ by using Lemma 2.9. Since $y \notin p^*cl(\{z\})$, there is a pre*-closed set H such that $z \in H$ and $y \notin H$. Since $x \in p^*cl(\{z\})$, we have $x \in H$. Also $y \notin H$ and $x \in H$ implies $y \notin p^*cl(\{x\})$. But $y \in p^*cl(\{y\})$, we get $p^*cl(\{x\}) \neq p^*cl(\{y\})$. The proof for the other case is similar.

(2) $\Rightarrow$ (1): Suppose $p^*cl(\{x\}) \neq p^*cl(\{y\})$. Then there exists $z \in X$ such that $z \in p^*cl(\{x\})$ and $z \notin p^*cl(\{y\})$ or $z \notin p^*cl(\{x\})$ and $z \in p^*cl(\{y\})$). Consider the case $z \in p^*cl(\{x\})$ and $z \notin p^*cl(\{y\})$. Then by Lemma 2.9, $x \in \Lambda_p^*(\{z\})$ and $y \notin \Lambda_p^*(\{z\})$. Since $y \notin \Lambda_p^*(\{z\})$, there is a pre*-open set G such that $z \in G$ and $y \notin G$. Since $x \in \Lambda_p^*(\{z\})$, $x \in G$. Now $y \notin G$ and $x \in G$ implies that $y \notin \Lambda_p^*(\{x\})$. Thus $\Lambda_p^*(\{y\}) \neq \Lambda_p^*(\{x\})$. The proof for the case $z \notin p^*cl(\{x\})$ and $z \in p^*cl(\{y\})$) is similar.

Lemma 2.11. Let (X, τ) be a topological space and $A \in P^*O(X, \tau)$.

Then $\Lambda_p^*(A) = \{x \in X : p^*cl(\{x\}) \cap A \neq \varnothing\}$.

Proof: Let $x \in \Lambda_p^*(A)$. Since $A \in P^*O(X, \tau)$, $A = \Lambda_p^*(A)$. Also $x \in p^*cl(\{x\})$ and hence $p^*cl(\{x\}) \cap A \neq \varnothing$. Conversely, let $x \in X$ such that $p^*cl(\{x\} \cap A \neq \varnothing$. If $x \notin \Lambda_p^*(A)$, then there exists $V \in P^*O(X, \tau)$ such that $A \subseteq V$ and $x \notin V$. Let $y \in p^*cl(\{x\} \cap A$. Then $y \in A$ and hence $y \in V$, since $A \subseteq V$. Since $y \in p^*cl(\{x\})$, $x \in \Lambda_p^*(\{y\})$. Therefore for every pre*-open set $G \supseteq \{y\}$ in (X, τ), $x \in G$. In particular, $x \in V$. This is a contradiction to $x \notin V$, so we get $x \in \Lambda_p^*(A)$.

Λ_p^*-CLOSED SETS AND ITS PROPERTIES

Definition 3.1. (1) Let A be a subset of a space (X, τ). Then A is called a Λ_p^*-closed set if $A = S \cap C$ where S is a Λ_p^*-set and C is a closed set.

(2) The complement of a Λ_p^*-closed set is called a Λ_p^*-open set.

(3) The collection of all Λ_p^*-open sets in (X, τ) is denoted by $\Lambda_p^* O(X, \tau)$.

The collection of all Λ_p^*-closed sets in (X, τ) is denoted by $\Lambda_p^* C(X, \tau)$.

(4) A point $x \in X$ is called a Λ_p^*-cluster point of A if for every Λ_p^*-open set U containing x, $A \cap U \neq \varnothing$.

(5) The set of all Λ_p^*-cluster points of A is called the Λ_p^*-cluster of A and is denoted by Λ_p^*-cl(A).

Proposition 3.2. (1) $A \subseteq \Lambda_p^*$-cl(A),

(2) $\Lambda_p^* - \text{cl}(A) = \cap \{F/A \subseteq F$ and F is Λ_p^*-closed$\}$,

(3) If $A \subseteq B$, then Λ_p^*-cl $(A) \subseteq \Lambda_p^*$-cl (B),

(4) A is Λ_p^*-closed if and only if $A = \Lambda_p^*$-cl(A),

(5) Λ_p^*-cl(A)= Λ_p^* -cl (Λ_p^*-cl (A)),

(6) Λ_p^*-cl(A) is Λ_p^*-closed.

Proof: (1) Let $x \notin \Lambda_p^*$-cl(A). Then x is not a Λ_p^*-cluster poin of A. So there exists a Λ_p^*-open set U containing x such that $A \cap U = \varnothing$ and hence $x \notin A$.

(2) Let $x \notin \Lambda_p^*$-cl(A). Then there exists a Λ_p^*-open set U containing x such that

$A \cap U = \varnothing$. Take $F = X \backslash U$. Then F is Λ_p^* -closed, $A \subseteq F$ and $x \notin F$ and hence $x \notin \cap \{F : A \subseteq F$ and F is Λ_p^* -closed$\}$. Similarly Λ_p^* -cl$(A) \subseteq \cap \{F : A \subseteq F$ and F is Λ_p^* - closed$\}$.

(3) Let $x \notin \Lambda_p^*$ -cl (B). Then there exists a Λ_p^* -open set U containing x such that $B \cap U = \varnothing$. Since $A \subseteq B$, $A \cap U \subseteq B \cap U = \varnothing$ and hence x is not a Λ_p^* -cluster point of A.

Therefore $x \notin \Lambda_p^*$ -cl(A).

(4) Suppose A is Λ_p^* -closed. Let $x \notin A$. Then $x \in X \backslash A$ and $X \backslash A$ is Λ_p^* -open. Take

$X \backslash A = U$. Then U is a Λ_p^* -open set containing x and $A \cap U = \varnothing$ and hence $x \notin \Lambda_p^*$ -cl(A). Therefore Λ_p^* -cl $(A) \subseteq A$. By using (1), we get $A = \Lambda_p^*$ -cl(A). Conversely, suppose $A = \Lambda_p^*$ -cl(A). Since $A = \cap \{F/A \subseteq F$ and F is Λ_p^* -closed$\}$, by using (2), we get A is Λ_p^* -closed.

(5) By using (2) and (4), we have Λ_p^* -cl $(A) \subseteq \Lambda_p^*$ -cl(Λ_p^* -cl(A)). Let $x \in \Lambda_p^*$ -cl (Λ_p^* -cl(A)). That implies x is a Λ_p^* -cluster point of Λ_p^* -cl(A).

That implies for every Λ_p^* -open set U containing x, Λ_p^* -cl $(A) \cap U \neq \varnothing$. Let $y \in \Lambda_p^*$ -cl$(A) \cap U$. Then y is a Λ_p^* -cluster point of A. Therefore for every Λ_p^* -open set G containing y, $A \cap G \neq \varnothing$. Since U is Λ_p^* -open and $y \in U$, $A \cap U \neq \varnothing$ and hence $x \in \Lambda_p^*$ -cl(A). Hence Λ_p^* -cl$(A) = \Lambda_p^*$ -cl(Λ_p^* -cl(A).

(6) Follows from (4) and (5).

Remark 3.3. (1) $\varnothing$ and X are both Λ_p^* -open and Λ_p^* -closed.

(2) By Proposition 3.2(6), Λ_p^* -cl(A) is the smallest Λ_p^* -closed set containing A.

Proposition 3.4 – Arbitrary intersection of Λ_p^* -closed sets is Λ_p^* -closed.

Proof: Let $A = \bigcap_{k \in I} A_k$ and $x \in \Lambda_p^*$ -cl(A). Then x is a Λ_p^* -cluster point of A. Hence for every Λ_p^* -open set U containing x, $A \cap U \neq \varnothing$. That implies $\bigcap_{k \in I} A_k \cap U \neq \varnothing$. This implies that $A_k \cap U \neq \varnothing$ for each $k \in I$. If $x \notin A$, then for some $i \in I$, $x \notin A_i$. Since A_i is Λ_p^* -closed,

$A_i = \Lambda_p^*$ -cl(A_i) and hence $x \notin \Lambda_p^*$ -cl(A_i). Therefore x is not a Λ_p^* -cluster point of A_i. So there exists a Λ_p^* -open set V containing x such that $A_i \cap V = \varnothing$. By this contradiction, $x \in A$.

Therefore Λ_p^* -cl(A) $\subseteq A$ and hence $A = \Lambda_p^*$ -cl(A). Therefore A is Λ_p^* -closed. That is $\bigcap_{k\in I} A_k$ is Λ_p^* -closed.

Proposition 3.5. Arbitrary Union of Λ_p^* -open sets is Λ_p^* -open.

Definition 3.6. Let (X, τ) be a topological space, and let $\underline{A} \subseteq X$. Then Λ_p^* -kernel of A is defined by Λ_p^* -ker(A) = $\cap$ {$G/G \in \Lambda_p^* O$ (X, τ) and $A \subseteq G$}.

Let (X, τ) be a topological space and A, B be subsets of X. Let x, $y \in X$. Then we have the following lemma.

Lemma 3.7. (1) $A \subseteq \Lambda_p^*$ -ker(A),

(2) If $A \subseteq B$, then Λ_p^* -ker(A) $\subseteq \Lambda_p^*$ -ker(A),

(3) Λ_p^* -ker(A)= Λ_p^* -ker (Λ_p^* -ker(A)),

(4) $y \in \Lambda_p^*$ -ker ({x}) if and only if $x \in \Lambda_p^*$ -cl({y}),

(5) Λ_p^* -ker(A) = {x : Λ_p^* -cl({x}) $\cap A \neq \varnothing$}.

Proof: (1) Let $x \notin \Lambda_p^*$ -ker(A). Then there exists $V \in \Lambda_p^* O$ (X, τ), such that $A \subseteq V$ and $x \notin V$ and hence $x \notin A$.

(2) Let $x \notin \Lambda_p^*$ -ker(B). Then there exists a $G \in \Lambda_p^* O$ (X, τ) such that $B \subseteq G$ and $x \notin G$. By our assumption $A \subseteq B$, $A \subseteq G$ and hence $x \notin \Lambda_p^*$ -ker(A).

(3) From (1) and (2), we have Λ_p^* -ker(A) $\subseteq \Lambda_p^*$ -ker(Λ_p^* -ker(A)).

Let $x \in \Lambda_p^*$ -ker(A). Then for every Λ_p^* -open set $G \supseteq \Lambda_p^*$ -ker(A), $x \in G$. Since $A \subseteq \Lambda_p^*$ -ker(A), for every Λ_p^* -open set $G \supseteq A$, $x \in G$. Hence $x \in \Lambda_p^*$ -ker(A). Therefore Λ_p^* -ker(Λ_p^* -ker(A) $\subseteq \Lambda_p^*$ -ker(A). Hence Λ_p^* -ker(A) = Λ_p^* -ker(Λ_p^* -ker(A)).

(4) It directly follows from the definition of Λ_p^* (S) and (1).

(5) Let $x \in \Lambda_p^*$ -ker(A). Then for every Λ_p^* -open set $G \supseteq A$, $x \in G$. Suppose Λ_p^* -cl({x}) $\cap A = \varnothing$. Then $A \subseteq X\backslash$(Λ_p^* -cl({x})). Then V is a Λ_p^* -open set containing A and $x \notin V$. By this contradiction, we get Λ_p^* -cl({x}) $\cap A \neq \varnothing$. Conversely, let $x \in X$ such that Λ_p^* -cl({x}) $\cap$

$A \neq \emptyset$. Let $y \in \Lambda_p^*$-cl$(\{x\}) \cap A$. Then y is a Λ_p^*-cluster point of $\{x\}$. Therefore for every Λ_p^*-open set U containing y, $U \cap \{x\} \neq \emptyset$ and hence $x \in U$.

If $x \notin \Lambda_p^*$-ker(A), then there exists a Λ_p^*-open set $V \supseteq A$ such that $x \notin$ cl$(\{x\}) \cap A \neq \emptyset$.

(S), and so (5) holds.

Theorem 3.8. For any points x and y in a space X, Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$ if and only if Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$.

Proof : Suppose Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$. Then there exists a point z in X such that $z \in \Lambda_p^*$-$ker(\{x\})$ and $z \notin \Lambda_p^*$-$ker(\{y\})$. By Lemma 3.7(4), $x \in \Lambda_p^*$-$cl(\{z\})$ and $y \notin \Lambda_p^*$-$cl(\{z\})$. By Proposition 3.2(3), Λ_p^*-$cl(\{x\}) \subseteq \Lambda_p^*$-$cl(\{z\})$ and $y \notin \Lambda_p^*$-$cl(\{z\})$ which implies that $y \notin \Lambda_p^*$-$cl(\{x\})$. This shows that Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$.

For the converse, suppose Λ_p^*-$cl(\{x\}) \neq \Lambda_p^*$-$cl(\{y\})$. Then there exists a point z in X such that $z \in \Lambda_p^*$-$cl(\{x\})$ and $z \notin \Lambda_p^*$-$cl(\{y\})$ which implies that by Definition 3.1(5), there exists a Λ_p^*-open set V containing z such that $x \in V$ and $y \notin V$. Thus V is a Λ_p^*-open set containing x but not y. If $y \in \Lambda_p^*$-$ker(\{x\})$, then By Lemma 3.7(4), $x \in \Lambda_p^*$-$cl(\{y\})$ and so for every Λ_p^*-open set G containing x, $G \cap \{y\} \neq \emptyset$, that is, $y \in G$, a contradiction. Hence $y \notin \Lambda_p^*$-$ker(\{x\})$ and hence Λ_p^*-$ker(\{x\}) \neq \Lambda_p^*$-$ker(\{y\})$.

Definition 3.9. Let X be a space and $x \in X$. Then we define a subset Λ_p^*-$\langle x \rangle$ of X as follows : Λ_p^*-$\langle x \rangle$ = Λ_p^*-$cl(\{x\}) \cap \Lambda_p^*$-$ker(\{x\})$.

Proposition 3.10. Let X be a space. Then the following properties hold:

(1) For each $x \in X$, Λ_p^*-ker (Λ_p^*-$\langle x \rangle$) = Λ_p^*-$ker(\{x\})$.

(2) For each $x \in X$, Λ_p^*-cl(Λ_p^*-$\langle x \rangle$) = Λ_p^*-$cl(\{x\})$.

(3) If U is a Λ_p^*-open set of X and $x \in U$, then Λ_p^*-$\langle x \rangle \subseteq U$.

(4) If F is a Λ_p^*-closed set of X and $x \in F$, then Λ_p^*-$\langle x \rangle \subseteq F$.

Proof : (1) Let $x \in X$. By Proposition 3.2(1) and Lemma 3.7(1), $\{x\} \subseteq \Lambda_p^*$-$cl(\{x\})$ and $\{x\} \subseteq \Lambda_p^*$-$ker(\{x\})$ and so by Definition 3.9, it follows that $\{x\} \subseteq \Lambda_p^*$-$\langle x \rangle$. By using Lemma 3.7(2), Λ_p^*-$ker(\{x\}) \subseteq \Lambda_p^*$-$ker$($\Lambda_p^*$-$\langle x \rangle$). For the reverse inclusion, if $y \notin \Lambda_p^*$-$ker(\{x\})$, then there exists a Λ_p^*-open set V such that $x \in V$ and $y \notin V$. It follows that $\Lambda_p^*\text{-}\langle x \rangle \subseteq \Lambda_p^*\text{-}ker(\{x\}) \subseteq \Lambda_p^*\text{-}ker(V) = V$ and so $\Lambda_p^*\text{-}ker(\Lambda_p^*\text{-}\langle x \rangle) \subseteq \Lambda_p^*\text{-}ker(V) = V$. Since $y \notin V$, $y \notin \Lambda_p^*\text{-}ker(\Lambda_p^*\text{-}\langle x \rangle)$.

Consequently, $\Lambda_p^*\text{-}ker(\Lambda_p^*\text{-}\langle x \rangle) \subseteq \Lambda_p^*\text{-}ker(\{x\})$.

(2) By Proposition 3.2(1) and Lemma 3.7(1), and Definition 3.9, we have $\{x\} \subseteq \Lambda_p^*\text{-}\langle x \rangle$. Then by Proposition 3.2(3), $\Lambda_p^*\text{-}cl(\{x\}) \subseteq \Lambda_p^*\text{-}cl(\Lambda_p^*\text{-}\langle x \rangle)$. On the other hand, by Definition 3.9, $\Lambda_p^*\text{-}\langle x \rangle \subseteq \Lambda_p^*\text{-}cl(\{x\})$ and so $\Lambda_p^*\text{-}cl(\Lambda_p^*\text{-}\langle x \rangle) \subseteq \Lambda_p^*\text{-}cl(\Lambda_p^*\text{-}cl(\{x\})) = \Lambda_p^*\text{-}cl(\{x\})$.

(3) Suppose U is a Λ_p^*-open set and $x \in U$. Then by Lemma 3.7(2), $\Lambda_p^*\text{-}ker(\{x\}) \subseteq \Lambda_p^*\text{-}ker(U) = U$ and so $\Lambda_p^*\text{-}\langle x \rangle \subseteq U$.

(4) Suppose F is Λ_p^*-closed and $x \in F$. Then $x \in \Lambda_p^*\text{-}cl(\{x\}) \subseteq F$. By Definition 3.9, we have $x \in \Lambda_p^*\text{-}\langle x \rangle$ and $\Lambda_p^*\text{-}\langle x \rangle \subseteq \Lambda_p^*\text{-}cl(\{x\})$ which implies that $\Lambda_p^*\text{-}\langle x \rangle \subseteq F$.

GENERALIZED PRE*-Λ-SETS

Following the lines of investigation of Maki in [11] one could now define generalized pre*-Λ-sets and generalized pre*-V-sets in the following way.

Definition 4.1. A subset S of a space (X, τ) is called

(i) a *generalized pre**-Λ-*set*, briefly g-Λp*-set, if $\Lambda_p^*(S) \subseteq P$ whenever $S \subseteq P$ and $P \in P^*C(X, \tau)$,

(ii) a *generalized pre**-V-*set*, briefly g-Vp*-set, if $V \subseteq V_p^*(S)$ whenever $V \subseteq S$ and $V \in P^*O(X, \tau)$.

Proposition 4.2. Let S be a subset of a space (X, τ).

(i) S is a generalized pre*-Λ-set if and only if S is a pre*-Λ-set,

(ii) S is a generalized pre*-V-set if and only if S is a pre*-V-set.

Proof: (i) Clearly, every pre*-Λ-set is a generalized pre*-Λ-set. Now let S be a generalized pre*-Λ-set. Suppose there exists $x \in \Lambda_p^*(S) \backslash S$. Observe that $\{x\}$ is open or pre*closed, and that $S \subseteq X \backslash \{x\}$. If $\{x\}$ is open, then $X \backslash \{x\}$ is closed, hence pre*closed, and so $\Lambda_p^*(S) \subseteq X \backslash \{x\}$, a contradiction to $x \in \Lambda_p^*(S)$. If $\{x\}$ is pre*closed, then $X \backslash \{x\}$ is pre*open and so $\Lambda_p^*(S) \subseteq X \backslash \{x\}$, a contradiction to $x \in \Lambda_p^*(S)$. Therefore $\Lambda_p^*(S) \backslash S = \phi$. And hence $S = \Lambda_p^*(S)$. Thus S is a pre*-Λ-set. (ii) This is proved in a similar way.

PROPERTIES OF PRE*-Λ-SETS AND PRE*-V-SETS

Proposition 5.1- Let (X, τ) be a space.

(1) $(X, \tau_*^{\Lambda p})$ and (X, τ_*^{Vp}) are always $T_{1/2}$ spaces,

(2) If (X, τ) is pre*-T_1, then both $(X, \tau_*^{\Lambda p})$ and (X, τ_*^{Vp}) are discrete spaces,

(3) The identity function $i : (X, \tau_*^{\Lambda p}) \to (X, \tau_*^{Vp})$ is continuous,

(4) The identity function $i : (X, \tau_*^{\Lambda p}) \to (X, \tau_*^{Vp})$ is contra-continuous.

Proof: (1) Let $x \in X$. Then $\{x\}$ is open or pre*closed in (X, τ). If $\{x\}$ is open, then it is pre*open, and hence $\{x\} \in \tau_*^{\Lambda p}$. If $\{x\}$ is pre*closed in (X, τ), then $X\backslash\{x\}$ is pre*open and so $X\backslash\{x\} \in \tau_*^{\Lambda p}$. That is $\{x\}$ is closed in $(X, \tau_*^{\Lambda p})$. Hence $(X, \tau_*^{\Lambda p})$ and (X, τ_*^{Vp}) are $T_{1/2}$ spaces.

(2) This follows from Theorem 3.8.

(3) and (4) are obvious.

Corollary 5.2. If (X, τ) is resolvable, then $(X, \tau_*^{\Lambda p})$ and (X, τ_*^{Vp}) are discrete.

Proof: We will show that $(X, \tau_*^{\Lambda p})$ is pre*-T_1. Let D and E be disjoint dense subsets of (X, τ), and let $x \in X$. Without loss of generality, $x \in D$. Then $X\backslash\{x\} = E \cup (D\backslash\{x\})$ is dense, hence pre*open and so $\{x\}$ is pre*closed.

Proposition 5.3. If $(X, \tau_*^{\Lambda p})$ is connected, then (X, τ) is pre*connected, *i.e.* X cannot be

Proof: Suppose that (X, τ) is not pre*connected. Hence there exist nonempty disjoint pre*open sets S, T in (X, τ) such that $S \cup T = X$. Since S and T are open in (X, τ_*^{Vp}), we have a contradiction.

Observe also that (X, τ_*^{Vp}) is connected if and only if (X, τ_*^{Vp}) is connected.

CONCLUSION

In this paper, we have introduced the concept of pre*-Λ-sets, pre*-V-sets, generalized pre*-Λ-sets, generalized pre*-V-sets, Λ_p^*-sets and V_p^*-sets and investigated some of their properties. Using these concepts, Further we characterize Λ_p^*-regular and Λ_p^*-normal spaces, Λ_p^*-homeomorphisms, Λ_p^*-connected and Λ_p^*-compact spaces.

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