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HEAT SOURCE EFFECT ON FLOW THROUGH POROUS MEDIUM INDUCED BY THE MOTION OF A PLATE MOVING WITH EXPONENTIALLY DECREASING VELOCITY

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The effect of heat source on MHD free convection and mass transfer flow through porous medium induced by the motion of a plate moving with exponentially decreasing velocity with respect to time with constant suction velocity and constant heat flux in the presence of a uniform magnetic field is analyzed. The effect of heat source parameter(S) with time on the velocity, temperature, concentration distribution and skin friction are discussed with the help of graphs.

KEYWORDS : Heat transfer, Mass transfer, heat source term, constant heat flux, normal magnetic field.

Introduction

The flow and temperature distribution through porous channels is of great importance in technological and biological flows such as motion of water waves over a shallow beach, the mechanics of cochlea in the human ear, food preservation, petroleum industry, cosmetic industry, polymer technology magneto hydrodynamic generators etc. MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. MHD flow has an application in metrology, solar physics and in motion of earth's core.

The effect of variable permeability on combined free and forced convection in porous media was studied by Chandrasekhara and Namboodiri [1]. Later on mixed convection in porous media adjacent to a vertical uniform heat flux surface was studied by Joshi and Gebhart [2]. Heat and mass transfer in a porous medium was discussed by Bejan and Khair [3]. The above problem was studied in presence of buoyancy effect by Trevisan and Bejan [4]. Lai and Kulacki [5] studied the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium.

The free convection effect on the flow of an ordinary viscous fluid past an infinite vertical porous plate with constant suction and constant heat flux was investigated by Sharma [6]. The study of two dimensional flow through porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of free convection current was studied by Sharma [7].

Convection in a porous medium with inclined temperature gradient was investigated by Nield [8]. The problem of mixed convection along an isothermal vertical plate in porous medium with injection and suction was studied by Hooper *et al* [9]. Acharya *et al* [10] have discussed magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux.

Varshney and Kaushlendra Kumar [11] have studied unsteady effect on MHD free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Jaypal Singh and C.B. Gupta [12] studied an unsteady effect on MHD free convection and mass transfer flow through porous medium induced by the motion of a plate moving with velocity decreasing exponentially with the time.

Ahmed S., [13] have studied the effects of unsteady free convective MHD flow through porous medium bounded by an infinite vertical porous plate. Hemanth Poonia and Chaudary, R.C., [14] analyzed the MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation. Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion have studied by Saxena *et al* [15]. Mukesh Chandra Shakya and Rajesh Johari [16] have discussed the effect of Hall current on MHD free convective flow of a viscoelastic dusty gas through a porous medium with heat source/sink. Thermo diffusion and mass transfer effects MHD flow of a dusty gas through porous medium have studied by Rajesh Kumar *et al* [17]. Free convection heat and mass transfer MHD flow in a vertical porous channel in the presence of chemical reaction was analysed by Barik *et al* [18]. The aim of this paper is to study the heat source effect on flow through porous medium induced by the motion of a plate moving with exponentially decreasing velocity.

Nomenclature

u	velocity component along the plate	V_0	steady suction velocity
v	velocity component normal to the plate	q	heat flux per unit area
t	time	m	mass flux per unit area
T	fluid temperature	g	acceleration due to gravity
C	fluid concentration	***Greek symbols***	
T_∞	temperature of the fluid in the free stream	β	coefficient of volume expansion
C_∞	concentration of the fluid in the free stream	β'	coefficient of concentration expansion
C_p	specific heat at constant pressure	σ	electrical conductivity
k	thermal conductivity	θ	nondimensional temperature
S	coefficient of heat source	ϕ	nondimensional concentration
K_0	porosity parameter	ν	kinematic viscosity
B_0	magnetic induction		

FORMULATION OF THE PROBLEM

Consider unsteady two-dimensional motion of viscous, incompressible, electrically conducting fluid through a porous medium occupying semi-infinite region of space bounded by a vertical infinite surface moving with velocity decreasing exponentially with time under the action of uniform magnetic field applied normal to the direction of flow. The effect of induced magnetic field is neglected. The magnetic Reynolds number is assumed to be small. Further, magnetic field is not strong enough to cause Joule heating. So, the term due to electrical dissipation is neglected in energy equation. Heat source term is introduced in the energy equation. The x-axis is taken along the surface in the upward direction and y-axis is taken normal to it. The fluid properties are assumed constant except for the influence of density in the body force term. As the boundary surface is infinite in length, all the variables are functions of y. Hence, by the usual boundary layer approximation, the basic equations for unsteady flow through porous medium are:

$$\frac{dv}{dy} = 0 \qquad \dots (1)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta'(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{K_0}u \qquad \dots (2)$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{K}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{S}{\rho C_P}(T - T_\infty) \qquad \dots (3)$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \qquad \dots (4)$$

with boundary conditions

$$u = -v\,(1 + \in e^{-nt}), \qquad -k\left(\frac{\partial T}{\partial y}\right) = q, \quad -D\left(\frac{\partial C}{\partial y}\right) = m \qquad \text{at } y = 0,\ t \geq 0$$

$$u \to 0, \qquad T \to T_\infty, \qquad C \to C_\infty \qquad \text{as } y \to \infty,\ t > 0 \qquad \dots (5)$$

METHOD OF SOLUTION

The equation of continuity (1) gives

$$v = \text{constant} = -V_0 \qquad \dots (6)$$

where $V_0 > 0$ corresponds to steady suction velocity at the surface. With the use of equation (5) now the equations (2), (3) and (4) are

$$\frac{\partial u}{\partial t} - V_0\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta'(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{K_0}u \qquad \dots (7)$$

$$\frac{\partial T}{\partial t} - V_0\frac{\partial T}{\partial y} = \frac{K}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{S}{\rho C_p}(T - T_\infty) \qquad \dots (8)$$

$$\frac{\partial C}{\partial t} - V_0\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \qquad \dots (9)$$

The boundary conditions of the problem are

$$u = V_0(1 + \in e^{-nt}), \qquad \frac{dT}{dy} = -\frac{q}{K}, \quad \frac{dC}{dy} = -\frac{m}{D} \qquad \text{at } y = 0, t \geq 0$$

$$u \to 0, \qquad T \to T_\infty, \qquad C \to C_\infty \qquad \text{as } y \to \infty, t > 0 \qquad \ldots(10)$$

By introducing the non-dimensional quantities

$$f(\eta) = \frac{u}{V_0} \qquad \eta = \frac{V_0 y}{\nu} \qquad \theta = \frac{(T - T_\infty)V_0 K}{q\nu} \qquad \phi = \frac{(C - C_\infty)V_0 D}{m\nu}$$

$$\Pr = \frac{\mu C_P}{\lambda} \qquad \text{Sc} = \frac{\nu}{D} \qquad \alpha = \frac{V_0^2 K_0}{\nu^2} \qquad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}$$

$$t^* = \frac{V_0^2}{\nu} t \qquad S^* = \frac{S\nu^2}{KV_0^2} \qquad \text{Gr} = \frac{g\beta q \nu^2}{V_0^4 K} \qquad \text{Gm} = \frac{g\beta' m \nu^2}{V_0^4 D}$$

equations (7), (8) and (9) reduced to

$$-\frac{\partial f}{\partial t} + \frac{\partial^2 f}{\partial \eta^2} + \frac{\partial f}{\partial \eta} - f(\alpha^{-1} + M) = -Gr\,\theta - Gm\,\phi \qquad \ldots(11)$$

$$-\Pr\frac{\partial \theta}{\partial t} + \frac{\partial^2 \theta}{\partial \eta^2} + \Pr\frac{\partial \theta}{\partial \eta} + S\,\theta = 0 \qquad \ldots(12)$$

$$-\text{Sc}\frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial \eta^2} + \text{Sc}\frac{\partial \phi}{\partial \eta} = 0 \qquad \ldots(13)$$

The corresponding boundary conditions become

$$f = 1 + \in e^{-nt} \qquad \theta' = -1, \qquad \phi' = -1 \qquad \text{at } \eta = 0, t \geq 0$$

$$f \to 0, \qquad \theta \to 0, \qquad \phi \to 0 \qquad \text{as } \eta \to \infty, t > 0 \qquad \ldots(14)$$

Assume that the solutions are of the form

$$f(\eta, t) = f_0(\eta) + \in f_1(\eta)\, e^{-nt}$$

$$\theta(\eta, t) = \theta_0(\eta) + \in \theta_1(\eta)\, e^{-nt}$$

$$\phi(\eta, t) = \phi_0(\eta) + \in \phi_1(\eta)\, e^{-nt} \qquad \ldots(15)$$

Using equation (15) into equations (11), (12) and (13) we get

$$f_0'' + f_0' - f_0(\alpha^{-1} + M) = -Gr\,\theta_0 - Gm\,\phi_0 \qquad \ldots(16)$$

$$f_1'' + f_1' - f_1(\alpha^{-1} + M - n) = -\text{Gr}\,\theta_1 - \text{Gm}\,\phi_1 \qquad \ldots(17)$$

$$\theta_0'' + \Pr\theta_0' + S\theta_0 = 0 \qquad \ldots(18)$$

$$\theta_1'' + \Pr\theta_1' + (S + n\Pr)\,\theta_1 = 0 \qquad \ldots(19)$$

$$\phi_0'' + Sc\,\phi_0' = 0 \qquad \ldots(20)$$

$$\phi_1'' + \text{Sc}\,\phi_1' + n\,\text{Sc}\,\phi_1 = 0 \qquad \ldots(21)$$

with corresponding boundary conditions

$$f_0 = 1, \quad \theta_0' = -1, \quad \phi_0' = -1 \quad \text{at } \eta = 0, t \geq 0$$

$$f_0 \to 0, \quad \theta_0 \to 0, \quad \phi_0 \to 0 \quad \text{as } \eta \to \infty, t > 0$$

$$f_1 = 1, \quad \theta_1' = 0, \quad \phi_1' = 0 \quad \text{at } \eta = 0, t \geq 0$$

$$f_1 \to 0, \quad \theta_1 \to 0, \quad \phi_1 \to 0 \quad \text{as } \eta \to \infty, t > 0 \qquad \ldots (22)$$

Solving equations (16) - (21) with boundary conditions (22) we get

$$f_0 = (1 + GrA_2 + GmA_3)\, e^{-A_1\eta} - A_2 Gr\, e^{-K_1\eta} - A_3 Gm\; e^{-Sc\eta} \qquad \ldots (23)$$

$$f_1 = e^{-A_4\eta} \qquad \ldots (24)$$

$$\theta_0 = \frac{1}{K_1}\; e^{-K_1\eta} \qquad \ldots (25)$$

$$\theta_1 = 0 \qquad \ldots (26)$$

$$\phi_0 = \frac{1}{Sc}\; e^{-Sc\eta} \qquad \ldots (27)$$

$$\phi_1 = 0 \qquad \ldots (28)$$

where $A_1 = \frac{1}{2}[1 + \{1 + 4(\alpha^{-1} + M)\}^{1/2}]$

$$A_2 = \frac{1}{K_1[K_1^2 - K_1 - (\alpha^{-1} + M)]}$$

$$A_3 = \frac{1}{Sc[Sc^2 - Sc - (\alpha^{-1} + M)]}$$

$$A_4 = \frac{1}{2}[1 + \{1 + 4(\alpha^{-1} + M - n)\}^{1/2}]$$

$$K_1 = \frac{1}{2}[\Pr + (\Pr^2 - 4S)^{-1/2}]$$

Hence, the equations for f, θ and ϕ will be

$$f(\eta, t) = (1 + GrA_2 + GmA_3)\, e^{-A_1\eta} - GrA_2\, e^{-K_1\eta} - GmA_3\, e^{-Sc\eta} + \in e^{-A_4\eta}\; e^{-nt} \qquad \ldots (29)$$

$$\theta(\eta, t) = \frac{1}{K_1}\; e^{-K_1\eta} \qquad \ldots (30)$$

$$\phi(\eta, t) = \frac{1}{Sc}\, e^{-Sc\,\eta} \qquad \ldots (31)$$

The skin friction coefficient at the surface is given by

$$\tau = \left(\frac{\tau_{xy}}{\rho V_0^2} \right)_{\eta=0} \quad \ldots (32)$$

$$\tau = -A_1 + Gr\, A_2 (K_1 - A_1) + Gm\, A_3 (Sc - A_1) - \in A_4\, e^{-nt} \quad \ldots (33)$$

RESULTS AND DISCUSSION

The effect of heat source on unsteady viscous, incompressible, electrically conducting fluid through a porous medium occupying a semi-infinite region of space bounded by a vertical surface moving with velocity decreasing exponentially with time have been studied. The values are chosen to be $Gr = 5.0$, $Gm = 2.0$, $M = 1.0$, $\alpha = 1.0$, $\varepsilon = 0.1$, $Pr = 2.5$, $Sc = 0.22$ and $n = 1.0$. Fluid velocity profile is plotted in Figs. 1 and 2. Fig 1 shows the variations of S with the increase in η. It is observed that increase in velocity accompanies a rise in S at $t = 0$.

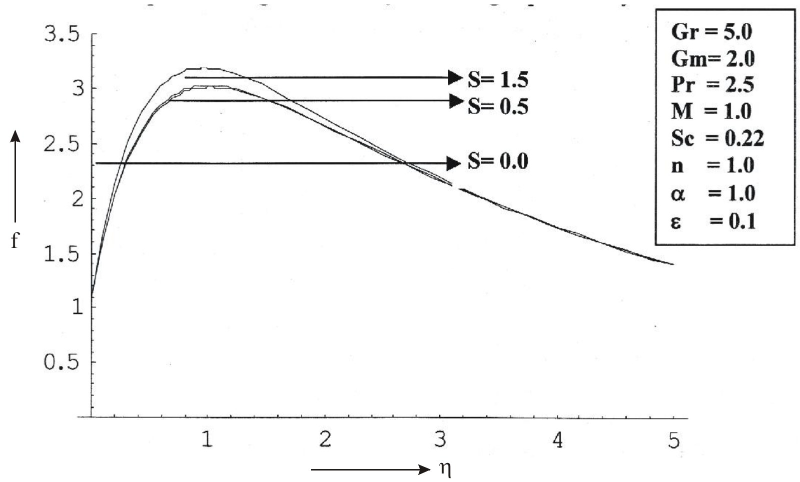

Fig. 1. Velocity profile for various *S* (*t* = 0).

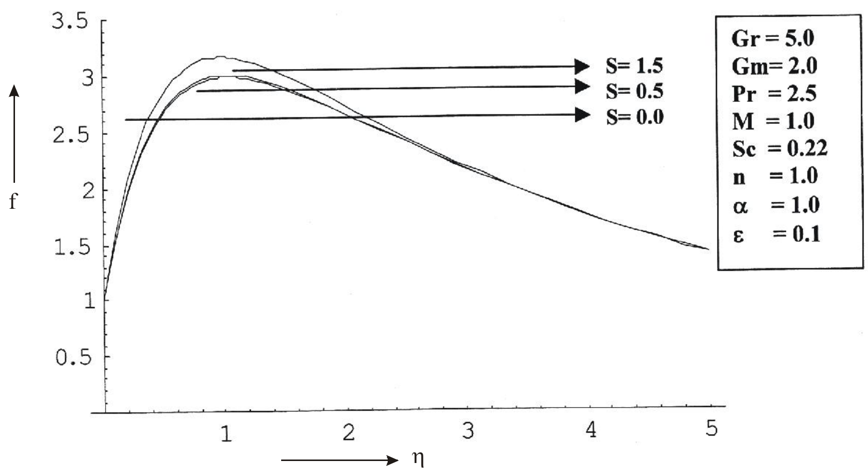

Fig. 2. Velocity profile for various *S* (*t* = 0)

Fig. 2 also shows the variations of S with the increase in η at $t = 6$. It is seen that, velocity increases with the increase in *S*. Fig 3 depicts the effect of *S* on temperature versus η. It is observed that the temperature increases with the increase in *S*.

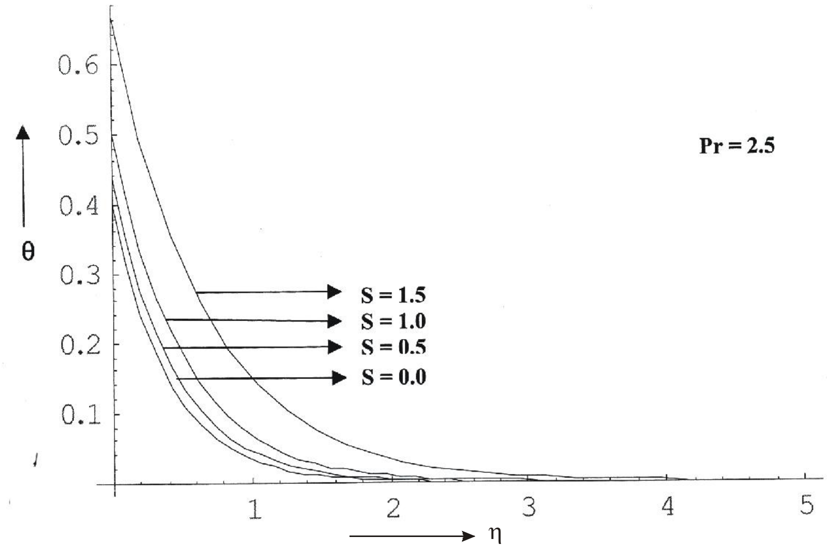

Fig. 3. Temperature profile for various *S*.

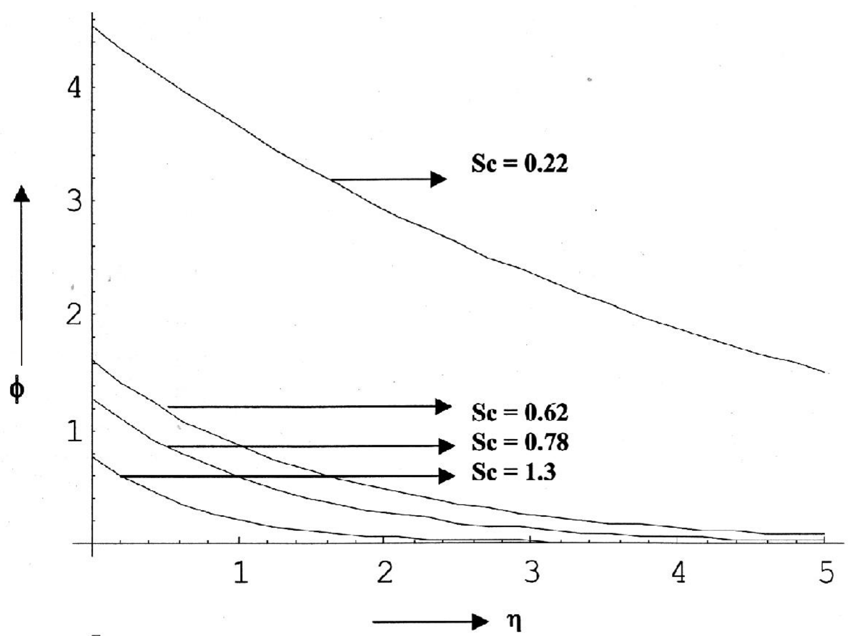

Fig. 4. Concentration profile for various Sc.

Fig. 4 shows the variations in concentration with η for the effect of *Sc*. As *Sc* increases, the concentration is lowered.

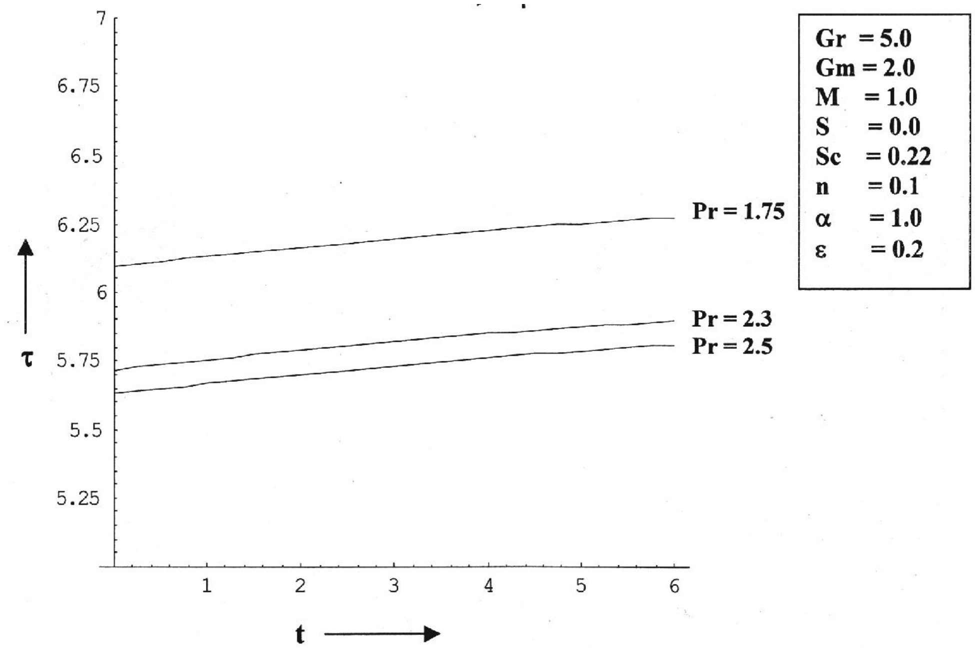

Fig. 5. Skin frictionprofile for various Pr

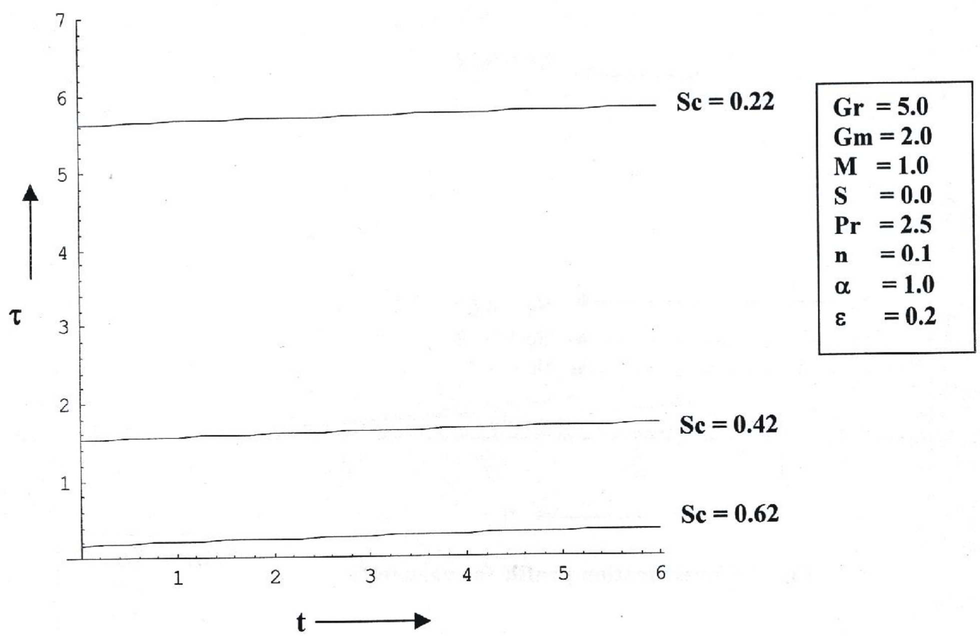

Fig. 6. Skin friction profile for various Sc.

By perusing Fig. 5 and Fig. 6 it is clear that when the parameter $S = 0.0$ it coincides with Jaypal Singh and C.B. Gupta. Fig. 7 depicts the variations in skin friction with time for different values of *S*. It shows that, as S increases, skin friction also increases.

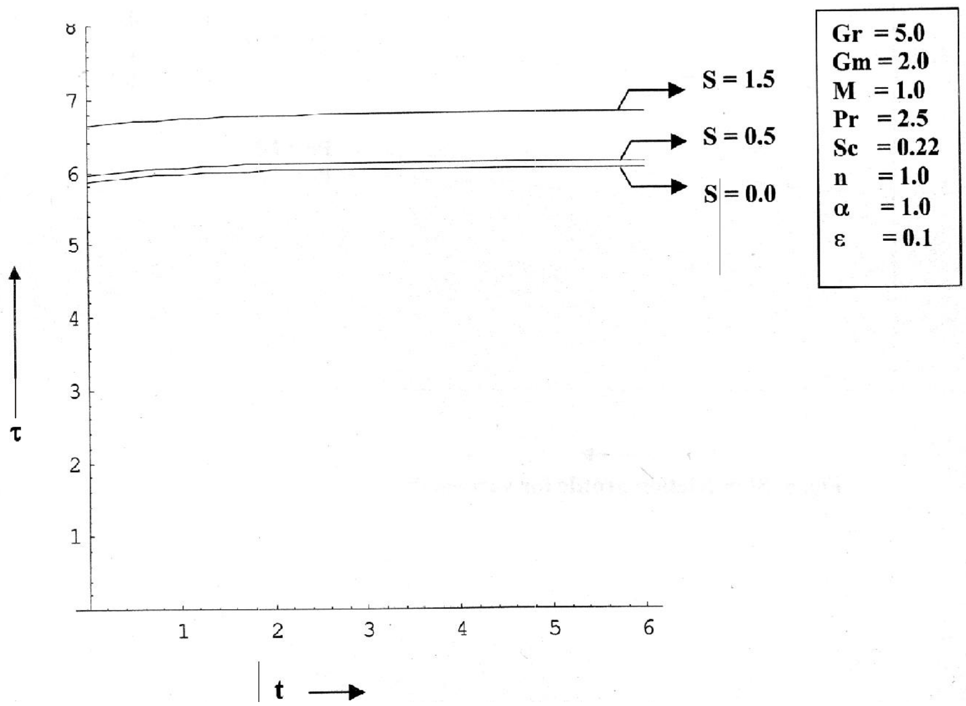

Fig. 7. Skin friction profile for varius *S*

Conclusion

The problem considered in this work is to study the effect of heat source on MHD flow through porous medium induced by the motion of a plate moving with exponentially decreasing velocity. From the results and discussion, the following conclusions can be drawn.

(i) The dimensionless velocity decreases due to the decrease of heat source parameter when $t = 0$ and $t = 6$.

(ii) An increase in heat source parameter results in increasing the temperature distribution.

(iii) Increase in Schmidt number results in decrease of the concentration profile.

(iv) Skin friction increases gradually with the increase in time.

(v) An increase in heat source parameter results in increasing the skin friction.

Current work in the absence of heat source term coincides with Jaypal Singh and C.B. Gupta [12]. Further, future works can also be done by discussing the dissipation effects, Hall effects, Dufour and Soret effect, effect of chemical reaction, an isothermal vertical plate in porous medium with injection and suction etc on MHD free convection and mass transfer flow through porous medium induced by the motion of the plate moving with velocity decreasing exponentially with time.

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