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AN UNSTEADY RADIAL FLOW OF A VISCOUS INCOMPRESSIBLE FLUID IN A POROUS MEDIUM AROUND A RADIALLY OSCILLATING SPHERICAL SURFACE

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This paper deals with an unsteady radial flow of a viscous incompressible fluid in a porous medium around a radially oscillating time dependent spherical surface. The momentum equation considered for the flow through a porous medium takes care of the fluid- inertia and the Newtonian stresses in addition to the classical Darcy's friction. Expression for the pressure distribution has been derived in terms of the expansion rate of the sphere-radius. Two special cases:

$$\text{(A) } r = \cos \alpha t \qquad \text{(B) } r = \frac{1}{(1+\varepsilon)}(1+\varepsilon \cos \alpha t)$$

are discussed in detail In this '*r*' is the radius of the surface of the sphere at time '*t*' and 'α' is the sphere radius oscillation parameter. The variation of the pressure for different values of the flow parameter 'α' and the Darcy number characteristic of the medium position at different instants of time in each of the cases has been discussed and illustrated.

KEYWORDS : Pressure, Darcy's number, Porous Medium, Expansion factor, Radius decay- parameter.

INTRODUCTION

Studies on radial flows of a viscous fluid were initiated in the year 1915 by Jeffery G.B. [5] and these were followed later by Hamael G. [4] and Harrison W. J. Such flows are discussed at length by Dryden H.L., Murnaghan F.D. and Batemen H. [3] in their classical work on Hydrodynamics. Recently Raisinghania M.D. [9] in his treatise on Fluid Dynamics discussed several types of radial flows of viscous fluids in a clear medium.

Flows through porous media have been a subject of considerable research activity for over the last one and half centuries, because of their wide range of application in diverse fields of science, engineering and technology. Studies in this area were initiated in 1856 by Darcy H [2] based on a series of experiments on flows of slurry fluids through channels. Darcy formulated an empirical law for fluid flows through porous media : The total volume of the fluid percolating in unit time is proportional to the hydraulic head and inversely proportional to the distance between the inlet and outlet .This law was later generalized by Brinkman H.C. [1] by , taking in to account for the stresses generated in the flow region. Later, Yamamoto K. and Yoshida Z. [10] further generalized the basic equations by the inclusion fluid inertia in addition to the Newtonian-Stresses developed in fluids in motion. Later Pattabhi Ramacharyulu N.Ch. [8] examined several flow problems through straight tubes of diverse cross sections. A general solution for an incompressible flow through porous media has been obtained by Narasinhacharyulu V. and Pattabhi Ramachryulu N. Ch. [6]. Recently the present authors [7] investigated an unsteady radial flow of a viscous incompressible fluid through a porous medium around a sphere whose radius (r) exponentially decreases with time. The present investigation is on the unsteady radial flow of viscous incompressible flow through a porous medium around a sphere whose surface is oscillating. A generalized momentum equation given by Yamamoto K. and Yoshida Z. [10] for the flows through porous medium has been solved for the radial flow. It is noticed that the flow is independent of a Newtonian viscous stresses. However the flow depends on Darcian friction. Expression for the pressure distribution has been obtained in terms of the radial velocity on the sphere- surface. The cases of the sphere radius at time are

(A) $r = \cos\alpha t$ and (B) $r = \frac{1}{(1+\varepsilon)}(1+\varepsilon\cos\alpha t)$ in the non dimensional form

MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Consider a spherical co-ordinate system R, θ, ϕ with a origin 'O' fixed at the center of the sphere. R is the radial distance from the origin, 'θ' the polar angle, 'ϕ' the azimuthal angle. The flow of a viscous incompressible fluid through a porous medium is governed by the modified Navier-Stokes equations suggested by Yamamoto K. and Yoshida Z. [10]:

$$\rho\frac{d\vec{q}}{dt} = -\nabla p + \mu\nabla^2\vec{q} - \frac{\mu}{k}\vec{q} \qquad \dots (1)$$

together with the equation of continuity

$$\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = 0 \qquad \dots (2)$$

Here '$\vec{q}$' represents the fluid velocity and 'p' is the fluid pressure. Further ρ is the fluid mass density, μ is the coefficient of Newtonian of viscosity and 'k' is the coefficient of Darcian porosity of the medium and all these coefficients are assumed to be constants. The term $\mu\nabla^2\vec{q}$ on the R.H.S. of (1) represents the contribution of the Newtonian Viscous-Stress and $-\frac{\mu}{k}\vec{q}$ is the classical Darcy-resistance to the flow.

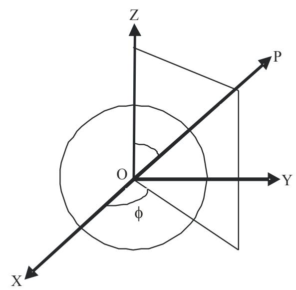

Fig. 1: Flow- Sketch

By radial and axi-symmetries

$$\frac{\partial \vec{q}}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial \vec{q}}{\partial \phi} = 0 \qquad \dots (3)$$

For the unsteady radial flow under investigation, the velocity field can be taken as

$$\vec{q} = U((R, T), 0, 0) \qquad \dots (4)$$

The continuity equation (2) now reduced to

$$\frac{1}{R^2}\frac{\partial (R^2 U)}{\partial R} = 0 \qquad \dots (5)$$

Momentum equation in the Radial-direction (R)

$$\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial R} = -\frac{1}{\rho}\frac{\partial P}{\partial R} - \left(\frac{\nu}{k}\right)U \qquad \dots (6)$$

where $\nu = \frac{\mu}{\rho}$

Momentum equation in the θ and φ-direction are

$$\frac{\partial P}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial P}{\partial \phi} = 0 \qquad \dots (7)$$

It can be noted from the equation (7) that

(i) The pressure (P) is a function of R and T only. (*i.e.* independent of θ and φ)

(ii) It is also independent of Newtonian viscous stresses and

(iii) The Newtonian viscous resistance on the porous media $\left(-\frac{\mu}{k}\vec{q}\right)$ influences the pressure distribution.

For simplicity the following non dimensional quantities are introduced in the foregoing analysis

$$R = R_0\, r;\ U = \frac{\mu u}{\rho R_0};\ T = \frac{\rho R_0{}^2 t}{\mu};\ P = \frac{\mu^2 p}{\rho R_0{}^2};\ D = \frac{R_0{}^2}{k} \qquad \dots (8)$$

also where 'D' is the Darcy porosity coefficient and where R_0 is the initial radius of the sphere. By definition, the radial velocity 'u' on the sphere of the surface is given by

$$u = \frac{dr}{dt} \quad \text{on the sphere-surface} \qquad \ldots (9)$$

The following are the basic equations in the non-dimensional form

Continuity Equation

$$\frac{1}{r^2}\frac{\partial(r^2 u)}{\partial r} = 0 \qquad \ldots (10)$$

Momentum equation in the radial direction

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} - Du \qquad \ldots (11)$$

From (10), we get

$$u = \frac{f(t)}{r^2} \qquad \ldots (12)$$

It follows from (9) and (12) that

$$f(t) = r^2 u \ \text{ on } \ r = 1 \qquad \ldots (13)$$

and from (11) and (12) we get the equation for the determination of the fluid pressure (P):

$$-\frac{\partial p}{\partial r} = \frac{f^1(t)}{r^2} - 2\frac{(f(t))^2}{r^5} + D\frac{f(t)}{r^2} \qquad \ldots (14)$$

this on integrating with respect to 'r' yields

$$p_\infty - p = -\frac{1}{r^2}\left(\frac{df}{dt}\right) + \frac{1}{2r^4}\frac{d^2 f}{dt^2} - \frac{D}{r}(f(t)) \qquad \ldots (15)$$

where p_∞ in the pressure at infinity *i.e.* $\lim_{r\to\infty} p = p_\infty$ this computed on the sphere surface given

$$P = r\frac{d^2 r}{dt^2} + \left(\frac{3}{2}\right)\left(\frac{dr}{dt}\right)^2 + Dr\left(\frac{dr}{dt}\right) \quad \text{where } \ P = p - p_\infty \qquad \ldots (16)$$

computed on the surface of the sphere

Case-A: $r = \cos\alpha t$ where α the sphere radius oscillation parameter is a constant in this case we get

$$f(t) = (\cos\alpha t)^2 \qquad \ldots (A.1)$$

Also

$$P = (\alpha^2 - (5\alpha^2 \cos 2\alpha t + 2D\alpha \sin 2\alpha t))/4 \qquad \ldots (A.2)$$

$$= (\alpha^2 - R\cos\,(2\alpha t - \phi))/4$$

with $$R = \sqrt{25\alpha^2 + 4D^2} \text{ and } \varphi = \tan^{-1}\left(\frac{2D}{5\alpha}\right) \qquad \ldots \text{(A.3)}$$

The maximum and minimum pressures on the sphere surface are

$$p_{\max} = \left(\frac{\alpha}{4}\right)(\alpha + \sqrt{25 + 4D^2}) \quad \text{and} \quad p_{\min} = \left(\frac{\alpha}{4}\right)(\alpha - \sqrt{25 + 4D^2}) \qquad \ldots \text{(A.4)}$$

$P_{\max}$ is always positive and $P_{\min}$ is positive whenever $\alpha^2 \succ 25 + 4D^2$.

Results and discussions

It is noticed from the figures (A.1)-(A.8) the variation of the pressure on the spherical surface verses Radius-oscillation parameter (α) for different values of porosity parameter (D), the number of oscillation beats increases as Radius-oscillation parameter increases for a specific range of Radius-oscillation parameter.

The variation of the pressure verses Darcy number (D). It is illustrated in figures (A.9-A.12) and it is noticed that as the Darcy number (D) increases for different values of Radius-oscillation parameter ($\propto$) the pressure gradually decreases this may be associated due to the increase in internal resistance in the porous medium.

Also it is noticed from figures (A.13)-(A.16) that the variation of the pressure verses time (t) on the spherical surface as time increasing for different values of porosity parameter (D) the chaotic oscillations are noted in each of the cases stated above.

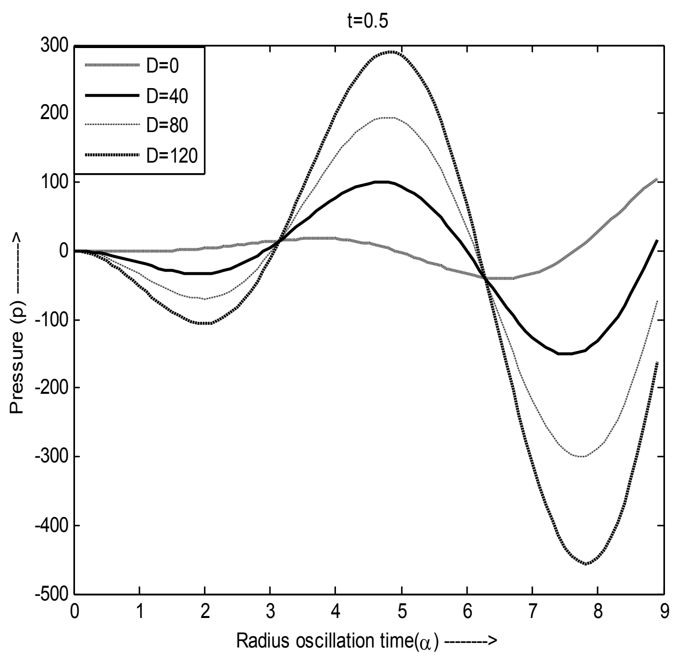

Fig. (A.1). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 0.5

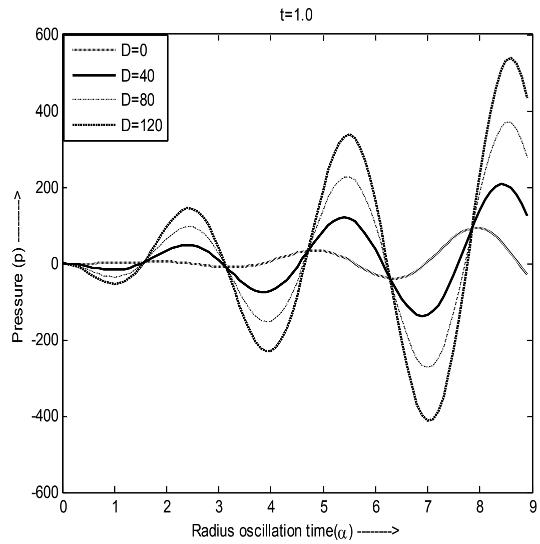

Fig. (A.2). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 1.0

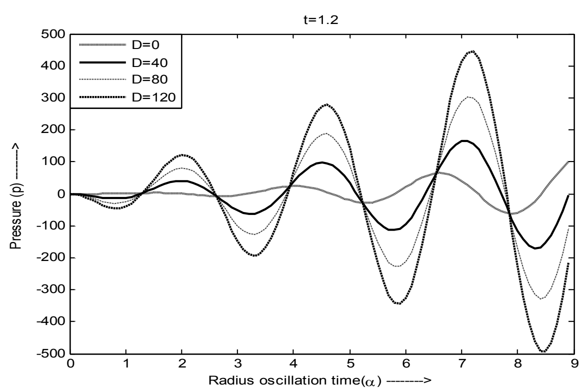

Fig. (A.3). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 1.2

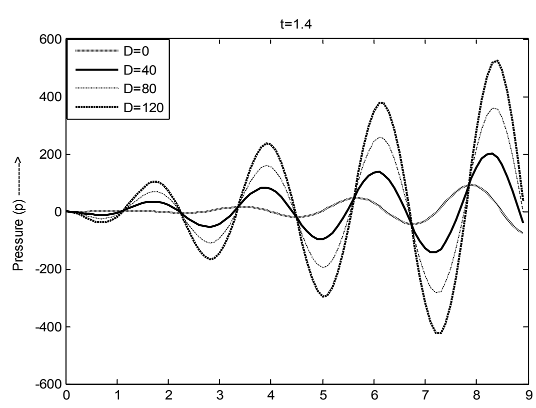

Fig. (A.4). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 1.4

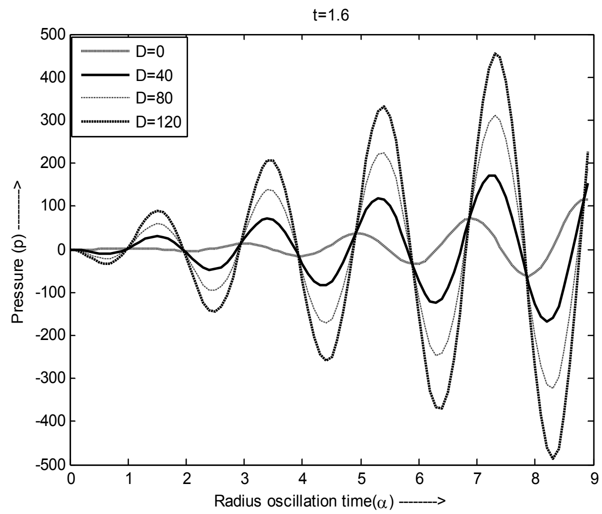

Fig. (A.5). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 1.6

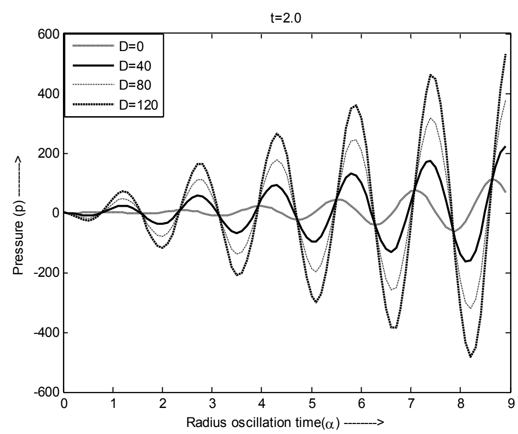

Fig. (A.6). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 2.0

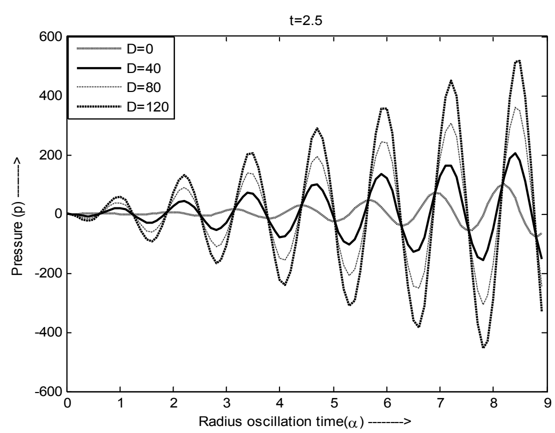

Fig. (A.7). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 2.5

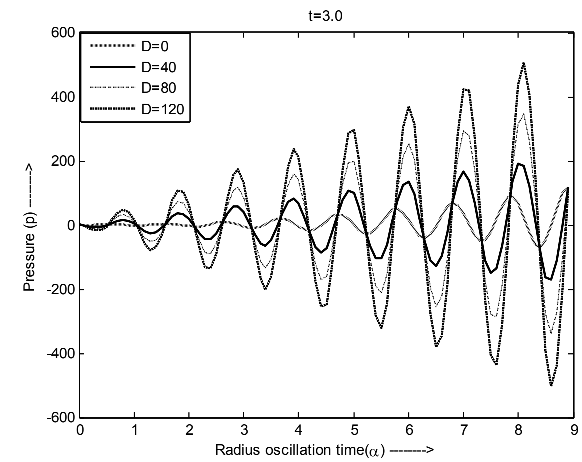

Fig. (A.8). Variation of the Pressure VS Radius oscillation time for different porosity at time *t* = 3.0

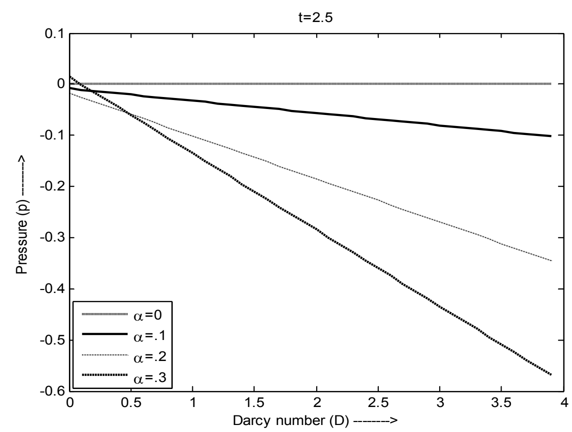

Fig. (A.9). Variation of the Pressure VS Darcy number for different alpha at time *t* = 2.5

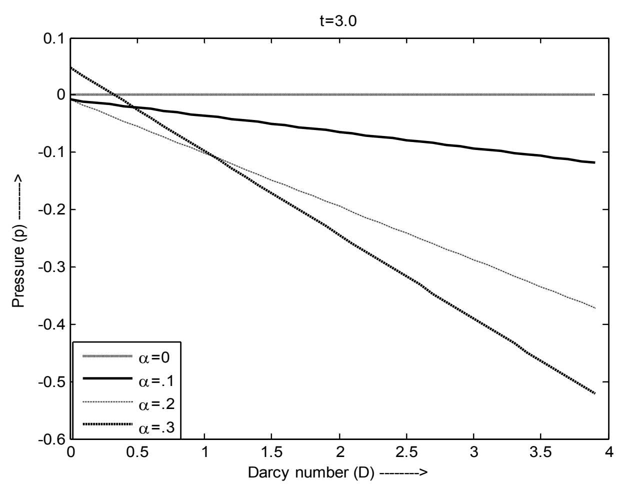

Fig. (A.10). Variation of the Pressure VS Darcy number for different alpha at time *t* = 3.0

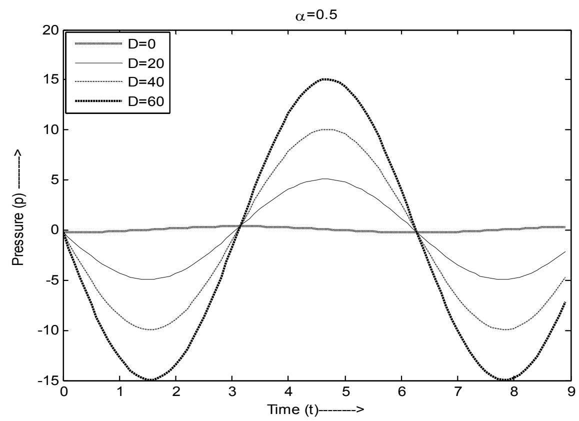

Fig. (A.11). Variation of the Pressure VS Time for different porosity at $\alpha = .5$

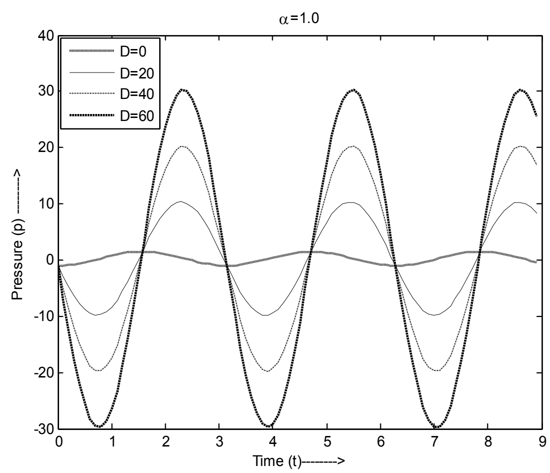

Fig. (A.12). Variation of the Pressure VS Time for different Darcy no at $\alpha = 1.0$

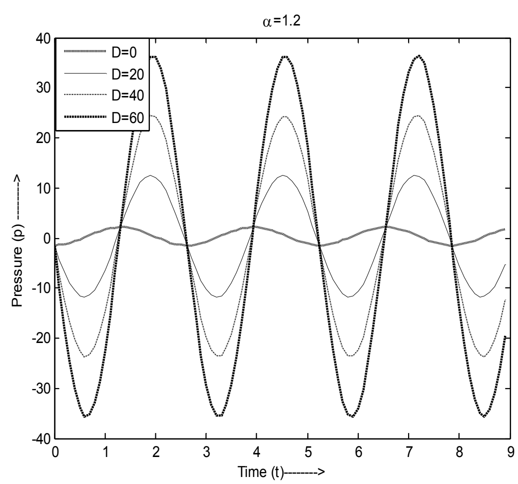

Fig. (A.13). Variation of the Pressure VS Time for different Darcy no at $\alpha = 1.2$

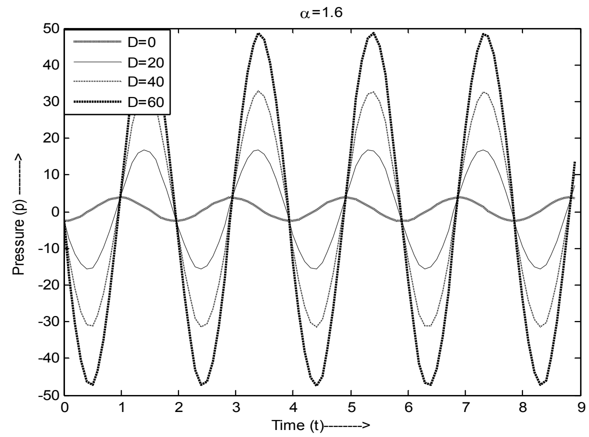

Fig. (A.14). Variation of the Pressure VS Time for different Darcy no at $\alpha = 1.6$

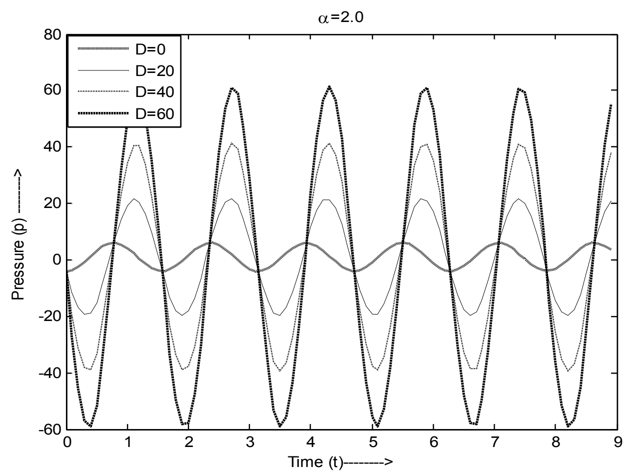

Fig. (A.15). Variation of the Pressure VS Time for different Darcy no at time $\alpha = 2.0$

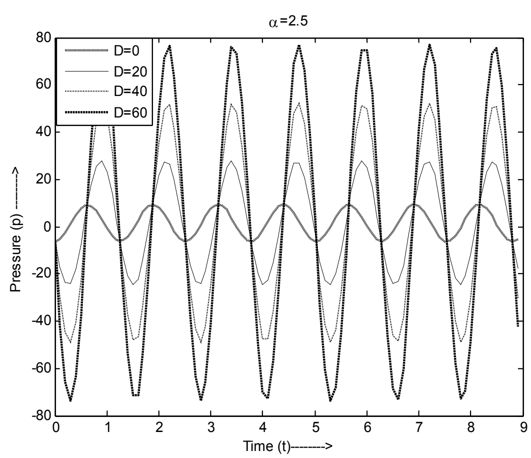

Fig. (A.16). Variation of the Pressure VS Time for different Darcy no at time $\alpha = 2.5$

Case-B : $r = \frac{(1+\varepsilon\cos\alpha t)}{(1+\varepsilon)} = r^*(1+\varepsilon\cos\alpha t)$

Here the sphere radius oscillating from its initial value $r^* = \frac{1}{(1+\varepsilon)}$

From equation (11) we get

$$f(t) = (r^*)^2(1+\varepsilon\cos\alpha t)^2 \quad \ldots (B.1)$$

Also $P = \frac{\varepsilon^2\alpha^2}{4} - \{\alpha\varepsilon(\alpha\cos\alpha t + D\sin\alpha t) + \alpha\varepsilon^2((1.25)\alpha\cos 2\alpha t + (.5)D\sin 2\alpha t)\}$ …(B.2)

$$= \frac{\varepsilon^2\alpha^2}{4} - R_1\cos(\alpha t - \phi_1) - R_2\cos(2\alpha t - \phi_2) \quad \ldots (B.3)$$

the amplitudes R_1, R_2 and phase lags ϕ_1, ϕ_2 respectively are given by

$$R_1 = \sqrt{\varepsilon^2\alpha^4 + D^2\varepsilon^2\alpha^2}\,; \qquad \phi_1 = \tan^{-1}\left(\frac{D}{\alpha}\right) \quad \ldots (B.4)$$

$$R_2 = (.5)(\varepsilon^2\alpha)\sqrt{25\alpha^2 + 2D^2}\;;\; \phi_2 = \tan^{-1}\left(\frac{2D}{5\alpha}\right) \quad \ldots (B.5)$$

Results and Discussions

It is noticed from the figures (B.1)-(B.6) the variation of the pressure on the spherical surface verses Radius-oscillation parameter (α) for different values of porosity parameter (D), the number of oscillation beats increases as Radius-oscillation parameter increases for a specific range of Radius-oscillation parameter.

The variation of the pressure verses Darcy number (D). It is illustrated in figures (B.7-B.10) and it is noticed that as the Darcy number (D) increases for different values of Radius-oscillation parameter (α) the pressure gradually decreases this may be associated due to the increase in internal resistance in the porous medium.

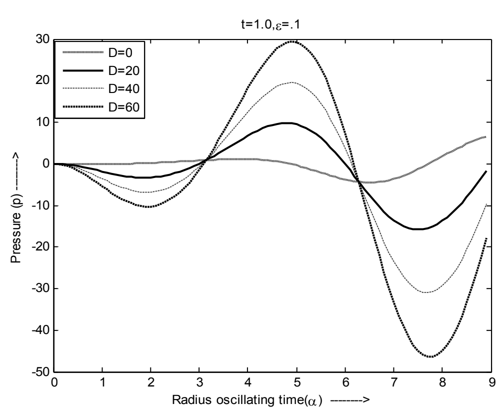

Fig. (B.1). Variation of the Pressure VS Radius oscillation time for different porosity at t = 1.0, ε = .1

Also It is noticed from figures (B.11)-(B.16) that the variation of the pressure verses time (t) on the spherical surface as time increasing for different values of porosity parameter (D) the chaotic oscillation are noted in each of the cases stated above.

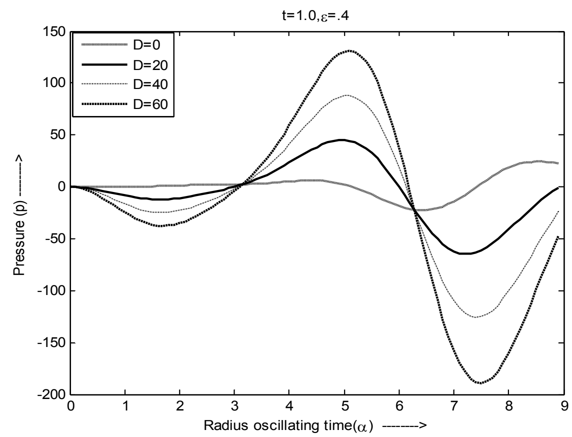

Fig. (B.2). Variation of the Pressure VS Radius oscillation time different for porosity at $t = 1.0$, $\varepsilon = .4$

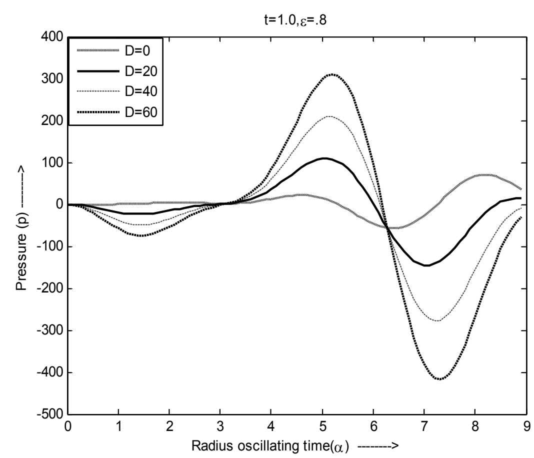

Fig. (B.3). Variation of the Pressure VS Radius oscillation time for different porosity at $t = 1.0$, $\varepsilon = .8$

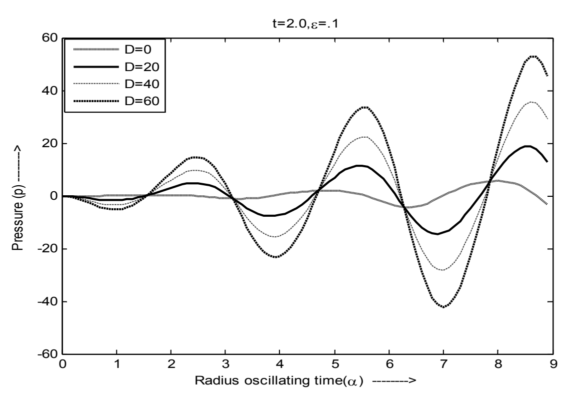

Fig. (B.4). Variation of the Pressure VS Radius oscillation time for different porosity at $t = 2.0$, $\varepsilon = .1$

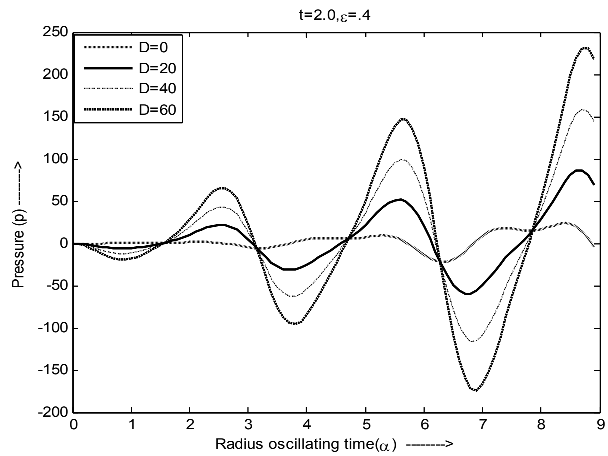

Fig. (B.5). Variation of the Pressure VS Radius oscillation time for different porosity at $t = 2.0$, $\varepsilon = .4$

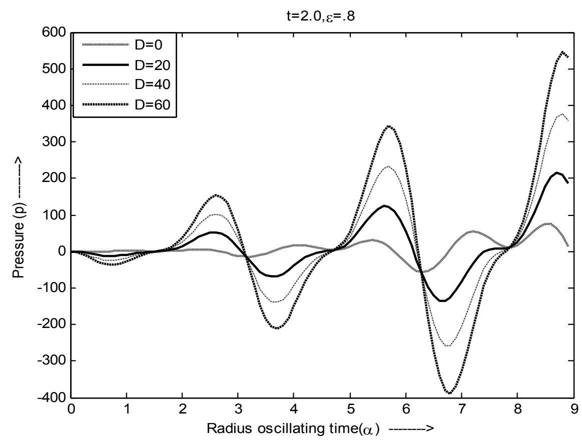

Fig. (B.6). Variation of the Pressure VS Radius oscillation time for different porosity at t = 2.0, $\varepsilon = .8$

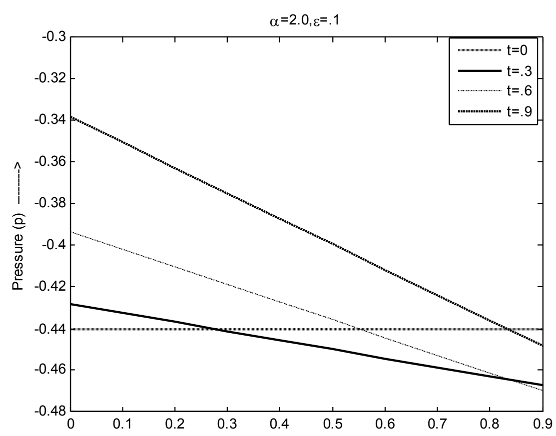

Fig. (B.7). Variation of the Pressure VS Darcy number for different time at $\alpha = 2.0$, $\varepsilon = .1$

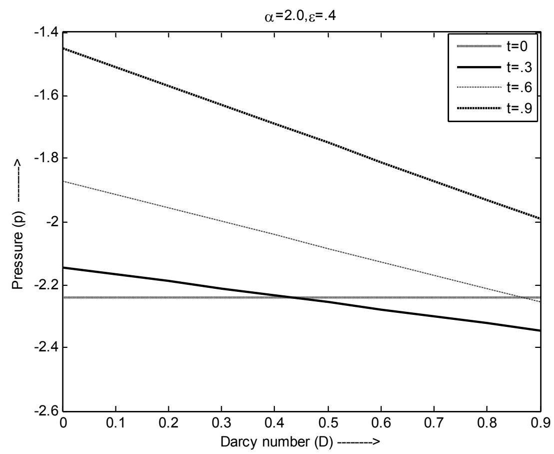

Fig. (B.8). Variation of the Pressure VS Darcy number for different time at $\alpha = 2.0$, $\varepsilon = .4$

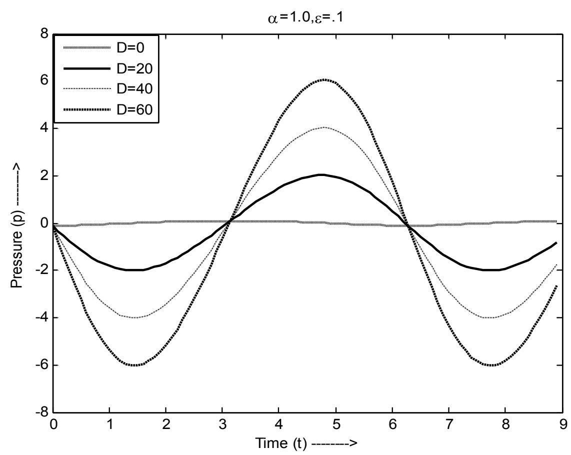

Fig. (B.9). Variation of the Pressure VS Time for different porosity at $\alpha = 1.0$, $\varepsilon = .1$

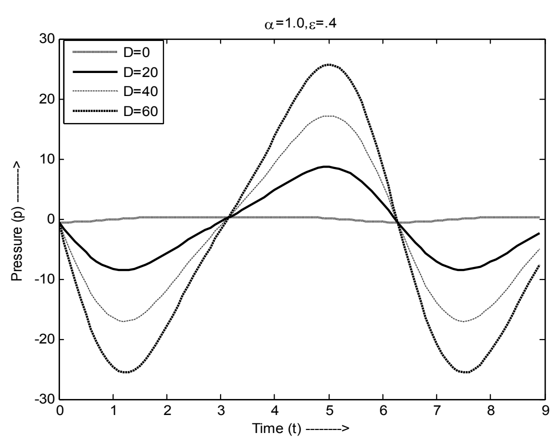

Fig. (B.10). Variation of the Pressure VS Time for different porosity at $\alpha = 1.0$, $\varepsilon = .4$

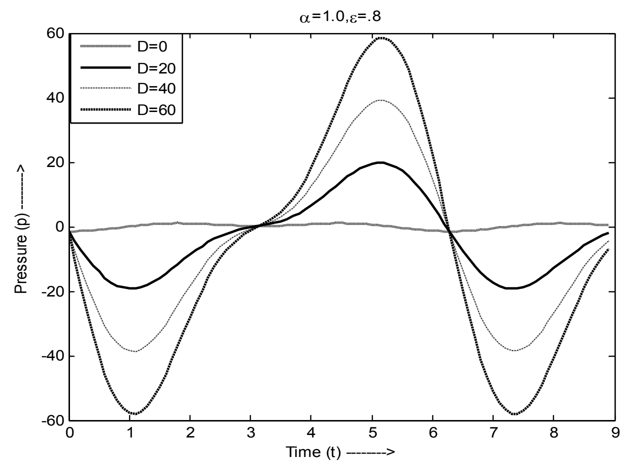

Fig. (B.11). Variation of the Pressure VS Time for different porosity at $\alpha = 1.0, \varepsilon = .8$

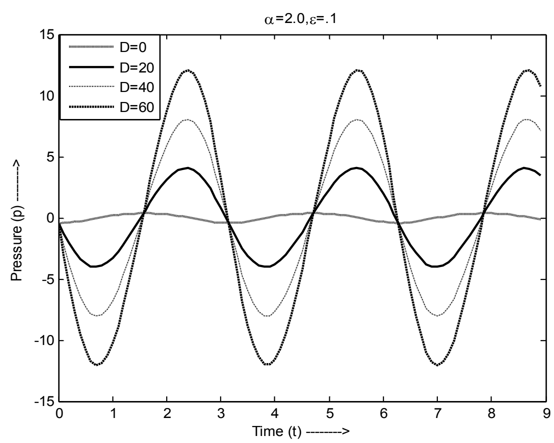

Fig. (B.12). Variation of the Pressure VS Time for different porosity at $\alpha = 2.0, \varepsilon = .1$

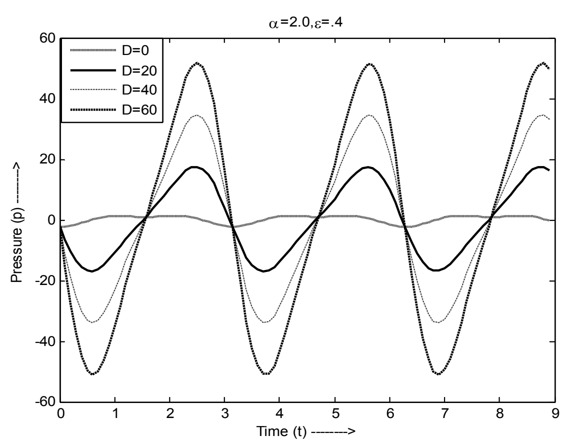

Fig. (B.13). Variation of the Pressure VS Time for different porosity at $\alpha = 2.0, \varepsilon = .4$

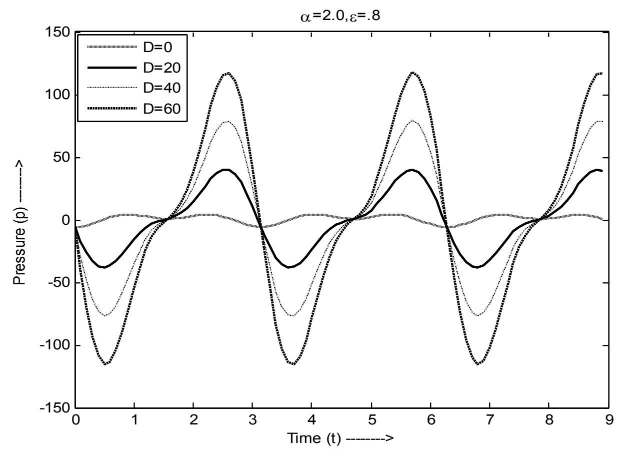

Fig. (B.14). Variation of the Pressure VS Time for different porosity at $\alpha = 2.0, \varepsilon = .8$

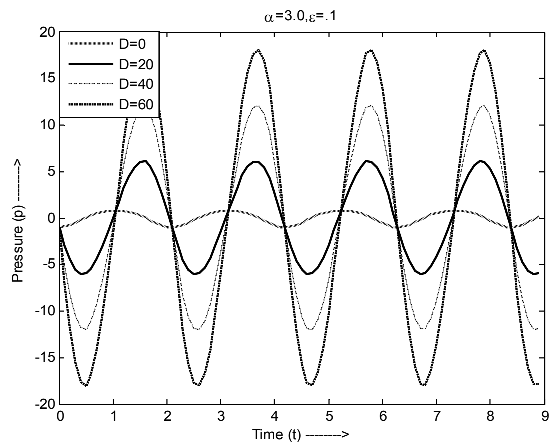

Fig. (B.15). Variation of the Pressure VS Time for different porosity at $\alpha = 3.0, \varepsilon = .1$

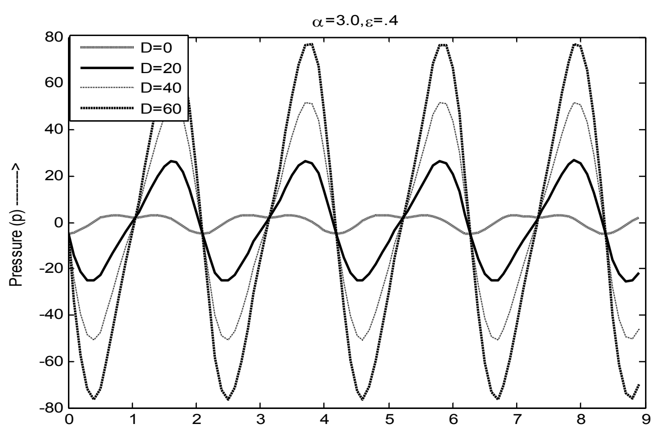

Fig. (B.16). Variation of the Pressure VS Time for different porosity at $\alpha = 3.0, \varepsilon = .4$

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