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ANALYSIS OF PERISTALTIC FLOW IN DISEASED ARTERY

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In the functioning of ureters, intestines, esophagus and in medical instruments such as the heart-lung machine, fluid is transported by a unique process, arteries of contractions of the wall propagates along the length of the tube causing fluid to be transported in the direction of the wave. This is called peristaltic pumping. In the paper, we have studied the effect of asymmetric Peristalsis on blood flow through an artery with stenosis. Here, we have considered the blood as a Newtonian fluid. Here, the basic equation is taken as Navier-stokes equation and we have transformed the stationary co-ordinates to moving Co-ordinates. The friction force and resistance to flow has been obtained.

KEYWORDS : Peristalsis, Newtonian fluid, Non-Newtonian fluid.

Nomenclature

C = Wave Propagation velocity.

h = Mean radius of the channel.

λ = Wavelength.

$\bar{q}$ = Dimensionless volume flow.

p = Pressure.

b_1 = Amplitude of the Peristaltic wave on upper wall.

b_2 = Amplitude of the Peristaltic wave on lower wall.

$\in_1, \in_2$ = Dimensionless Amplitude.

Introduction

Peristaltic motion of blood (or other fluid) in animal or human bodies has been considered by many authors. It is an important mechanism for transporting blood, where the cross-section of the artery is contracted or expanded periodically by the propagation of progressive wave. Peristaltic motion occurs widely when stenosis is formed in the functioning of ureter, chyme movement in the intestine, movement of egg in the fallopian tube, the transport of spermatozoa in the cervical canal, transport of bile in bile duct, transport of cilia etc. [Abd EI Hakeem *et. al*] [1]. However, such motion in the case of blood is somewhat different from above. For blood, the motion is highly speedy, Reynolds number is large and the peristalsity is maintained automatically.

Shapiro *et al*. [2] introduced an idea for peristaltic pumping with long wavelength and low Reynolds number. Jaffrin and Shapiro [3] showed a comprehensive developing of a mathematical model of peristaltic flow. Takabatake *et al*. [4, 5] applied numerical method, with upwined finite-difference technique to solve the problem of peristaltic flow in circular cylindrical tube. Burns and Parkes [6], Fung and Yih [7], Chow [8] and Mishra and Pande [9] followed the perturbation technique to study the peristaltic flow in circular cylindrical tube.

Applying numerical technique, Brown and Hung [10] considered the curvature effect of peristaltic flow. Rao and Usha [11] studied the motion of two-layered Newtonian fluid through a circular cylindrical tube, while Utkin *et al*. [12] solved numerically the system of equations of motion and the wall. Blood is a non-Newtonian fluid and is a suspension of cells in plasma. Not only blood, but many other fluids of animal or human are, in fact, non-Newtonian. However, blood can be treated as a Newtonian fluid if the radius of the artery is greater than 0.25 mm. [13].

Recently, M. Brust *et al* [14] investigated about the rheology of human blood plasma. A comparative study of viscoelastic behaviour and Newtonian behaviour of human blood plasma has been taken into account by them. S. Maiti and J. C. Misra [15] investigated the non-Newtonian characteristics of peristaltic flow of blood in micro vessels. A Lamura *et al* [16] studied the dynamics and rheology and vesicle suspension in wall bounded shear flow whereas Omid David *et al* [17] dealt in vivo quantification of clot formation in extra corporeal circuits. Laura Facchini *et al* [18] developed a simple model of filtration and macromolecule transport through microvascular walls. S. Maiti and J. C. Misra [19] studied the peristaltic flow of a fluid in a porous channel. They completed a study having the relevance to flow of bile within ducts in a pathological state. In the present paper, the effect of asymmetric peristalsis on blood flow through a channel have been investigated. Here we used the perturbation theory to Solve differential equation.

BASIC EQUATIONS AND FORMULATION OF THE PROBLEM:

Let the Equation of the tube surface be given by :

$$h(x,t) = a\left[1 + \in \cos\left\{\left(\frac{2\pi}{\lambda}\right)(x - ct)\right\}\right]$$

where a = undisturbed radius and $\in$ = the amplitude ratio.

In the present communication, the flow of blood through a channel is considered. Here, Blood is taken as a Newtonian fluid with the assumptions of infinite wave length and inertia-free flow, the Navier-stokes equation reduces to :

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2} \qquad \dots (1)$$

The boundary conditions are :

$$u = -c \quad \text{at} \quad y = h + b_1 . \cos\left(\frac{2\pi}{\lambda}.x\right)$$

$$u = -c \ \text{ at } \ y = -h - b_2.\cos\left(\frac{2\pi}{\lambda}.x\right) \qquad \ldots (2)$$

where c is the wave propagation velocity, h is the mean radius of the channel, b_1 is b_2 are the amplitudes of the peristaltic wave on the upper and lower walls respectively. λ is the wavelength.

Let us transform the stationary Co-ordinates to moving Co-ordinates as

$$x = X - c.t, \quad y = Y$$

and $$u\,(x, y) = u\,(X - c.t,\ Y) - c$$

for velocity component in the laboratory frame u.

We shall solve equation (1) by applying perturbation theory for differential equations. We expand u, p and q as

$$u = u_0 + \in u_1 + 0(\in^2)$$

$$p = p_0 + \in p_1 + 0(\in^2) \qquad \text{where } P = \frac{dp}{dx}$$

$$q = q_0 + \in q_1 + 0(\in^2)$$

Substituting above expressions in equations (1) and in boundary conditions (2). We get the following :

$$\frac{dp_0}{dx} + \in \frac{dp_1}{dx} + o(\in^2) = \mu\left\{\frac{\partial^2 u_0}{\partial y^2} + \in \frac{\partial^2 u_1}{\partial y^2} + o(\in^2)\right\}$$

$$P_0 + \in P_1 + o(\in^2) = \mu\left\{\frac{\partial^2 u_0}{\partial y^2} + \in \frac{\partial^2 u_1}{\partial y^2} + o(\in^2)\right\}$$

where $P_0 = \frac{dp_0}{dx}$

On equating the like coefficients

$$P_0 = \mu \frac{\partial^2 p_0}{\partial y^2} \qquad \ldots (3)$$

and $$P_1 = \mu \frac{\partial^2 p_1}{\partial y^2} \qquad \ldots (4)$$

Solution of Order $\in^0$

On integrating twice equation (3)

$$u_0 = P_0.\frac{y^2}{2\mu} + A.y + B$$

where A and B are arbitrary constants here, boundary conditions are

$$u = -c \qquad \text{at } \ y = h_1(x)$$

$$u = -c \qquad \text{at } \ y = h_2(x)$$

$$-c = \frac{P_o}{2\mu}.h_1^2 + A.h_1 + B$$

$$-c = \frac{P_0}{2\mu}.h_2^2 + A.h_2 + B$$

$$A = \frac{-P_0}{2\mu}(h_2 + h_1)$$

and $$B = -c + \frac{P_0}{2\mu}.h_1.h_2$$

Hence, $$u_0 = \frac{P_0}{2\mu}\left\{y^2 - y(h_2 + h_1) + h_1.h_2\right\} - c \qquad \dots (5)$$

Solution of Order $\in^1$

On integrating twice equation (4)

$$u_1 = P_1.\frac{y^2}{2\mu} + C_1.y + D$$

where C_1, D are arbitrary constants.

Here boundary conditions are

$$u = 0 \quad \text{at} \quad y = h_1(x)$$

and $$u = 0 \quad \text{at} \quad y = h_2(x)$$

$$\Rightarrow \quad 0 = \frac{P_1}{2\mu}.h_1^2 + C_1.h_1 + D$$

$$0 = \frac{P_1}{2\mu}.h_2^2 + C_1.h_2 + D$$

$$C_1 = \frac{-P_1}{2\mu}(h_2 + h_1)$$

and $$D = \frac{P_1}{2\mu}(h_1.h_2)$$

Hence, $$u_1 = \frac{P_1}{2\mu}\left\{y^2 - (h_2 + h_1)y + h_1.h_2\right\} \qquad \dots (6)$$

Now, $$q_0 = \int_{h_2(x)}^{h_1(x)} u_0 \; dy$$

$$= \int_{h_2(x)}^{h_1(x)} \left\{\frac{P_0}{2\mu}\left\{y^2 - y(h_2 + h_1) + h_1.h_2\right\} - c\right\} dy$$

$$q_0 = \frac{P_0}{12\mu}\left[\left(\frac{h_1^3 - h_2^3}{3}\right) - \frac{\left(h_1^2 - h_2^2\right)}{2}.(h_2 + h_1) + h_1.h_2(h_1 - h_2)\right] - c(h_1 - h_2) \qquad \dots (7)$$

$$\Rightarrow \frac{dp_0}{dx} = \frac{12\mu\{q_0 + c(h_1 - h_2)\}}{-(h_1 - h_2)^3} \qquad \ldots (8)$$

Similarly,

$$q_1 = \int_{h_2(x)}^{h_1(x)} \frac{P_1}{2\mu}\left\{y^2 - (h_2 + h_1)y + h_1.h_2\right\}$$

$$\Rightarrow q_1 = \frac{P_1}{2\mu}\left\{-(h_1 - h_2)^3\right\} \qquad \ldots (9)$$

$$\Rightarrow \frac{dp_1}{dx} = \frac{2\mu q_1}{-(h_1 - h_2)^3} \qquad \ldots (10)$$

$$u = u_0 + \in u_1 + 0(\in^2)$$

$$\Rightarrow \quad u = \frac{P_0}{2\mu}\left\{y^2 - y(h_2 + h_1) + (h_1.h_2)\right\} - c + \in \frac{P_1}{2\mu}\left\{y^2 - (h_2 + h_1)y + h_1.h_2\right\} \qquad \ldots (11)$$

and $q = q_0 + \in q_1 + o(\in^2)$

$$= \left\{\frac{dp_0}{12\mu dx}\left[-(h_1 - h_2)^3\right] - c(h_1 - h_2)^3\right\} + \in .\frac{dp_1}{dx}.\left[-\frac{(h_1 - h_2)^3}{12\mu}\right] + o(\in^2)$$

$$\Rightarrow \quad q = \frac{-1}{12\mu}(h_1 - h_2)^3\left(\frac{dp_0}{dx} + \in \frac{dp_1}{dx}\right) - c(h_1 - h_2) + O(\in^2)$$

$$q = \frac{-1}{12\mu}.H^3.\frac{dp}{dx} - cH \qquad \text{where } H = h_1 - h_2 \qquad \ldots (12)$$

Now,

$\bar{q}$ = dimensionless volume flow = $\frac{q}{2hc}$

$$= \frac{-1.H^3}{2\mu.2hc}.\frac{dp}{dx} - \frac{H}{2hc} \qquad \ldots (13)$$

The instantaneous volume flow rate

$$Q = \int_{h_2}^{h_1} (u + c)\,dy = q + cH \qquad \ldots (14)$$

The time-mean volume flow at each cross section is

$$\bar{Q} = \frac{1}{T}\int_0^T Qdt$$

$$= \bar{Q} = q + 2hc \qquad \ldots (15)$$

The pressure change per wavelength is

$$\Delta_{P_\lambda} = -\int_0^\lambda \frac{dp}{dx}.dx \qquad \text{where } \frac{dp}{dx} = \frac{dp_0}{dx} + \in \frac{dp_1}{dx}$$

$$\Rightarrow \quad \frac{h^2}{12\mu c\lambda}\Delta_{p_\lambda} = \frac{(\in_1 + \in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}}.\left[-3+\left\{\frac{8}{(\in_1+\in_2)^2}+1\right\}.\theta\right]$$

where $\theta = \frac{\bar{Q}}{2hc}$

The dimensionless pressure rise $(\Delta_{P_\lambda})_{\theta=0}$ for zero time-mean flow and the dimensionless time mean flow θ_0 for the zero pressure rise are given by

$$\frac{h^2}{12\mu c\lambda}\left(\Delta_{p_\lambda}\right)_{\theta=0} = \frac{-3(\in_1+\in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}} \qquad \dots (16)$$

and

$$\theta_0 = \frac{3(\in_1+\in_2)^2}{8+(\in_1+\in_2)^2} \qquad \dots (17)$$

Friction force at the wall a cross one wavelength

i.e.

$$F' = -\int_0^\lambda \left(\frac{H}{2}\right)^2.\frac{dp}{dx}.dx = \frac{6\mu c\lambda q}{\sqrt{4-(\in_1+\in_2)^2}} + 3\mu c\lambda$$

Dimensionless friction force *i.e.* $F = \frac{F'}{\mu\lambda c} = \frac{6\bar{q}}{\sqrt{4-(\in_1+\in_2)^2}} + 3$

Resistance to flow $R = \frac{P_0 - P_\lambda}{\theta}$

$$= \frac{12\mu c\lambda}{h^2\theta}\frac{(\in_1+\in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}}.\left[-3+\left\{\frac{8}{(\in_1+\in_2)^2}+1\right\}\theta\right]$$

When there is no. Peristaltic wave, then $R_0 = \frac{3\mu c\lambda}{h^2}$

Dimensionless Resistance to flow is $\bar{R} = \frac{R}{R_0} = \frac{-4(\in_1+\in_2)^2}{\left[4-(\in_1+\in_2)^2\right]^{\frac{5}{2}}}.\left[\frac{3}{\theta} - \left\{\frac{8}{(\in_1+\in_2)^2}+1\right\}\right]$

RESULTS AND DISCUSSION

To see the effects of various parameters on friction, flux, resistance to flow, the following values of the parametres are taken

$\in_1$ and $\in_2 = 0, 0.25, 0.4, 0.6, 0.8, 0.9$

$\bar{q} = 0, 0.25, 0.5, 0.75, 1, 1.25$

$\Delta_{P_\lambda} = 0, 0.0025, 0.0050, 0.0075, 0.01, 0.0125$

it has been observed that

(i) θ increase as $\in_1$ and $\in_2$ increases

(ii) θ increase as Δ_p increases

(iii) F increase as $\bar{q}$ increases

(iv) F increase as $\in_1$ and $\in_2$ increases for a given value of $\bar{q}$

(v) As θ increases, $\bar{R}$ increases

Also : Figure (1.1) depicts the perisaltic Transport through a channel.

Figure (1.2) depicts the variation of F with $\in_1$ for different $\bar{q}$, $\in_2 = 0.8$

Figure (1.3) depicts the variation of Flux θ_0 with $\in_2$ for different $\in_2, \Delta_p = 0$.

Figure (1.4) depicts the variation of Resistance to flow with $\in_1$ for different θ, $\in_2 = 0.6$.

Figure (1.5) depicts the variation of F with $\bar{q}$ for different $\in_1 + \in_2$.

GRAPHS

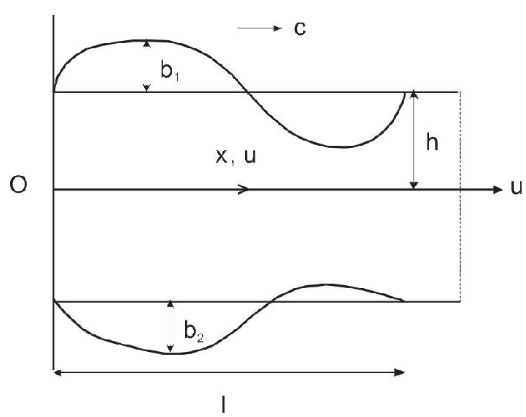

Fig. 1.1. Peristaltic Flow through a Channel

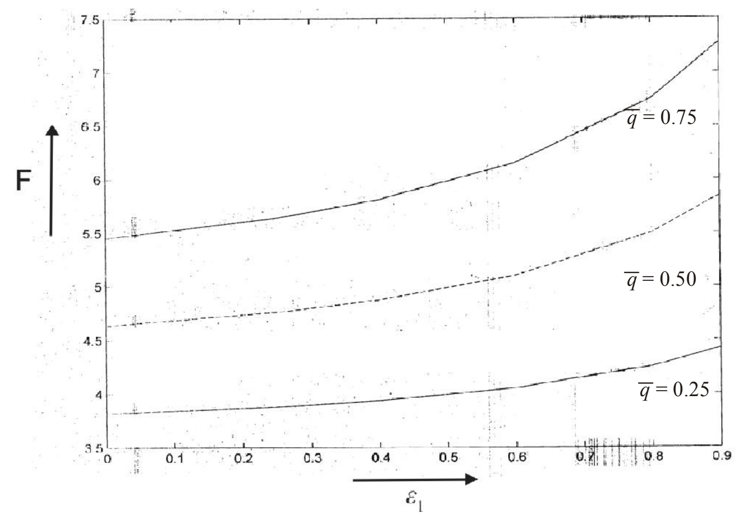

Fig. 1.2. Variation of '*F*' with 'ε_1' for different $\overline{q}$; $\varepsilon_2 = .8$ (Fixed)

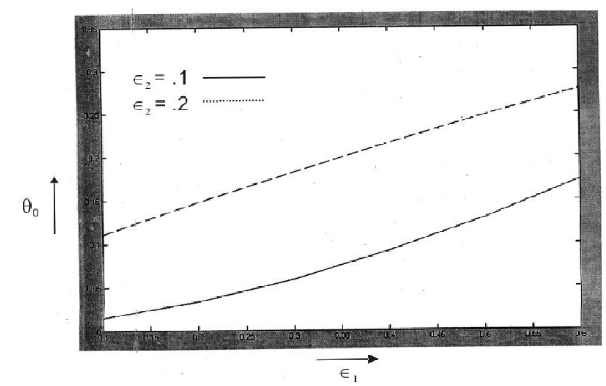

Fig. 1.3. Variation of θ_0 with e1 and $\Delta P_\lambda = 0$ and for different ε_2.

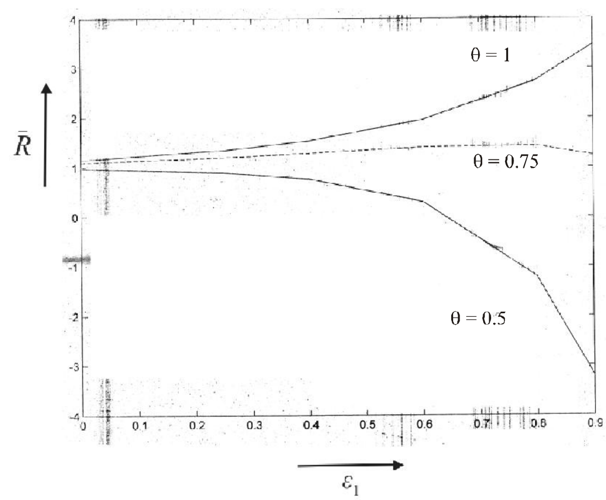

Fig. 1.4. Variation of $\overline{R}$ with 'ε_1' for 'θ'; $\varepsilon_2 = .6$ (Fixed)

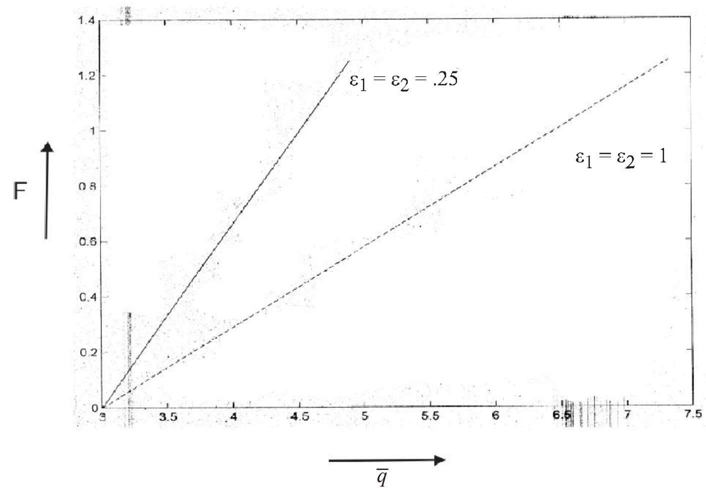

Fig. 1.5. Variation of 'F' with $\overline{q}$ for different values of $\varepsilon_1 = \varepsilon_2$.

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