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NEIGHBOURLY IRREGULAR GRAPHS AND SEMI NEIGHBOURLY IRREGULAR GRAPHS

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A connected graph *G* is said to be neighbourly irregular (NI) if no two adjacent vertices of G have the same number of vertices at a distance one away from them [6]. A connected graph *G* is said to be semi neighbourly irregular (SNI) if no two adjacent vertices of *G* have the same number of vertices at a distance two away from them [8]. This paper suggests a method to construct NI graphs which contains every graph of order $n \geq 2$ as a induced subgraph. Also this paper includes a few properties possessed by NI graphs and SNI graphs.

INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. For basic definitions and terminologies we refer Harary [5] and J.A. Bondy and U.S.R. Murty [3]. We denote the vertex set and edge set of a graph G by $V(G)$ and $E(G)$ respectively. The **degree** of a vertex v is the number of edges incident at v. A graph G is **regular** if all its vertices have the same degree .

For a connected graph G, the **distance** $\boldsymbol{d(u, v)}$ between two vertices u and v is the length of a shortest (u, v) path. Therefore, the degree of a vertex v is the number of vertices at a distance 1 from v, and it is denoted by $d(v)$. This observation suggests a generalization of degree. That is, $d_d(v)$ is defined as the number of vertices at a distance d from v in a graph G. Hence $d_1(v) = d(v)$ and $N_d(v)$ denote the set of all vertices that are at a distance d away from v in a graph G. That is, $N_1(v) = N(v)$.

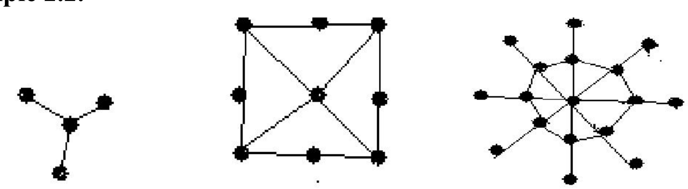
A connected graph G is neighbourly irregular (NI) if no two adjacent vertices of G have the same degree. This concept was studied by *S*. Gnana Prakasam and S.K. Ayyaswamy [6]. A connected graph G is said to be semi neighbourly irregular graph (SNI) if no two adjacent vertices of G have the same number of vertices at a distance two away from them. That is, $d_2(u) \neq d_2(v)$, for all uv in $E(G)$. This concept was studied by N.R. Santhi Maheswari and C. Sekar [8]. In this paper, we investigate some more results on NI graphs and SNI graphs.

NEIGHBOURLY IRREGULAR GRAPH

In this section, we construct neighbourly irregular graphs containing the given graph as an induced subgraph and also we can see neighbourly regular strength of G. Also we will determine minimum number of points needed to construct neighbourly irregular graph containing some particular graphs.

Definition 2.1. A connected graph G is neighbourly irregular (NI) if no two adjacent vertices of G have the same degree[6].

Example 2.2.



The following facts are known from literature.

Fact 1 [6]. The complete bipartite graph $K_{m,\ n}$ is Neighborly Irregular if and only if $m \neq n$.

Fact 2 [6]. If v is a vertex of maximum degree in a Neighborly Irregular graph, then at least two of the adjacent vertices of v have the same degree.

Fact 3 [6]. If a graph G is Neighborly Irregular then no P_4 contains internal vertices of same degree in G.

Fact 4 [6]. Any graph of order n can be made to be an induced subgraph of a Neighborly Irregular graph of order at most $(n + 1)\ C_2$.

Fact 5 [6]. We have also seen that there exists a Neighbourly Irregular graph $K_{n1,\ n2....nm}$ (NI) of order n, for any +ve integer n and $(n_1, n_2, ...n_m)$ be a partition of n with distinct parts and given some properties of $K_{n1,\ n2....\ nm}$ (NI) of order n.

Observation : 2.3

1. There is no neighbourly irregular graph of order one and two.
2. Only one neighbourly irregular graph of order three is P_3.
3. There is only one neighbourly irregular graph of order four is $K_{1.3}$.
4. There are only four neighbourly irregular graph of order five.

2.4. Neighbourly irregular graphs containing a given graph as an induced subgraph

Konig [7] proved that if G is any graph, whose maximum degree is r, then it is possible to add new points and to draw new lines joining either two new points or a new point to an existing point, so that the resulting graph H is a regular graph containing G as an induced subgraph. Paul Erdos and Paul Kelly [4] determined the smallest number of new vertices which must be added to a given graph G to obtain such a graph.

Theorem 2.5. Every graph G as an induced subgraph of neighbourly irregular graph.

Proof : Let G be a given graph .

Case 1 : If G is a r-regular graph, then we can construct neighbourly irregular graph N containing given G.

Case 2 : If G is not a regular graph whose maximum degree is r, then we can construct r-regular graph H whose degree of regularity is the maximum degree of G [7].

Let $V(H) = \{h_i / (1 \leq i \leq n)\}$.

The desired graph N has the vertex set $V(N) = V(H)\ U\ V(F)\ U\ V(T)$,

where $V(F) = \{f_i/(1 \leq i \leq n-1))\}$ and $V(T) = \{t_i/(1 \leq i \leq n + r - 1)\}$.

Let $E(N) = E(H)\ U\ \{h_i f_j / (1 \le i \le n,\ 1 \le j < i)\}\ U\ \{f_i t_j / (1 \le i \le n-1), (1 \le j \le n + r - 1)\}$.

Here, degree of h_i in $N = r + i - 1$, for $1 \le i \le n$ and degree of f_i in $N = n + r + j - 1$, for $(1 \le j \le n - 1)$ and degree of t_i in $N = n - 1$.

Therefore, the desired graph N is the neighbourly irregular graph of order $3n + r - 2$ containing every r-regular graph H of order n as an induced subgraph. This graph N is the neighbourly irregular graph containing every graph as an induced subgraph.

Theorem 2.6. For $n \ge 1$, the minimum order of neighbourly irregular graph containing the regular complete bipartite graph $K_{n,\,n}$ of order $2n$ as an induced subgraph is $2n + 1$.

Proof : By attaching one pendant vertex to one vertex of any one of the partition set of $K_{n,\,n}$, we get neighbourly irregular graph of order $2n + 1$.

Theorem 2.7. For any $n > 3$, every n-cycle is an induced subgraph of neighbourly irregular graph.

Proof:

Case (i) : n is even ($n \ge 4$)

By attaching one pendant vertex to alternate vertices of n-cycle (n is even), we get neighbourly irregular graph of order $(3n)/2$.

Case (ii) : n is odd ($n \ge 5$).

Let C_n be an odd n-cycle. $V(C_n) = \{v_1, v_2, v_3, v_4, \ldots. v_n\}$. By attaching two pendant vertices to v_n, and attaching one pentant vertex to v_i, for $i = 1, 3, 5, 7, \ldots\ n - 2$, we get neighbourly irregular graph of order $(3n + 3)/2$.

Theorem 2.8. For any $n \ge 3$, every path P_n is an induced subgraph of Neighbourly irregular graph.

Proof :

Case (i) : *n* is odd.

If $n = 3$, P_3 is neighbourly irregular graph. Let P_n ($n \ge 5$) be a path of length $n - 1$.

$V(P_n) = \{v_1, v_2, v_3, v_4, \ldots. v_n\}$. By attaching one pendant vertex to v_i, $i = 3, 5, 7 \ldots n - 2$. We get Neighbourly irregular graph of order $3(n - 1)/2$.

Case (ii) : *n* is even.

If $n = 4$, attach one pendant vertex to v_3, to get neighbourly irregular graph of smallest order 5. Let P_n ($n \ge 6$) be a path of length $n - 1$. $V(P_n) = \{v_1, v_2, v_3, v_4, \ldots. v_n\}$.

By attaching one pendant vertex to v_i, i =3, 5, 7, 9 … $n - 1$. We get neighbourly irregular graph of order $(3n - 2)/2$.

Theorem 2.9. For $n \ge 2$, the smallest order of neighbourly irregular graph containing K_n as an induced subgraph of order $2n - 1$.

Proof : A Neighbourly irregular graph of order $2n - 1$ which contain K_n as an induced sub graph is $SNIK_n$ [8].

NEIGHBOURLY IRREGULAR TREES

Definition 3.1. A tree T is said to be neighbourly irregular tree if no two adjacent vertices of T have the same degree.

Result 3.2. For any $n \geq 3$, there exist a NI Star.

Result 3.3. If we subdivide each edge of $K_{1,\,n}$ $(n \geq 3)$ one time .The resulting graph is NI.

Definition 3.4. Bistar-$B_{n,m}$, $(n \neq m)$

K_2 with n pendent edges at one end point and m pendent edges at another end point is called $B_{n,\,m}$ $(n \neq m)$ Star (tree).

Result 3.5. $B_{n,\,m}$, $(n \neq m)$ tree is NI.

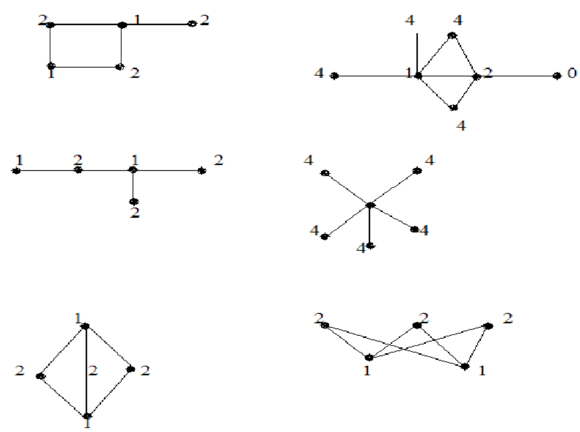
Result 3.6. Bistar $B_{n,\,n}$ $(n \geq 2)$ is not NI, but subdivide the edge $v_1 v_2$, we get NI tree of order $2n + 3$.

Result 3.7. If we subdivide each edge of $B_{n,\,n}$ $(n \geq 2)$ one time , then the resulting graph is NI.

SEMI NEIGHBOURLY IRREGULAR GRAPHS.

4.1. Definition: A connected graph G is said to be Semi neighbourly irregular (SNI) if no two adjacent vertices of G have the same number of vertices at a distance two away from them. That is, $d_2(u) \neq d_2(v)$, for all uv in $E(G)$ and $d_2(v)$-denote the number of vertices at a distance two away from v in G.

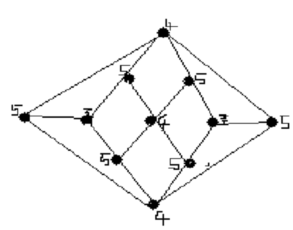
4.2. Example of SNI Graphs



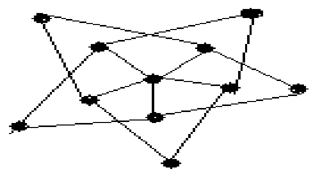
(ii) Any complete bipartite Graph $K_{m,\,n}$ is SNI graph only when $m \neq n$ [8].

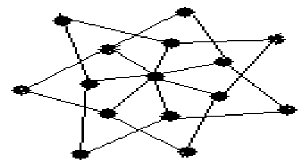
(iii) Any complete 'm' partite graph $K_{n1,\,n2\ldots\ldots nm}$ is SNI iff $n_1 \neq n_2 \neq n_3 \ldots . \neq n_m$ [8].

(iv) Hershel graph is SNI



(v) Graph is obtained from star by joining alternate two pentant vertices to one new vertex is SNI graph.

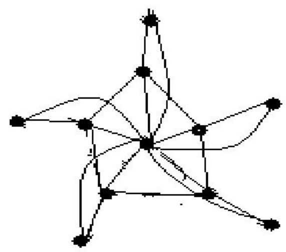




4.3. Examples of Graphs which are not SNI.

(i) $K_{n1,\, n2\, \ldots.\, nm}$ is not SNI only when atleast any two n_i's are same .

(ii) Flower graph is obtained from a helm by joining each pendent vertex to the central vertex of the helm. Flower graph is not SNI**.**



The following facts are known from literature

Fact 1. [8] For any $n \geq 3$, there exists atleast one SNI graph of order n.

Fact 2. [8] Any vertex v is adjacent with vertex u of degree n and non adjacent with vertices which are adjacent with u. Then v is at a distance two away from at least $n - 1$ vertices.

Fact 3. [8] If a graph G is SNI, then no P_4 (path on 4 vertices) contains internal vertices having same number of vertices at a distance two away from them.

Fact 4. [8] For n ≥ 1, the minimum order of Semi neighbourly irregular graph containing the complete bipartite graph $K_{n,n}$ of order 2n as an induced subgraph is $2n + 1$.

Fact 5. [8] For any $n \geq 3$, every 'n'-cycle is an induced subgraph of SNI of order $2n$.

Fact 6. [8] For any $n \geq 3$, every path P_n is an induced subgraph of SNI graph of order $2n - 3$.

Fact 7. [8] For $n \geq 2$, the smallest order of $SNIK_n$ graph of order $2n - 1$ containing K_n as an induced sub graph **.**

Fact 8. [8] Every graph of order $n \geq 2$ as an induced subgraph of SNI graph of order $4n - 1$.

Fact 9. [8] A minimal covering edge family of SNI graph of order $4n - 1$ containing given graph of order n as an induced sub graph has cardinality $3n - 1$.

Fact 10. [8] A minimal vertex covering number of SNI graph of order $4n - 1$ containing given graph of order n as an induced sub graph is $2n - 1$.

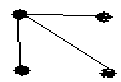
4.4. SNI graphs of order up to six :

1. There is no SNI graph of order one and two.

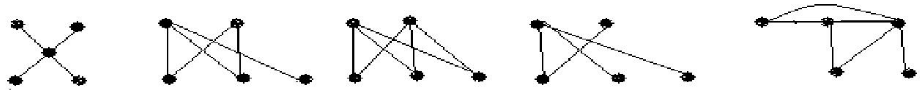
2. Path P_3 is the only SNI graph of order three.

P_3

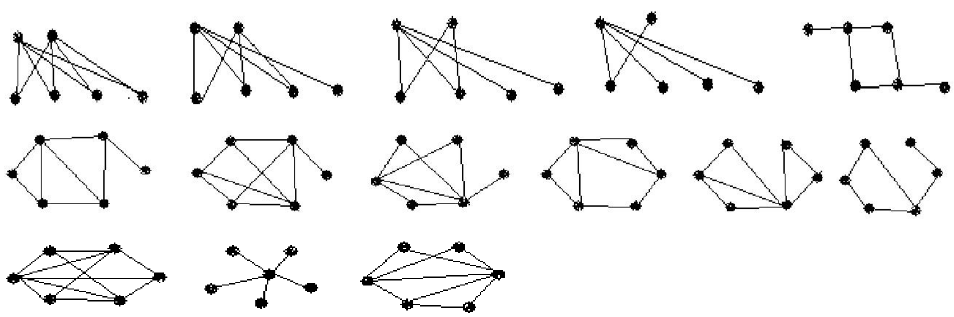
3. Star $K_{1,3}$ is the only SNI graph of order four.



4. SNI graphs of order five are given below



5. SNI graphs of order six are given below



SOME RESULTS RELATED WITH NEIGHBOURLY IRREGULAR GRAPH AND SEMI NEIGHBOURLY IRREGULAR GRAPH

In this section, we will see some results connecting with Neighbourly irregular graphs and Semi neighbourly irregular graphs.

Result 5.1. Graph G is Neighbourly irregular graph of diameter two if and only if G is Semi neighbourly irregular graph of diameter two.

Proof : Let G is Neighbourly irregular graph of diameter two if and only if $d(u) \neq d(v)$, for all $uv \in E(G)$ if and only if $n - 1 - d(u) \neq n - 1 - d(v)$, for all $uv \in E(G)$ if and only if $d_2(u) \neq d_2(v)$, for all $uv \in E(G)$ if and only if G is Semi neighbourly irregular graph of diameter two.

Result 5.2. Graph G is not Neighbourly irregular graph of diameter two if and only if G is not Semi neighbourly irregular graph of diameter two.

Proof : G is not Neighbourly irregular graph of diameter two if and only if $d(u) = d(v)$, for some $uv \in E(G)$ if and only if $n - 1 - d(u) = n - 1 - d(v)$, for some $uv \in E(G)$ if and only if $d_2(u) = d_2(v)$, for all $uv \in E(G)$ if and only if G is not Semi neighbourly irregular graph of diameter two.

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