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ANALYSIS OF FAILURE RATE OF ATM OF A BANK

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This paper deals with the failure time of the electronic device ATM associated with the Bank queueing system. The Weibull distribution is used to model the life data of the machine which can deal with increasing, decreasing and constant failure rates.

KEYWORDS : Weibull probability distribution, median rank, rate function, failure rate, random variable, parameter estimation, cumulative probability distribution, Weibull plot.

Weibull distribution is a continuous probability distribution given by Waloddi Weibull. The Weibull probability distribution has been applied to many random phenomenons. Weibull analysis is a method for modeling data containing values greater than zero, such as failure data. This analysis can make predictions about a product's life, compare the reliability of competing product designs and statistically establish warranty policies. An application of this has been considered as a model for life time or time to failure in electrical components.

History in Telephone system we provide communication paths between pairs of customers on demand. The permanent communication path between two telephone sets would be expensive and impossible. So to build a communication path between a pair of customers, the telephone sets are provided a common pool, which is used by telephone set whenever required and returns back to pool after completing the call. So automatically calls experience delays when the server is busy. To reduce the delay we have to provide sufficient equipment. To study how much equipment must be provided to reduce the delay we have to analyze queue at the pool. In 1908 Copenhagen Telephone Company requested Agner K. Erlang to work on the holding times in a telephone switch. Erlang's task can be formulated as follows. What fraction of the incoming calls is lost because of the busy line at the telephone exchange? First we should know the inter arrival and service time distributions. After collecting data, Erlang verified that the Poisson process arrivals and exponentially distributed service were appropriate mathematical assumptions. He had found steady state probability that an arriving call is lost and the steady state probability that an arriving customer has to wait. Assuming that arrival rate is λ, service rate is μ and $\rho = \frac{\lambda}{\mu}$ he derived formulae for loss and deley.

The probability that an arriving call is lost (which is known as Erlang B-formula or loss formula).

$$P_n = \frac{\frac{\rho}{n!}}{\sum \frac{\rho^k}{k!}} = B(n, \rho)$$

The probability that an arriving has to wait (which is known as Erlang C-formula or deley formula).

$$P_n = \frac{n}{n - \rho(1 - B(n, \rho))} B(n, \rho)$$

Erlang's paper "On the rational determination of number of circuits" deals with the calculation of the optimum number of channels so as to reduce the probability of loss in the system.

Whole theory started with a congestion problem in tele-traffic. The application of queueing theory scattered many areas. It include not only tele-communications but also traffic control, hospitals, military, call-centers, supermarkets, computer science, engineering, management science and many other areas.

Introduction

A lot of our time is consumed by unproductive activities. Travelling has its own demerits one of them being wastage of time getting caught in traffic jams. A visit to the Post office or bank is very time consuming as huge crowds are waiting to be serviced. A simple shopping chore to a supermarket leads us to face long queues. In general we do not like to wait. People who are giving us service also do not like these delays because of loss of their business. These waits are happening due to the lack of service facility. To provide a solution to these problems we analyze queueing systems to understand the size of the queue, behaviour of the customers in the queue, system capacity, arrival process, service availability, service process in the system. After analyzing the queueing system we can give suggestions to management to take good decisions.

A queue is a waiting line. Queueing theory is mathematical theory of waiting lines. The customers arriving at a queue may be calls, messages, persons, machines, tasks etc. we identify the unit demanding service, whether it is human or otherwise, as a customer. The unit providing service is known as server. For example (1) vehicles requiring service wait for their turn in a service center. (2) Patients arrive at a hospital for treatment. (3) Shoppers are face with long billing queues in super markets. (4) Passengers exhaust a lot of time from the time they enter the airport starting with baggage, security checks and boarding.

Queueing theory studies arrival process in to the system, waiting time in the queue, waiting time in the system and service process. And in general we observe the following type of behavior with the customer in the queue. They are :

Balking of Queue : Some customers decide not to join the queue due to their observation related to the long length of queue, in sufficient waiting space. This is called Balking.

Reneging of Queue : This is the about impatient customers. Customers after being in queue for some time, few customers become impatient and may leave the queue. This phenomenon is called as Reneging of Queue.

Jockeying of Queue Jockeying is a phenomenon in which the customers move from one queue to another queue with hope that they will receive quicker service in the new position.

Important concepts in Queueing theory

Little law

One of the feet of queueing theory is the formula Little law. This is

$$N = \lambda T$$

This formula applies to any system in equilibrium (steady state).

Let λ is the arrival rate

T is the average time a customer spends in the system

N is the average number of customers in the system

Little law can be applied to the queue itself.

i.e. $$N_q = \lambda T_q$$

where λ is the arrival rate

T_q the average time a customer spends in the queue

N_q is the average number of customers in the queue

Classification of queuing systems

Input process : If the occurrence of arrivals and the offer of service strictly follow some schedule, a queue can be avoided. In practice this is not possible for all systems. Therefore the best way to describe the input process is by using random variables which we can define as "Number of arrivals during the time interval" or "The time interval between successive arrivals"

Service Process : Random variables are used to describe the service process which we can define as "service time" or "no of servers" when necessary.

Number of servers : Single or multiple servers

Queue length : 1 to ∞

System capacity : Finite or Infinite

Queue discipline : This is the rule followed by server in accepting the customers to give service. The rules are

- FCFS (First come first served).
- LCFS (Last come first served).
- Random selection (RS).
- Priority will be given to some customers.
- General discipline (GD).

Notation for describing all characteristics above of a queueing model was first suggested by David G Kendall in 1953.

- *A*/*B*/*P*/*Q*/*R*/*Z*

where *A* indicates the distribution of inter arrival times

B denotes the distribution of the service times

P is the number of servers

Q is the capacity of the system

R denotes number of sources

Z refers to the service discipline

Examples of queueing systems that can be defined with this convention are M/M/1

M/*D*/*n*

G/*G*/*n*

where *M* stands for Markov

D stands for deterministic

G stands for general

The Weibull cumulative distribution function and density functions

The two parameter Weibull cumulative probability distribution with shape parameter b and scale parameter a is

$$F(t,a,b) = 1 - \exp\left[-\left(\frac{t}{a}\right)^{b}\right].$$

The Weibull probability distribution function with shape parameter *b* and scale parameter *a* is

$$f(t,a,b) = \frac{d}{dt}(F\ (t,a,b))$$

$$f(t,a,b) = \frac{b\left(\exp\left[-\left(\left(\frac{t}{a}\right)^{b}\right)\right]\right)\left(\frac{t}{a}^{b-1}\right)}{a}$$

Weibull shape parameter *b* indicates that the failure rate is increasing, constant or decreasing. A $b < 1$ indicates that the product has a decreasing failure rate. This scenario is typical of "infant mortality" and indicates that the product is failing during its "burn-in" period. A $b = 1$ indicates constant failure rate. A $b > 1$ indicates an increasing failure rate. The Weibull characteristic life, called measure of scale or spread in the distribution data. It is so happens that 'a' equals the number of cycles at which 63.2 percent of product has failed. The Weibull distribution is linked with so many other distributions, for instance exponential distribution when b = 1. The exponential distribution is special case of gamma and Weibull distributions, where gamma and Weibull are not interchangeable. The fit of data to a Weibull distribution can be visually assessed using a Weibull plot. With some effort the Weibull cumulative distribution function can be transformed so that it appears in the form of straight line $y = mx + c$.

$$F(x) = 1 - e^{-\left(\frac{x}{a}\right)^{b}}$$

$$1 - F(x) = e^{-\left(\frac{x}{a}\right)^{b}}$$

$$\ln(1 - F(x)) \left(\frac{x}{a}\right)^b$$

$$\ln \left(\ln\left(\frac{1}{1 - F\ (x)}\right)\right) = b\ln(x) - b \text{ in } (a)$$

where $$y = \ln \left(\frac{1}{1 - \mathrm{F}\ (\mathrm{x})}\right), m = b \text{ and } c = -b\ln(a)$$

The graph of cumulative probability distribution with time *t* is same as the graph of ln (*t*) with $\ln\left(\ln\left(\frac{1}{1-F(x)}\right)\right)$ on Weibull plot. The shape parameter *b* can also directly be obtained from the slope of the straight line.

Median rank : Estimate of the proportion of the population that will fail by the number of cycles listed. This can be accomplished using several different methods, the most common method of which is median rank. The Median Ranks method is used to obtain an estimate of the unreliability for each failure. The median rank is the value that the true probability of failure, should have at the *j*th failure out of a sample of *N* units at the 50% confidence level.

The rank can be found for any percentage point, *P*, greater than zero and less than one, by solving the cumulative binomial equation for *Z*. This represents the rank, or unreliability estimate, for the jth failure in the following equation for the cumulative binomial:

$$P = \sum_{k=j}^{N} \binom{N}{k} x^k (1 - x)^{N-k}$$

The median rank is obtained by solving this equation for *x* at *p* = 0.50 is

$$0.50 = \sum_{k=j}^{N} \binom{N}{k} x^k (1 - x)^{N-k}$$

For example, if *N* = 4 and we have four failures, we would solve the median rank equation for the value of *z* four times; once for each failure with *j* = 1, 2, 3 and 4.

Rank Order	**Median Ranks (50%) Sample Size**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	50	29,29	20,63	15,91	12,94	10,91	9,43	8,30	7,41	6,70
2		70,71	50,00	38,57	31,38	26,44	22,85	20,11	17,96	16,23
3			79,37	61,43	50,00	42,14	36,41	32,05	28,62	25,86
4				84,09	68,62	57,86	50,00	44,02	39,31	35,51
5					87,06	73,56	63,59	55,98	50,00	45,17
6						89,09	77,15	67,95	60,69	54,83
7							90,57	79,89	71,38	64,49
8								91,70	82,04	74,14
9									92,59	83,77
10										93,30

Another quick, and less accurate, approximation of the median ranks is also given by:

$$MR = \frac{j - 0.3}{N + 0.4}$$

This approximation of the median ranks is also known as *Benard's approximation*.

The random variable "time to failure" is our assumption for the ATMs of the Andhra bank of JNTUH branch. The following are the failure timings of the respective ATM.

Rank	**Failure time *t***	**Median rank %**
1	30	12,94
2	79	31,38
3	161	50,00
4	251	68,62
5	347	87,06

Probability paper has scales that transform the cumulative probability distribution into a linear scale. Log-Log scales are very popular. If the data is plotted on this type of paper and results in a straight line, this supports the assumption that the distribution under study is appropriate.

The Weibull cumulative distribution is defined as :

$$F(t, a, b) = 1 - \exp\left[-\left(\left(\frac{t}{a}\right)^b\right)\right]$$

R (*t*, *a*, *b*) is called the Survival function. It gives the fraction of products that still operate after time *t*. That is probability of failure of the product up to the time *t* (*F* (*t*, *a*, *b*)) is subtracted from 1.

$$R(t, a, b) = 1 - F(t, a, b)$$

$$R(t, a, b) = 1 - F(t, a, b) = \exp\left[-\left(\frac{t}{a}\right)^b\right]$$

$$\frac{1}{1 - \mathrm{F}(\mathrm{t}, \mathrm{a}, \mathrm{b})} = e^{\left(\frac{t}{a}\right)^b}$$

r (*t*, *a*, *b*) is the failure rate function which lists the failure rate or speed at which the products are failing over the life time of a product.

$$r(t, a, b) = \frac{f(t, a, b)}{R(t, a, b)} = \frac{b\left(\frac{t}{a}\right)^{b-1}}{a}$$

The values are tabulated below.

j	t_j	ln (t_j)	ln (ln (1/(1 – 0.3)/(*n* + 0.4)))
1	30	3.4011	– 1.9744
2	79	4.3694	– 0.9727
3	161	5.0814	– 0.3665
4	251	5.5254	0.3447
5	347	5.8493	0.7145

In terms of variables x_j = ln (t_j) and y_j = ln (ln (1/(1 – (*j* – 0.3)/(*n* + 0.4))), the Weibull parameters are given by formulae

$$b = \frac{k\sum_{j=1}^{k} xjyj - \sum_{j=1}^{k} yj \sum_{j=1}^{k} xj}{k\sum_{j=1}^{k} xj^2 - \left(\sum_{j=1}^{k} xj\right)^2}$$

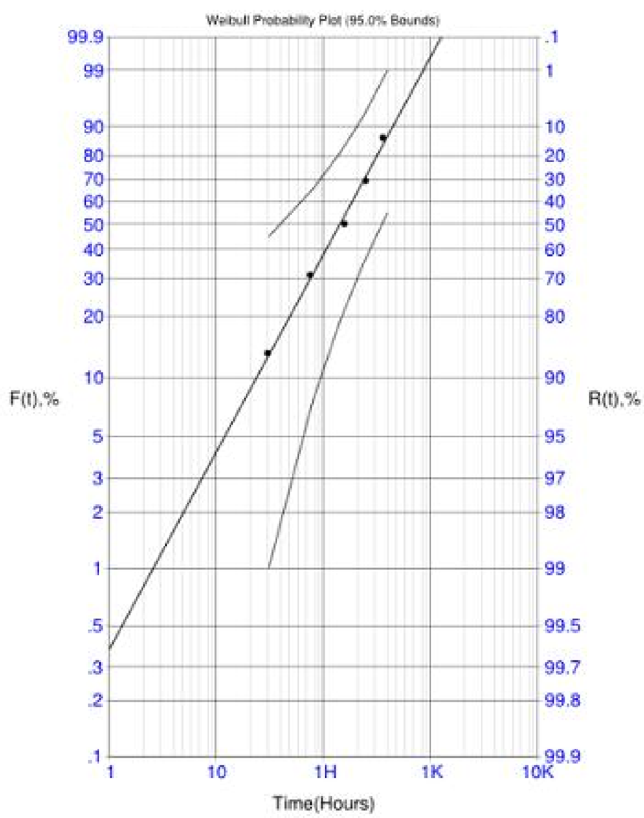
$$a = e^{\frac{(\sum_{j=1}^{k} yj)(\sum_{j=1}^{k} xj^2) - (\sum_{j=1}^{k} xj)(\sum_{j=1}^{k} xjyj)}{-bk\sum_{j=1}^{k} xj^2 - (\sum_{j=1}^{k} xj)^2}}$$

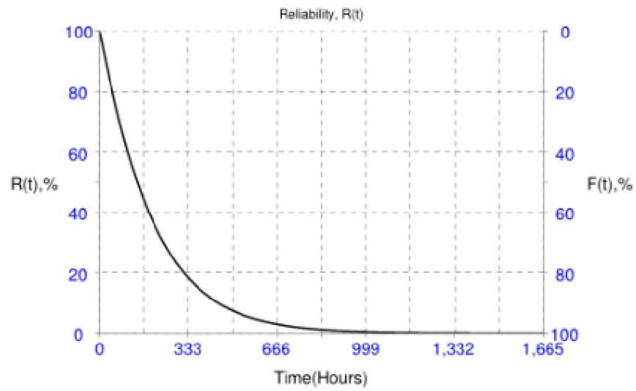
By using the formulae, calculating the parameters of Weibull probability distribution:

$\ln(t_j) = x_j$	$\ln(\ln(1/(1-(j-0.3)/(n+0.4))) = y_j$	$x_j y_j$	$\sum_{j=1}^{k} xj)$	$\sum_{j=1}^{k} xjyj)$	$\sum_{j=1}^{k} xj^2$	$\sum_{j=1}^{k} yj$
3.4011	–1.9744	– 6.715523789	24.22	– 7.857	121.22	– 2.455
4.3694	– 0.3665	– 4.250101391				
5.0814	– 0.3665	– 1.862400357				
5.5254	0.1447	0.799905428				
5.8493	0.7145	4.170910266				

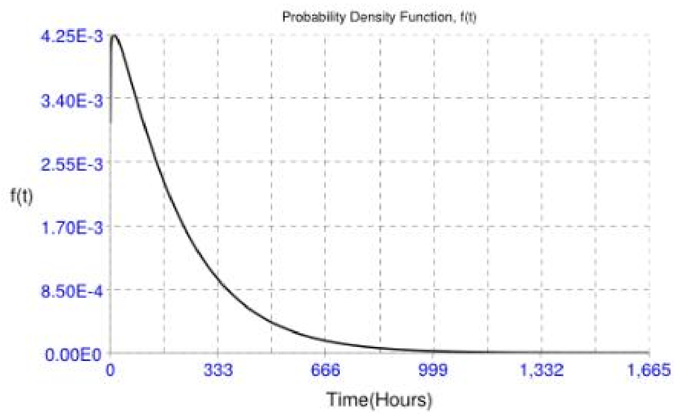
With the calculations above we got the parameters *a* = 202.62 which is scale parameter and *b* = 1.05 the shape parameter. The parameters can also be calculated through the graph of cumulative probability curve taken directly on Weibull plot.

The cumulative distribution function and the survival function graphs are drawn below.

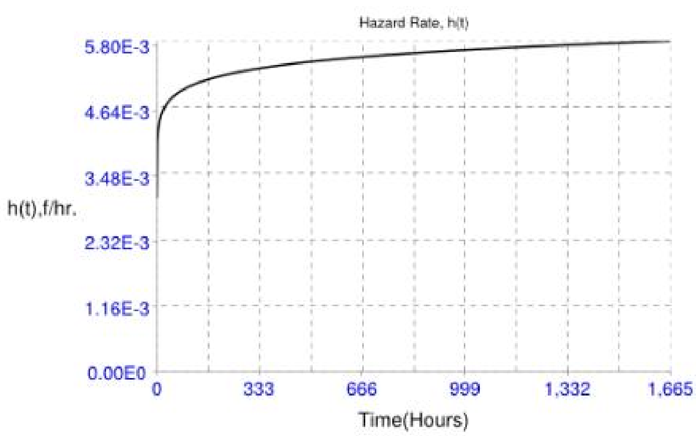




The graph of the probability distribution function is given below.



The failure rate function graph is given below.



Now we have found the failure rate function and also expected number of failures. The expected number of failures is the *n* times the cumulative distribution function. That is

$$N\left\{1 - e^{-\left(\frac{t}{a}\right)^{b}}\right\}$$

The expected numbers of failures within given time are tabulated below.

Time in hours	**Expected number of failures**
50	1
100	2
150	2
200	3
250	4
300	4
350	4
400	5

Conclusion

In this paper we have studied number of failures and failure rate of Bank's ATM by using Weibull probability distribution. In this study, five failure occurrence times are considered and extrapolated the probability of the failure occurrences by using Weibull distribution. We have observed failure rate is slightly proportional to time in Hazard graph. We can further study by considering the type of failure and its time and by considering Weibull distribution to work on preventive actions.

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