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BOOLEAN LIKE NEAR RINGS

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The concepts of Boolean like near-ring and special boolean like ring are introduced. It is shown that these two concepts are equivalent. An example of a Boolean like near ring which is not a Boolean like ring is furnished. It is also shown that a special boolean like ring with identity is a boolean like ring and conversly. It is proved that the set I of all nilpotent elements in a boolean like near-ring N is an ideal and N/I is a boolean ring. Finally it is proved that a proper ideal in a special boolean-like ring R with $R^2 = R$ is maximal if and only if it is prime. In addition, some interesting results are proved.

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Introduction

In [2] Foster, A.L. introduced the concept 'Boolean ring'. He extended many properties, both ring and logical, of Boolean rings to Boolean like rings. Swaminathan, V. [9] continued the study of Boolean-like rings initiated by A.L. Foster. He investigated some aspects of the ideal theory of Boolean like rings and also obtained a subdirect product representation of Boolean-like rings in terms of two element fields and particular four-element Boolean-like rings. In [5] K. Venkateswarlu *et al* introduced the concept of Boolean-like semi ring which is a generalization of Boolean-like rings of A.L. Foster and obtained various properties of ideals.

In this paper we introduce the concept of Boolean-like near ring, which is a generalization of Boolean like ring. An example of a Boolean-like near-ring, which is not a Boolean-like ring is given. In section 3, it is proved that a Boolean-like near-ring is a Boolean near-ring if and only if it is a Boolean ring. In section 4, we prove that every Boolean like near-ring with identity is a Boolean-like ring. In addition we prove that if a Boolean-like near-ring N has no nonzero idempotent elements; then N is a zero ring.

In section 5, we prove that in a Boolean like near-ring N the set I of all nilpotent elements of N forms an ideal and N/I is a Boolean ring. Further the set J of all idempotent elements of N

forms a subnear-ring. We also prove that every Boolean-like near-ring is a commutative ring. In section 6, we introduce the concept of special Boolean-like ring and prove that a near-ring is a Boolean-like near-ring if and only if it is a special Boolean-like ring. In addition, we prove that in a special Boolean-like ring R such that $R^2 = R$, an ideal $P \neq R$ is maximal if and only if it is prime.

Throughout the paper we consider only left near-rings. Henceforth, by a near-ring we mean a left near-ring.

Preliminaries

A (left) near-ring is a nonempty set N together with two binary operations + and . such that (i) $(N, +)$ is a group, (ii) $(N, .)$ is a semigroup and (iii) $n_1 (n_2 + n_3) = n_1 n_2 + n_1 n_3$ for all $n_1, n_2, n_3 \in N$ (left distributive law) [7]. If we take $(n_1 + n_2) n_3 = n_1 n_3 + n_2 n_3$ instead of (iii), we get a (right) near-ring. A (right) near ring N is called a boolean (right) near ring if $n^2 = n$ for all $n \in N$ [7]. D.J. Hansen and Jiang Luh [3] proved that every boolean (right) near-ring satisfies (right) weak commutative law: $xyz = xzy$ for all x, y, z. According to Foster, A.L. [9] a boolean like ring is a commutative ring with unity 1 and is of characteristic 2 with $a(1 + a) b(1 + b) = 0$ for all a, b. As per Venkateswarlu, K. *et al* [5] a boolean like semiring is a (left) near-ring such that $a + a = 0$ for all a, and (ii) $ab (a + b + ab) = ab$ for all a, b. As per Subrahmanyam, N.V. [8], a boolean semiring is a (left) boolean near-ring N such that $(N, +)$ is abelian.

Throughout the paper, it will be assumed that the near-rings are (left) near-rings.

Basic definitions and results

We now introduce the concept of a Boolean like near-ring.

Definition 3.1: A near-ring N is said to be a Boolean like near ring if the following conditions hold:

(i) $a + a = 0$ for all $a \in N$ (*i.e.*, Characteristic of N is 2)

(ii) $ab (a + b + ab) = ba$, for all $a, b \in N$, and

(iii) $abc = acb$, for all $a, b, c \in N$. (right weak commutative law)

It is well known that every Boolean ring with unity is a Boolean like ring and every Boolean like ring is a Boolean like near ring.

Following example shows that every Boolean like near-ring, need not, in general be a Boolean like ring.

Example 3.2 : Let $N = \{0, a, b, c\}$ be the Klein's four group. Addition and multiplication are given in the following tables [3].

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	0	0
c	0	a	0	a

Then N is a Boolean like near ring but not a Boolean like ring.

Let us recall that a near ring N is called a Boolean near ring if $a^2 = a$, for all $a \in N$.

We show that the conditions (i) and (ii) of def 1.1 are equivalent for boolean near-rings.

Theorem 3.3 : A Boolean near-ring N has characteristic 2 if and only if $ab\,(a + b + ab) = ba$ for $a, b \in N$.

Proof: Hansen and Luh [3] proved that a (left) Boolean near-ring satisfies (left) weak commutative law,

i.e., $$abc = bac.$$

Suppose N has characteristic 2 then

$$\begin{aligned} ab\,(a + b + ab) &= aba + abb + abab \\ &= baa + abb + aabb \quad \text{(by left weak commutativity } i.e., abc = bac) \\ &= ba + ab + ab \\ &= ba \quad \text{(since } N \text{ has characteristic 2)} \end{aligned}$$

Conversly, suppose that $ab\,(a + b + ab) = ba$, for all $a, b \in N$

By taking $b = a$, we get that

$$aa\,(a + a + aa) = aa$$
$$\Rightarrow\ aaa + aaa + aaaa = aa$$
$$\Rightarrow\ a + a + a = a$$
$$\Rightarrow\ a + a = 0$$

Therefore, N is of characteristic '2'.

It is well known that a Booolean ring is a Boolean (left) near-ring satisfying the right weak commutative law. The converse is proved in the following:

Lemma 3.4 : A Boolean near-ring N with right weak commutativity is a Boolean ring.

Proof: By [3], a Boolean near ring satisfies (left) weak commutative law *i.e.*, $abc = bac$.

Therefore, $aba = baa = ba$ by (left) weak commutative law.

Also, $ab = aab = aba$ by (right) weak commutative law.

Therefore, $ab = ba$, for all $a, b \in N$.

Hence multiplication in N is commutative.

By using two distributive laws, we have

$$\begin{aligned} (a + b)\,(b + a) &= (a + b)\,b + (a + b)\,\text{a} \\ &= ab + bb + aa + ba \qquad \ldots (1) \end{aligned}$$

$$\begin{aligned} (a + b)\,(b + a) &= a\,(b + a) + b\,(b + a) \\ &= ab + aa + bb + ba \qquad \ldots (2) \end{aligned}$$

From (1) and (2),

$$ab + bb + aa + ba = ab + aa + bb + ba$$

$$\Rightarrow \qquad b + a = a + b$$

Therefore, addition is commutative.

Hence N is a Boolean ring.

By the above lemma, we have the following:

Corollary3.5 : A Boolean near-ring satisfies (right) weak commutative law if and only if it is a Boolean ring.

Since every Boolean like near-ring satisfies the (right) weak commutative law, we get the following:

Corollary 3.6 : A Boolean near-ring is a Boolean like near-ring if and only if it is a Boolean ring.

ELEMENTARY RESULTS

In this section, N stands for a Boolean like near-ring, unless otherwise stated. We prove some interesting results which are useful in proving the main theorem.

In section 3,we mentioned that every Boolean like ring is a Boolean like near-ring with identity. We prove in this section the converse, namely every Boolean like near ring having an identity element is a Boolean like ring.

Lemma 4.1 : If $a \in N$, then a^2 is an idempotent.

Proof : By def, $ab\,(a + b + ab) = ba$ for $a, b \in N$.

Substituting $b = a$, we get that $aa\,(a + a + aa) = aa$ *i.e.*, $aaaa = aa$ and so $a^4 = a^2$

Therefore, $a^2 = a^4 = (a^2)^2$. Thus a^2 is an idempotent.

Corollary 4.2 : If $a \in N$, then a is nilpotent iff $a^2 = 0$

Proof : If a is nilpotent, then $a^k = 0$ for some $k > 1$.

Choose an even integer $m > 1$ such that $a^m = 0$. Then $m = 2n$, for some $n \geq 1$

Therefore $0 = a^m = a^{2n} = (a^2)^n = a^2$, since a^2 is an idempotent. Converse is trivial.

Note 4.3: Since characteristic of N is 2, addition in N is commutative.

Lemma 4.4 : For any $a, b \in N$, $(a + b)^2 = (a + b)^2\,(a^2 + b^2)$

Proof : By lemma 4.1 and by using the definition 3.1 repeatedly, we get that

$$\begin{aligned}
(a + b)^2 &= (a + b)^4 = (a + b)^2 (a + b)^2 = (a + b)^2\,[(a + b)\,a + (a + b)\,b] \\
&= (a + b)^2\,[(a + b)\,a + b\,(a + b)] \\
&= (a + b)^2\,[a\,(a + b)\,[a + a + b + a\,(a + b)] + (a + b)^2\,[(a + b)\,b\,[a + b + b + (a + b)\,b] \\
&= (a + b)^2\,[(a + b)\,a\,[b + (a + b)\,a] + (a + b)^2\,[(a + b)\,b\,[a + (a + b)\,b] \\
&= (a + b)^2\,[(a + b)\,ab + (a + b)^2\,a^2] + (a + b)^2\,[(a + b)\,ba + (a + b)^2\,b^2] \\
&= (a + b)^2\,[(a + b)\,ab + (a + b)^2\,(a^2 + b^2) + (a + b)\,ab] \\
&= (a + b)^2\,[(a + b)^2\,(a^2 + b^2)] \\
&= (a + b)^4\,(a^2 + b^2) \\
&= (a + b)^2\,(a^2 + b^2)
\end{aligned}$$

Corollary 4.5 : If $a \in N$, then $a + a^2$ is nilpotent.

Proof : By lemmas 4.4 and 4.1 $(a + a^2)^2 = (a + a^2)^2\,(a^2 + a^4) = (a + a^2)^2\,(a^2 + a^2) = 0$,

Therefore $(a + a^2)$ is nilpotent.

Corollary 4.6 : If $a, b \in N$, then $(a + a^2)\,(b + b^2) = 0$

Proof : We know that $xy\,(x + y + xy) = yx$, for all $x, y \in N$.

Therefore

$$(a + a^2)\,(b + b^2) = (b + b^2)\,(a + a^2)\,(b + b^2 + a + a^2 + (b + b^2)\,(a + a^2)\,)$$
$$= (b + b^2)^2\,(a + a^2) + (b + b^2)\,(a + a^2)^2 + (b + b^2)^2\,(a + a^2)^2 = 0,$$

since $(c + c^2)^2 = 0$, for all $c \in N$, by corollaries 4.5 and 4.2.

We now prove that every Boolean like near-ring with identity is a boolean like ring.

Corollary 4.7 : If N has the identity 1, then N is a Boolean like ring.

Proof : By definition, $abc = acb$ for $a, b, c \in N$. Then $1bc = 1cb$ and this implies that $bc = cb$.

Thus multiplication is commutative. Since the characteristic of N is 2, $(N, +)$ is an abelian group.

So, N is a commutative ring with 1.

For $a, b \in N$, $a\,(1 + a)\,b\,(1 + b) = (a + a^2)\,(b + b^2) = 0$ [by cor.4.6]

Therefore N is a Boolean like ring.

Corollary 4.8 : If a and b are nilpotent elements in N, then $ab = 0$

Proof : Since a and b are nilpotent, by corollary 4.2, $a^2 = 0$ and $b^2 = 0$

Again by corollary 4.6, $(a + a^2)\,(b + b^2) = 0$ and this implies that $ab = 0$.

Lemma 4.9 : If N has no nonzero idempotent elements, then $ab = 0$ for all $a, b \in N$

Proof : If $a \in N$, then by lemma 4.1, a^2 is an idempotent. By hypothesis, $a^2 = 0$

Therefore a is nilpotent. Thus, every element of N is nilpotent. Hence

If $a, b \in N$, then a and b are nilpotent elements and by corollary 4.8, $ab = 0$.

Therefore, $ab = 0$ for $a, b \in N$.

Corollary 4.10: If N has no nonzero idempotent elements, then N is a trivial ring.

MAIN THEOREM

In this section, N stands for a Boolean like near-ring. We prove in the section that the set I of all nilpotent elements of N is an ideal and N/I is a Boolean ring. Main result of the section is that every Boolean like near-ring is a commutative ring.

From now onwards, we consider nontrivial near-rings. So, by cor. 4.10, we may assume that N contains nonzero idempotent elements.

We now prove the main theorem. Before that we prove some results.

Lemma 5.1: For any $b \in N$, b^2 is a central idempotent.

Proof : By lemma 4.1 b^2 is an idempotent element.

By def 3.1 $ac\,(a + c + ac) = ca$, for all $a, c \in N$.

Substituting $c = b^2$ in this equation, we get that $ab^2\,(a + b^2 + ab^2) = b^2a$. Then $ab^2\,a + ab^4 + ab^2\,ab^2 = b^2a$, and by lemma 4.1, this implies that $a^2\,b^2 + ab^2 + a^2b^2 = b^2a$. Thus, $ab^2 = b^2a$.

Theorem 5.2 : The map $\varphi : N \rightarrow N$ defined by $\varphi(a) = a^2$ for all $a \in N$ is a near-ring homomorphism.

Proof : For $a, b \in N$ $\varphi(a + b) = (a + b)^2$.

By using lemma 4.4, lemma 5.1, def 3.1(iii) and note 4.3

We get that

$$
\begin{aligned}
(a + b)^2 &= (a + b)^2 (a^2 + b^2) = (a + b)^2 a^2 + (a + b)^2 b^2 \\
&= a^2(a + b)^2 + b^2 (a + b)^2 \\
&= a^2 [(a + b)(a + b)] + b^2 [(a + b)(a + b)] \\
&= a^2 [(a + b) a + (a + b) b] + b^2 [(a + b) a + (a + b) b) \\
&= a [a^2 (a + b) + ab (a + b)] + b [ba (a + b) + b^2 (a + b)] \\
&= a [a^3 + a^2 b + a^2 b + ab^2] + b [ba^2 + b^2 a + b^2 a + b^3] \\
&= [a^4 + a^2 b^2] + [b^4 + b^2 a^2] \\
&= [a^4 + a^2 b^2 + b^4 + b^2 a^2] = a^4 + b^4 = a^2 + b^2 = \varphi(a) + \varphi(b)
\end{aligned}
$$

Therefore, $\varphi(a + b) = \varphi(a) + \varphi(b)$

Also $\varphi(ab) = (ab)^2 = (ab)(ab) = abab = a^2 b^2 = \varphi(a)\varphi(b)$

Therefore φ is a homomorphism.

Lemma 5.3 : If $a \in N$, then a can be represented uniquely as $a = n + e$, where n is nilpotent and e is an idempotent.

Proof : By corollaries 4.5 , 4.1 and def 3.1, $a = (a + a^2) + a^2$ where $(a + a^2)$ is nilpotent and a^2 is an idempotent.

Suppose $a = n + e$, where n is nilpotent and e is an idempotent.

By lemma 5.1, e is a central element.

Then, $a^2 = (n + e)^2 = (n + e)^2 (n^2 + e^2) = (n + e)^2 e^2 = [e (n + e)]^2 = (en + e)^2$

$= (en + e)(en + \mathrm{e}) = (en + e) en + (en + e) e = en (en + e) + e (en + e)$

$= e^2 n^2 + e^2 n + e^2 n + e^2 = e$

Therefore, $e = a^2$. Also, $(a + a^2) + a^2 = n + a^2$ and this implies that $a + a^2 = n$.

Thus, the representation is unique.

The following result follows from the above lemma.

Corollary 5.4 : If N is Boolean like near ring without nonzero nilpotent elements, then N is a Boolean ring.

Theorem 5.5 : The set I of all nilpotent elements of a Boolean like near-ring N forms an ideal and N/I is a boolean ring.

Proof : By cor. 4.2, an element $a \in N$ is nilpotent iff $a^2 = 0$. Therefore, $I = \{a \in N / a^2 = 0\}$

$\Rightarrow$ $I = \ker \varphi$, where φ is the near-ring homomorphism defined in theorem 5.2. Hence I is an ideal. Since N is a Boolean like near-ring, N/I is also a boolean like near-ring. Further N/I has no nonzero nilpotent elements. By corollary 5.4., N/I is a boolean ring.

Corollary 5.6 : The set of all idempotent elements of N form a boolean ring.

Proof: By lemma 4.1, for any $a \in N$, a^2 is an idempotent. Therefore $\{a^2 / a \in N\}$ is the set of all idempotents. Hence $\{a^2 / a \in N\} = \varphi(N)$, where φ is the homomorphism defined in theorem 5.2. But $N/I \cong \varphi(N)$. By theorem 5.5, N/I is a boolean ring and so $\varphi(N)$ is also boolean ring.

We now prove the main theorem.

Theorem 5.7 : Every Boolean like near ring is a commutative ring.

Proof : Let N be a Boolean like near ring.

Since $(N, +)$ is abelian, it suffices to prove that multiplication in N is commutative.

Let $x, y \in N$. By lemma 5.3, $x = n + e$, $y = m + f$ where n, m are nilpotent elements and e, f are idempotent elements.

Then $xy = (n + e)(m + f) = (n + e)m + (n + e)f = (n + e)m + nf + ef$, since f is central.

Similarly, $yx = (m + f)(n + e) = (m + f)n + (m + f)e = (m + f)n + me + fe$

Clearly $ef = fe$, since e, f are central elements.

To show $xy = yx$, it suffices to show that $(n + e)m + nf = (m + f)n + me$

Consider, $(n + e)m = m(n + e)[m + (n + e) + m(n + e)]$

$= m^2(n + e) + m(n + e)^2 + m^2(n + e)^2$ (by def 3.1(iii))

$= m(n + e)^2$, since $m^2 = 0$

$= m(n^2 + e^2)$, by theorem 5.2

$= me$

Similarly $(m + f)n = nf$. Therefore $(n + e)m + nf = me + nf = nf + me = (m + f)n + me$.

This completes the proof.

According to Venkateswarlu, K., *et al* [5] a Boolean like semiring is a near-ring N such that

(i) $a + a = 0$ for all $a \in N$, and (ii) $ab(a + b + ab) = ab$ for all $a, b \in N$

Corollary 5.8 : Every Boolean like near-ring is a Boolean like semiring.

Proof : If N is a Boolean like near-ring, then N is a commutative ring, by theorem 5.7

Therefore, $ab(a + b + ab) = ba$ (by definition)

$= ab$ (since multiplication is commutative)

Therefore, N is a Boolean like semiring.

But every boolean like semiring need not, in general, be a boolean like near-ring. This can be seen from the following example [5].

Example 5.9 : Let $K = \{0, a, b, c\}$ be the Klein's four group operations + and . are defined as follows:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
c	0	a	b	c

K is a Boolean like semi ring but not a boolean like near-ring, since $cab \neq cba$

Special boolean-like rings

We introduce, in the section, the concept of a special boolean-like ring and prove that this concept is equivalent to the concept of a boolean like near-ring.

Definition 6.1: A commutative ring R is called a special boolean-like ring if

(i) $r + r = 0$ for all $r \in R$

(ii) Every element $a \in R$ can be expressed as $a = n + e$, where n is a nilpotent element and e is an idempotent element.

(iii) $n_1 n_2 = 0$ for all nilpotent elements n_1, n_2 in R.

Note 6.2 : (1) Every boolean ring is a special boolean ring.

(2) By (ii) of the above definition, we get that every special boolean-like ring with no nonzero nilpotent elements is a boolean ring.

(3) If n is a nilpotent element of R, then by taking $n_1 = n_2 = n$ in (iii) of the above definition we get that $n^2 = 0$.

By theorem 5.6, lemma 5.4 and corollary 6.8, we get the following

Theorem 6.3: Every Boolean – like near-ring is a special Boolean-like ring.

We now prove the converse of the above result

Theorem 6.4 : Every special Boolean-like ring is a Boolean-like near-ring.

Proof : Let R be a special Boolean-like ring. To prove the theorem, it suffices to prove that $xy\,(x + y + xy) = yx$ for all $x, y \in R$. If $x, y \in R$, then by definition, $x = n + e$, $y = m + f$ where n, m are nilpotent elements and e, f are idempotents. Since the characteristic of R is 2,

$x + x^2 = (n + e) + (n + e)^2 = (n + e) + (n^2 + e^2) = (n + e) + e = n$, as $n^2 = 0$ by note 6.2 (2),

Similarly, one can prove that $y + y^2 = m$.

Hence $(x + x^2)\,(y + y^2) = mn = 0$, by (iii) of the definition 6.1 and this implies that $xy + xy^2 + x^2y + x^2y^2 = 0$

$\Rightarrow xy^2 + x^2y + x^2y^2 = xy = yx$, since R is commutative

$\Rightarrow xy\,(x + y + xy) = yx$.

This completes the proof.

By combining the above two theorems, we get the following.

Theorem 6.5 : Let N be a near-ring. Then N is a boolean like near-ring if and only if N is a special boolean-like ring.

Corollary 6.6 : If a special boolean –like ring R has the identity 1 then it is a boolean-like ring.

Proof : By the theorem 6.4, R is a Boolean like near-ring. Since R has 1, the result follows by Corollary 4.7.

By (III theorem 2.19.11) we have the following.

Theorem 6.7 : If R is a special Boolean-like ring such that $R^2 = R$, then every maximal ideal in R is prime.

We now prove the converse. Before that, we prove a lemma.

Lemma 6.8 : Let B be a Boolean ring $\neq (0)$. If B has no nonzero zero divisors, then B is a field.

Proof : Let $u \neq 0$ be an arbitrary but a fixed element in B. For any $r \in B$, $x\,(r + ur) = 0$.

By hypothesis, $r + ur = 0$, *i.e.*, $ur = r$. Thus, u is the identity element in B, which we denote by 1.

For $a \in B$, $a\,(1 + a) = 0$ and hence either $a = 0$ or $a = 1$.

$\therefore B = \{0, 1\}$ is a two element field.

Theorem 6.9: Let R be a special Boolean-like ring. If P is a prime ideal in R such that $P \neq R$, then P is a maximal ideal.

Proof : Since P is a prime ideal, R/P has no nonzero zero divisors, since $(x + P)\,(y + P) = P$

$\Rightarrow xy + P = P$

$\Rightarrow xy \in P \Rightarrow x \in P$ or $y \in P \Rightarrow x + P = P$ or $y + P = P$.

Since R is a special Boolean like ring, R/P is also a special Boolean-like ring. By note 6.2(1) R/P is a Boolean ring. By lemma 6.8, R/P is a field. Thus, P is a maximal ideal of R.

By combining the above two theorems, we get the following.

Theorem 6.10 : Let R be a special Boolean-like ring such that $R^2 = R$. Then an ideal $P \neq R$ is maximal in R if and only if P is prime in R.

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