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SMARANDACHE-BOOLEAN-NEAR-RINGS AND BOOLEAN-I-ALGEBRA

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In this paper we introduced Smarandache-2-algebraic structure of Boolean-near-ring namely Smarandache-Boolean-near-ring. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exist a proper subset M of N, which is embedded with a stronger algebraic structure A_1, stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, from the unit element if any, from the whole set. We define Smarandache-Boolean-near-ring and obtain some of its characterization through Boolean-ring and Lattice Ordered groups II. For basic concept of near-ring we refer to G. Pliz.

KEYWORDS : Boolean ring, Boolean-near-ring, Smarandache-near-ring, Boolean-I-algebra, Lattice ideals and Clans.

Introduction

The study of Boolean-near-ring is one of the generalized structure of rings. The study and research on near-rings is very systematic and continuous. Near-rings newsletters containing complete and updated bibliography on the subject of near-rings are published periodically by a team of editors. Then motivated by several researchers we wish to study and analyse the substructure in Smarandache-near-rings. The substructure in near-rings play vital role in the study of near-rings. Unlike other algebraic structure we see in case of near-rings we have the substructure playing vital role in the study and analyse of near-rings. Apart from the sub near-rings and ideals of near-rings we have special substructure like N-groups, filter and modularity in near-rings. It is these study in the context of Smarandache-Boolean-near-rings will yield several interesting results. Also the Smarandache substructure in Boolean-near-rings will also yield very many results in the direction.

For the study we would be using the book of Pilz Gunter, Near-rings (1997) published by North Holland Press, Amesterdam [10], Special Algebraic Structure by Florentin Smarandache, University of New Mexico, USA (1991) [18], Smarandache Algebraic Structure by Raul Padilla, Universidade do Minho, Portugal (1999) [13], Blackett [3] discusses the near-ring of affine transformations on a vector space where the near-ring has a unique maximal ideal. Gonshor [8] defines abstract of affine near-rings and completely determines the lattice of ideals for these near-rings. The near-rings of differential transformations is seen in [4]. For near-rings with geometric interpretation [10] or [18] and several research papers on Boolean-near-rings. We would first study and characterize the ideals and sub Boolean-near-rings in Smarandache-Boolean-near-rings. Also to study and analyse those Boolean-near-rings, which are Smarandache-Boolean-near-ring and find the conditions for Smarandache-boolean-near-rings. Yet another major substructure in Boolean-near-rings is the notion of filters. We would extend and study the notion of Smarandache filters given in Smarandache-Boolean-near-rings.

Further the notion of Smarandache ideals in near-ring would be studied, characterized and analysed for Smarandache-Boolean-near-rings. Both the notions viz. *N*-groups and ideals in near-ring and Smarandache-boolean-near-rings would be compared and contrasted. Also the nice notion of modularity in near-rings, which are basically built using concepts of idempotents, will be studied and analysed in Smarandache modularity in Boolean-near-ring.

Finally, Smarandache-Boolean-near-rings has constructed from Boolean-ring by an algorithmic approach through its substructures and Smarandache-Boolean-near-ring has introduced some application.

In order that New notions are introduced in algebra to better study the congruence in number theory by Florentin Smarandache [18]. By <proper subset> of a set *A* we consider a set *P* included in *A*, and different from *A*, different form the empty set, and from the unit element in *A* – if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures $S_1 << S_2$ if: both are defined on the same set;: all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1 laws, or S_2 has more laws than S_1.

For example : Semi group >> Monoid << group << ring << field, or Semi group << commutative semi group, ring << unitary, ring etc. They define a General special structure to be a structure *SM* on a set *A*, different form a structure *SN*, such that a proper subset of *A* is an structure, where *SM* << *SN* >

PRELIMINARIES

Definition 1.1. A left near-ring *A* is a system with two binary operations, addition and multiplication, such that

(i) The elements of *A* form a group $(A, +)$ under addition,

(ii) The elements of *A* form a multiplicative semi-group,

(iii) $x\,(y + z) = xy + xz$, for all $x, y, z \in A$

In particular, if *A* contains a multiplicative semi-group *S* whose elements generate $(A, +)$ and satisfy

(iv) $(x + y)\, s = xs + ys$, for all $x, y \in A$ and $s \in S$, then we say that *A* is a distributively generated near-ring.

Definition 1.2. A near-ring $(B, +, .)$ is Boolean-Near-Ring if there exists a Boolean-ring $(A, +, \Lambda, 1)$ with identity such that **.** is defined in terms of $+$, Λ and 1, and for any $b \in B$,

$$b.b = b$$

Definition 1.3. A near-ring $(B, +, .)$ is said to be idempotent if $x^2 = x$, for all $x \in B$. If $(B, +, .)$ is an idempotent ring, then for all $a, b \in B$,

$$a + a = 0 \quad \text{and} \quad a.b = b.a$$

Definition 1.4. A Boolean-near-ring $(B, +, .)$ is said to be Smarandache-Boolean-near-ring whose proper subset A is a Boolean-ring with respect to same induced operation of B.

Definition 1.5. A lattice $A = (A : \cup, \cap)$ with a binary operation '–' is called a Boolean -l-algebra if it satisfies the following properties :

(i) $a \cup b - c = (a - c) \cup (b - c)$

(ii) $a - (b \cap c) = (a - b) \cup (a - c)$ and

$a - (b \cup c) = (a - b) \cap (a - c)$

(iii) If $a \leq b$ then $c - b = (c - a) - (b - a)$

(iv) If $a \geq b \cup c$ then $a - b \geq a - c$ implies $c \geq b$, for all $a, b, c \in A$

Definition 1.6. A boolean-ring $(B, \cap, +, -)$ is called a Boolean-l-algebra if we define $a - b = a + a \cap b$.

Definition 1.7. Any Dually Residuated lattice Semi-group A is a Boolean-l-algebra if it satisfies the following conditions :

(i) $a - (b \cup c) = (a - b) \cap (a - c)$

(ii) $a \geq b \cup c$ and $a - b \geq a - c$ then $c \geq b$ for all $a, b, c \in A$.

MAIN THEOREMS ON SMARANDACHE-BOOLEAN-NEAR-RING WITH BOOLEAN-L-ALGEBRA

Theorem 2.1. Let $(B; \cup, \cap, +, -)$ is a Boolean-near-ring, B is a Smarandache-Boolean-near-ring if and only if there exists a proper subset $(A, \cup, \cap, +, -)$ of B with $a - b = a + a \cap b$ satisfies $x \leq a$ implies $x \cap (a - x) = 0$

Part I : Assume that $(B; \cup, \cap, +, -)$ is a Smarandache-Boolean-near-ring, then by definition, there exists a proper subset $(A, \cup, \cap, +, -)$ of B which is a Boolean-ring.

Proof : Since $(A, \cup, \cap, +, -)$ is a Boolean-ring with $a - b = a + a \cap b$, then we have $x \leq a$ implies $x \cap (a - x) = 0$.

The first part is clear, automatically.

Part II : Assume that there exists a proper subset $(A, \cup, \cap, +, -)$ of B with $a - b = a + a \cap b$ satisfies $x \leq a$ implies $x \cap (a - x) = 0$.

Prove that B is a Smarandache-Boolean-near-ring.

It is enough to prove that A is a Boolean-ring.

Proof: First we will prove that 0 is the least example of A.

Since $x \leq a$, for all $a \in A$

For, $a \leq a$, for all $a \in A$

$$\Rightarrow \quad 0 = x \cap (a - x) = a \cap (a - a) \text{ [by our hypothesis]}$$

$$= a \cap 0$$

$\therefore$ $a - 0 = a$, since $a \geq 0$ then $a - 0 = a$, for all $a \in A$.

Secondly, we will prove that if $x \leq a$ then $a - x = (a - x) - x$

For if $x \leq a$ then $a - x = (a - x) - \{(a - x) \cap x\}$ [since $a - (a \cap b) = (a \cup b) - b$]

$= \{(a - x) - (a - x)\} \cup \{(a - x) - x\}$

$= 0 \cup \{(a - x) - x\}$

Therefore, $a - x = (a - x) - x, \quad \text{if } x \leq a.$

Next to prove that $x \cup (a - x) = a$, if $x \leq a$.

For $\{x \cup (a - x)\} - x = (x - x) \cup \{(a - x) - x\}$ [since $a \cup (b - c) = (a - c) \cup (b - c)$]

$= 0 \cup \{(a - x) - x\}$

$= ((a - x) - x)$ [since $(a - x) - x = (a - x)$, if $x \leq a$]

$= a - x$

$\therefore \quad \{x \cup (a - x)\} - x = a - x$ [since 0 is the least element of A]

It follows that, $x \cup (a - x) = a$

Finally our aim is to show that A is a Boolean-ring.

If A is distributive and let $x < z < y$, then

$z \cap \{x \cup (y - z)\} = (z \cap x) \cup \{z \cap (y - z)\}$ [since A is distributive, $a, b, c \in A$ and $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$]

$= (z \cap x) \cup 0$ [by hypothesis]

$= z \cap x$

$= x$, if $x < z < y$

and, $z \cup \{x \cup (y - z)\} = x \cup z \cup (y - z)$ [since A is a distributive lattice then $a \cup b = a \cup c$ and $a \cup b = a \cap c$ which implies $b = c$ for all $a, b, c \in A$]

$= x \cup \{z \cup (y - z)\}$ [since $x \leq a$ implies $x \cup (a - x) = a$]

$= y$

$\therefore \quad z \cap \{x \cup (y - z)\} = \text{y}$

Hence A is a relatively complemented and therefore A is a Boolean-ring and it follows that B is a Smarandache-Boolean-ring.

Theorem 2.2. Let $(B; \cup, \cap, +, -)$ is a Boolean-near-ring, B is a Smarandache-Boolean-Near-ring if and only if there exists a proper subset of $(A; \cup, \cap, +, -)$ of B with $a - b = a + a \cap b$ which is a Boolean-l-algebra, for each a and $b \in A$.

Part I : Assume that

(i) B is a Boolean-near-ring and

(ii) There exists a proper subset A of B with $a - b = a + a \cap b$ which is a Boolean-l-algebra.

To prove that, B is a Smarandache-Boolean-Near-ring

It is enough to prove that A is a Boolean ring.

Proof : If $x \leq a$ then $a = a \cup x$

$= x \cup (a - x)$, [since $x \leq a$]

Since $(A; \cup, \cap, +, -)$ is a Boolean-l-algebra and by known theorem 1, "Let $(B; \cup, \cap, +, -)$ is a Boolean-near-ring, B is a Smarandache-Boolean-near-ring if and only if there exists a proper subset $(A; \cup, \cap, +, -)$ of B with $a - b = a + a \cap b$ satisfies $x \leq a$ implies $x \cap (a - x) = 0$"

Also, $a - x = \{x \cup (a - x)\} - x$ [by theorem 1]

$= (a - x) - \{x \cap (a - x)\}$ [since $(a \cup b) - a = b - (a \cap b)$]

$\therefore$ $a - x = (a - x) - \{x \cap (a - x)\}$, for each $a \in A$.

Since, by theorem 1,

If $x \leq a$ implies $x \cap (a - x) = 0$ then it follows that A is a Boolean-ring.

Hence B is a Smarandache-Boolean-Near-Ring.

Part II : Suppose $(B; \cup, \cap, +, -)$ is a Smarandache-Boolean-Near-Ring.

Then to prove that there exists a proper subset $(A; \cup, \cap, +, -)$ of B with

$a - b = a + a \cap b$ which is a Boolean-l-algebra.

Proof : Since, there exists a proper subset A of B which is a Boolean-Ring and $x \leq a$ implies $x \cap (a - x) = 0$.

Now, $a - x = (a - x) - 0$

$\Rightarrow$ $a - x = (a - x) - \{x \cap (a - x)\}$

$\Rightarrow$ $a - x = \{x \cup (a - x)\} - x$ [since $b - (a \cap b) = (a \cup b) - a$]

Further, if $x \leq a$ then

$a = x \cup (a - x)$ [by theorem 1 and $x \leq a$]

$a = a \cup x$

It follows that $(A; \cup, \cap, +, -)$ is a Boolean-l-algebra.

Theorem 2.3. Let $(B; \cup, \cap, +, -)$ is a Boolean-near-ring, there exists a proper subset $(A, \cup, \cap, +, -)$ of B which is a Boolean-l-algebra in which $a - a \cap b \cap c = a$ implies $a \cap b \cap c = 0$. Then B is a Smarandache-Boolean-Near-Ring.

Assume that $(B; \cup, \cap, +, -)$ is a Boolean-near-ring and there exists a proper subset $(A; \cup, \cap, +, -)$ of B which is a Boolean-l-algebra with $a - a \cap b \cap c = a$ implies $a \cap b \cap c = 0$.

Then to prove that B is a Smarandache-Boolean-Near-Ring.

It is sufficient to prove that A is a Boolean-ring.

Proof: First we will show that, if $x \leq a$ then $x \cup (a - x) = a$

For, $\{x \cup (a - x)\} - x = (x - x) \cup \{(a - x) - x\}$

[By the result $a \cup (b - c) = (a - c) \cap (b - c)$]

$= 0 \cup \{(a - x) - x\}$

$= \{(a - x) - x\}$ [By if $x \leq a$ then $(a - x) - x = a - x$]

$= a - x$

$\therefore$ $\{x \cup (a - x)\} - x = a - x$

$\Rightarrow$ $x \cup (a - x) = a$, for all $x \leq a$

If $0 \leq x \leq a$ then

$$a - a \cap x \cap (a - x) = \{a - (a \cap x)\} \cup \{a - (a - x)\}$$
$$= (a - x) \cup \{a - (a - x)\}$$
$$= (a - x) \cup a$$
$$= a, \quad \text{for all } a \in A.$$

By known theorem 1,

"Let (B; $\cup$, $\cap$, +, –) be a Boolean-near-ring; B is a Smarandache-Boolean-Near-Ring if there exists a proper subset A of B with $a - b = a + a \cap b$ satisfies $x \leq a$ implies $x \cap (a - x) = 0$"

Hence $x \cap (a - x) = 0$ and so that $x \cup (a - x) = a$.

$\therefore$ A is section complemented and a Boolean-ring.

$\therefore$ B is a Smarandache-Boolean-Near-Ring.

Theorem 2.4. Let (B; $\cup$, $\cap$, +, –) be a Smarandache-Boolean-Near-Ring if and only if there exists A = (A; $\cup$, $\cap$, +, –) is a Dually Residuated lattice ordered semi-group with
$a - (b \cup c) = (a - b) \cap (a - c)$, where A is a proper subset of B.

Part I : Assume A is a Dually Residuated lattice ordered semi-group with

$$a - (b \cup c) = (a - \mathrm{b}) \cap (a - c)$$

Then to prove that B is a Smarandache-Boolean-Near-Ring.

We need to prove A is a Boolean-ring.

Proof: By theorem 1,

"Let (B; $\cup$, $\cap$, +, –) is a Boolean-near-ring. Then B is a Smarandache-Boolean-Near-Ring if and only if there exists a proper subset (A, $\cup$, $\cap$, +, –) of B with $a - b = a + a \cap b$ satisfies $x \leq a$ implies $x \cap (a - x) = 0$".

Then we have, A is a Boolean-ring. Hence B is a Smarandache-Boolean-Near-Ring.

Part II : Assume that B is a Smarandache-Boolean-Near-Ring.

Proof: Since (A; $\cup$, $\cap$, +, –) is a Dually Residuated lattice ordered semi-group then (A; $\cup$, $\cap$, +, –) is a Boolean-l-algebra.

Then we need to prove A is a Boolean-l-algebra, using A is clean.

Let $\Sigma = \{(a, b) \in A \times A / a \leq b\}$ and let $\sigma : \Sigma \rightarrow A$ be defined by $\sigma(a, b) = b - a$.

And, let C_1, C_2, C_3, C_4 and C_7 are satisfied in A by using clan [9].

$\therefore$ A is a Boolean-l-algebra.

Theorem 2.5. Let B is a Boolean-l-algebra with $a - (b \cup c) = (a - b) \cap (a - c)$, there exist a proper subset A is a Boolean-ring. Then B is a Smarandache-Boolean-Near-Ring, if it following are equivalent :

(i) B is a Boolean-ring.

(ii) (a, b, c) A iff (a, b, c) B

(iii) (a, b, c) B and (a, c, b) B imply $b = c$

(iv) Metric betweenness has transitivity t_1.

Proof : Proof for (i) $\Rightarrow$ (ii) :

We can assume $a \geq c$, then $a = (a - c) + c = a*b + b*c + c$

$$\geq (b - a) + (b - \text{c}) + c \geq (b - a) + b.$$

and by using definition 6, $0 - (b - a) = b - \{b + (b - a)\} \geq b - a$ (or) $a \geq b$.

So that by the property (iv) of definition 6, $b \geq c$, since $a - c \geq a - b$ (or) $(a, b, c)\, A$

Let $a \cup c \geq b \geq a \cap c$, then,

$$\begin{aligned} a*c &= a \cup c - a \cap c \\ &= (a \cup c - b) + (b - a \cap c) \\ &= (a \cup c*b) + (b*a \cap c) \\ &= a*b + b*c \text{ [By using } (a, b, c)\, B \Leftrightarrow (a \cup c, b, a \cap c)\, B] \end{aligned}$$

Hence $(a, b, c)\, B$.

Proof of (ii) $\Rightarrow$ (iv) :

This proof is obvious.

Proof of (i) $\Rightarrow$ (iii) :

Assume that B is a Boolean-ring.

Prove that $(a, b, c)\, B$ and $(a, c, b)\, B \Rightarrow b - c$.

Let $(a, b, c)\, B$ and $(a, c, b)\, B$, we need to prove $b = c$.

Then $a \cup c \geq b \geq a \cap c$ and $a \cup b \geq c \geq a \cap b$

Hence $a \cup b = a \cup c$ and $a \cap b = a \cap c$ so that $b = c$.

Proof of (iii) $\Rightarrow$ (i) :

Let $a \geq b \cup c$ and $a - b \geq a - c$

Since $(a, b, b \cap c)\, A$, we have

$$\begin{aligned} a*b \cap c + b \cap c*b &= a - (b \cap c) + (b - b \cap c) \\ &= (a - b) + b - b \cap c \\ &= a*b + b*b \cap c \text{ [since } (a, b, c)\, B \Rightarrow a*b + b*c = a*c] \\ &= a*b \cap c \\ &= a - (b \cap c) \\ &= a - b \\ &= a*c, \text{ so that } (a, b \cap c, b)\, B \end{aligned}$$

Hence $\quad b \cap c = b$ (or) $c \geq b$

Therefore, the condition (iv) of definition 1, holds in A and consequently, A is a Boolean-ring.

Some definitions and theorems on Smarandache-Boolean-near-rings

Definition 3.1. A normal sub group $(I, +)$ of $(B, +)$ is a left ideal if $B\, I \subseteq I$ and is an ideal If $(I, +, .)$ is the kernel of a near-ring homomorphism.

Definition 3.2. A Special Boolean-near-ring $(B, +, .)$ and $b \in B$, define

$$P(b) = \{a \in B/a \cap b = a\}$$

If $A \subseteq B$ and $b \in B$, define $A\,(b) = \{a \cap b/a \in A\}$ and $A\,(b) \subseteq P(b)$.

Theorem 3.3. Let $(B, +, \cap, 1)$ be a Boolean-near-ring whose proper subset $(A,+, \cap, 1)$ be a Boolean-ring with identity. Fix $x \in B$ and define a multiplication on B by $a.b = (a \cup x) \cap b$.

Then $(B, +, .)$ is a Smarandache-Boolean-Near-ring if any only if $x = 0$.

Part I : Assume that $(B, +, .)$ is a Smarandache-Boolean-near-ring.

(*i.e.*) A Boolean-near-ring $(B, +, .)$ which is a Boolean-ring.

We want to show that $x = 0$. For a, b and $c \in B$, we have

Proof :

$$a.(b.c) = (a \cup x) \cap [(b \cup x) \cap c]$$

$$a.(b.c) = [(a \cup x) \cap (b \cup x)] \cap c \quad \text{and,}$$

$$(a.b).c = \{[(a \cup x) \cap b] \cup x\} \cap c$$

$$= \{(a\cup x) \cap (b \cup x)\} \cap c$$

So that $a.(b.c) = (a.b).c$

Also to show that the distributive under multiplication :

(*i.e.*) to show that $a.(b + c) = (a.b) + (a.c)$

For all a, b and $c \in B$, then

$$a.(b + c) = (a \cup x) \cap (b + c)$$

$$= [(a\cup x) \cap b]+ [(a\cup x) \cap c]$$

$$= (a.b)+ (a.c)$$

$\Rightarrow \quad a.(b + c) = (a.b) + (a.c)$, for all $a, b, c \in B$,

Hence $x = 0$.

Part II : Consider $x = 0$, for all $x \in B$.

Then to prove that $(B, +, .)$ is a Smarandache-boolean-near-ring.

If $x = 0$ then to prove that $(B, +, .) = (B, +, \cap, 1)$.

It is enough to prove that the proper subset A of B is a Boolean-ring.

Proof : Since by the definition of idempotent and idempotent ring then for $x \in B$, an arbitrary idempotent element.

$$(x + x).x = 0.x \text{ (Since by definition of idempotent ring)}$$

$$= (0 \cup x) \cap x$$

$$= (x \cap x)$$

$\Rightarrow \quad (x + x).x = x$

and $\quad (x.x) + (x.x) = x + x$

$\Rightarrow \quad (x + x).x = 0$

Hence the right distributivity under multiplication is satisfied,

so that $\quad (B, +, .)$ is not a ring, if $x \neq 0$

Also $b.b = (b \cup x) \cap b$

$\Rightarrow$ $b.b = b$, for all $x \in B$

Hence $(B, +, .)$ is a Smarandache-Boolean-near-ring.

Theorem 3.4. Let I be an Ideal of Boolean-near-ring $(B, +, .)$. Then B is a Smarandache-Boolean- near-ring if and only if $P(x) \subseteq I$.

Part I : We assume that B is a Smarandache-Boolean-near-ring.

Proof : Since B is a Smarandache-Boolean-near-ring, then by the definition, a proper subset is Boolean-ring, B/I is such a proper subset.

Therefore, B/I is a Boolean-ring.

Then the right distributive law holds so that,

$$[(a + I) + (b + I)]\,(c + \mathrm{I}) = [(a + I)\,(c + I)] + [(b + I)\,(c + I) \qquad \ldots (1)$$

Thus,

$$(a + b).c + I = (a.c + b.c) + I$$

If a, b and $c \in B$, then

$$(a + b).c + a.c + b.c = \{[(a + b) \cup x + (a \cup x) + (b \cup x)]\} \cap c$$

Now,

$$\begin{aligned}(a + b) \cup x + (a \cup x) + (b \cup x) &= (a + b)\,x + \{[(a \cup x) \cap b' \cap x') \cup (a' \cap x \cap (b \cup x)]\} \\ &= (a + b)\,x + (a + b) \cap x' \\ &= \{[(a + b)\,x] \cap [(a + b) \cap x']\} \cup \{[(a + b)\,x]' \cap [(a + b) \cap x']\} \\ &= \{[(a + b) \cap [(a + b)']\,x\} \cup \{[(a + b)\,x]' \cap [(a + b) \cap x']\} \\ &= (0 \cup x) \cup (0 \cap x') \\ &= x\end{aligned}$$

Hence $(a + b).c + a.c + b.c = x \cap c \in I$ $\qquad \ldots (2)$

Since c is arbitrary, we have $P(x) \subseteq I$

Then to prove that B is a Smarandache-Boolean-near-ring.

Proof : Let B/I is proper subset of Boolean-near-ring $(B, +, .)$

To prove that B/I is a Boolean-ring.

Since $P(x) \subseteq I$, then the equation (1) is valid if any only if $(a + b).c + a.c + b.c \in I$

Hence B/I is a Boolean-ring.

Thus, every proper subset of B is a Boolean- ring and therefore B is a Smarandache-Boolean- near- ring.

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